Ie Collects Patterns in 'Random World'

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listed a few sequences. He spent s searching libraries and enter- numbers on computer punch s. When other mathematicians d about the project, they contrib- sequences by the dozens.

Handbook of Integer Se- ces" appeared in 1973 and be- an instant classic, at least by extremely finite standards of ematics books. Unfortunately, it became instantly outdated. New nces poured in.

Dr. Sloane's collection is fully uterized, and he is planning a edition, although many se- es still lie uncollated in cartons floor. In theory, of course, the er of sequences is infinite. "The could get very thick indeed," he "I've had to draw a line and away sequences that I consid- ot so interesting."

has certain rules. Sequences consist entirely of whole num- They must be infinitely long, disqualifies the famous se- 14, 18, 23, 28, 34, 42, 50, 59 . . . — al stops on the West Side IRT.

some sequences, a simple for- calculates any desired term. To nth number in the sequence of squares (1, 4, 9, 16 . . .), just ly n by itself. For other se- s, the process is less direct — lculates the nth term from the immediately before it, using a known as a "recurrence." the best-known sequences ted by a recurrence is the cci sequence, in which each the sum of the two preceding. many other sequences, though, a formula nor a recurrence is even when they are easy to An example, is the Mersenne — prime numbers that are than a power of 2, such as 3, 7, ding the next Mersenne prime starts to tax the limits of ational ingenuity. It is not own for certain whether they e requirement of being an infi- quence, but Dr. Sloane gives he benefit of the doubt."

sequences can be defined in conceivable ways, not all of et them into the book. There e mind-twisting possibilities. uldn't have *the first sequence t in the book*," said Ronald L. head of mathematical re- t Bell Labs.

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Neil, Let me try to kill two th one sloane. Here is a new e: 1, 2, 3, 8, 10, 54, 42, 944, 5112

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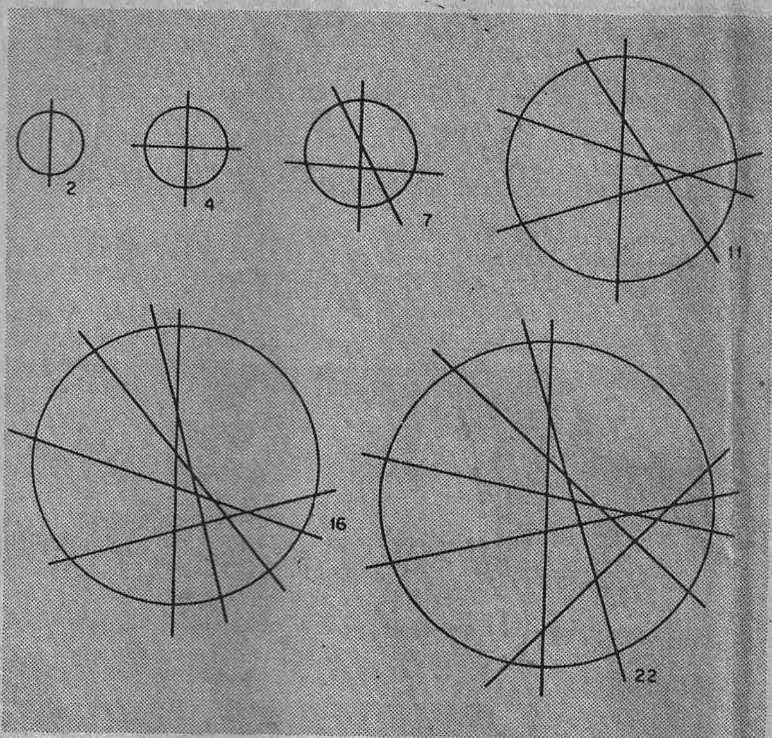
curator of this bizarre mu- r. Sloane, a wiry native of ho is also a serious rock sometimes regrets the time ore earnest mathematics. "I spend too much time on it," I should have done more ab- ngs."

Sequences and Pancakes

Many sequences arise in geometric problems, such as finding the largest number of pancake pieces that can be made by n straight cuts: 2, 4, 7, 11, 16, 22 . . . The sequence is generated by $\frac{1}{2}n(n+1)+1$.



Source: A Handbook of Integer Sequences by N. J. A. Sloane

He has worked in coding theory, algebra and combinatorics. Perhaps his most intense interest and his best contribution to hard-core mathematics lie in the area of "sphere packing" — a surprisingly large field of study devoted to questions of how best to arrange many identical balls so that they take up the least volume.

Sphere packing leads to some hard questions. In three dimensions, there is an obvious, very good arrangement: the regular, symmetrical array used for piling oranges or cannonballs. Many crystals, at the molecular level, favor the same array. But mathematicians have never managed to prove that some other arrangement would not be even denser.

Where sphere packing really gets lively, though, is in imaginary spaces of more dimensions than the usual three. Higher dimensions have a lot of room to play around in.

For example, the "kissing number" — the number of spheres that can be arranged around one central sphere — rises rapidly. In two dimensions, it is 6, as anyone can quickly see by placing some pennies on a table. In three dimensions, it is 12, but there is room left over, and some mathematicians long thought that there might somehow be a way to squeeze in a 13th.

Dr. Sloane keeps handfuls of pennies and ball bearings within easy reach. His most important contributions to sphere packing have been clever ideas about spaces of 8 and 24 dimensions, realms where the mind truly boggles.

Almost to his disappointment, the elegant geometries of sphere packing in higher dimensions have given rise to practical applications. Communications engineers, for example, devising strategies for efficiently transmitting the greatest possible information in a given "bandwidth," find that they are engaged in sphere packing. They want to squeeze in as many bits as possible, yet for the sake of clean communication they must keep them a certain distance apart.

In 24 dimensions, as it happens, the kissing number is 196,560 — part of a sequence that Dr. Sloane would add to his handbook, if only more terms were known for sure.

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Dear Dr. Sloane, . . . It would seem to me that since the Mega Test is a "take home" test, a person could find all the "series" answers on that test just by consulting your book. I threw away that stupid test, so I can't check to see whether it's true. . . .

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Mathematicians have tried similar catalogues of real numbers, for those who quickly need to identify pi the cube root of 12, but such things tend not to be quite so useful. "There are just too many real numbers," as Dr. Sloane said.

Number sequences are special, somehow. Although in principle they are infinitely numerous, in reality the interesting ones seem relatively few. They capture a kind of logic about the world — a flexible, not quite cut-and-dried logic — which may be why designers of intelligence tests have always had a weakness for them, rightly or wrongly.

The logic of sequences can be tricky. The Sloane handbook lists no less than 22 different sequences that begin 1, 2, 3, 4, 5 . . . But a longer lead-in — Dr. Sloane generally insists on 10 numbers — almost always suffices to specify a sequence uniquely.

One reason is that sequences so often correspond to some simple question about geometry or combinatorics — how many ways objects can branch or fold or slice or combine. There are sequences for knots, trees, graphs and beads on necklaces. Sometimes finding the next member of such a sequence is a famous unsolved problem, yet the sequence is clearly measuring something fundamental.

So if you tell Dr. Sloane that you have come across a sequence in some interesting physical or mathematical context and that it begins 2, 4, 8, 15, 26, 42, 64, 93, he will lay heavy odds that the next term will be 130. You have rediscovered a sequence that happens to be the greatest number of pieces you can get with successive slices through a cake.

Your problem may have nothing to do with cake; it may have nothing to do with geometry at all, as far as you can tell. No matter. When nature organizes itself into sequences, it seems to be a creature of habit.