Jacob Bronowski (1908–1974): *The Mathematical Gazette* and *retrodigitisation*

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Peter Braza and Tong Jing-Cheng, in an article [1] in *The Mathematical Gazette* in 1999, considered moving the leading digit of an integer to the rear so as to form a new integer that is a multiple of the original. Subsequently, Jeremy King wrote in [2] to point out a connection between this and recurring fractions (recurring decimals in our usual base). As it happens, this exchange neatly recapitulates an earlier discussion in the *Gazette* in the early 1950s that still merits recalling, although it might also be noted that Bob Burn had outlined an approach in another article [3] as recently as 1991.

The issue of *The New Statesman* for 24 December, 1949 (p. 761), included a challenge from Jacob Bronowski (1908–1974): find the least integer (in base 10) such that moving the leading digit to the rear produces a new integer one and a half times the original. This puzzle carried the warning that computation might prove lengthy, and, indeed, the answer runs to 16 digits. While no source was given, such problems had long been popular; for example, Maurice Kraitchik (1882–1957) poses several in which the first or final digit and the multiplier are specified in his popular book *Récréations Mathématiques* [4] originally published in 1930, with English translation in 1942. Further details of cognate interest were collated by Leonard Dickson (1874–1954) in the first volume of his *A History of the Theory of Numbers* [5, Chap. VI, esp. pp. 174–179]. But the first traced appearance of this type of problem is in an algebra textbook [6, pp. 73, 176] in 1854, not cited by Dickson.

Bronowski's problem was given an airing [7] in the *Gazette* in 1952 by J. H. Clarke, whose approach was somewhat along the lines adopted more recently in [1]. This attracted three rejoinders [8, 9, 10] in 1955 from D. E. Littlewood (1903–1979), E. J. F. Primrose (1920–1998) and R. Sibson (1912–1989), all noting the connection with recurring fractions in one way or another — 20/17 in the case of Bronowski's problem in *The New Statesman* — with Primrose also developing more of the general theory of such questions. The topic was capped the following year by the then Editor of the *Gazette*, R. L. Goodstein (1912–1985), who noted [11] that Littlewood's approach gave a more succinct solution than Primrose's generalization. The key role here of recurring fractions invites us to push back yet further, to the early years of the *Gazette*, when Lt.-Col. Allan J. C. Cunningham (1842–1928) published tables [12] for the first hundred integers of the periods of their reciprocals in bases 2, 3 and 5.

In these latter days of Research Assessment Exercises (RAEs), it is perhaps worth remembering that Littlewood, Primrose and Goodstein were academic mathematicians of some distinction (see the obituary notices [13, 14, 15]). Dudley Littlewood, who served as professor of mathematics at the University College of North Wales (later University of Wales), in Bangor, from 1948 until retirement in 1973, is best known for work on group characters and the matrix representation of groups, especially the symmetric group. Eric Primrose was a regular contributor to the *Gazette* throughout a career spent entirely at University College, Leicester (later University of Leicester), from 1947 until 1982, during which time he showed great deftness in the construction of geometrical designs. Louis Goodstein was Primrose's professorial colleague at Leicester from 1948 until retirement in 1977, being the first specialist in mathematical logic to hold a mathematical chair in the UK. He published prolifically in the *Gazette*, serving as Editor in the period 1956–1962. Naturally, his *Presidential Address* [16] to the annual conference of the Mathematical Association in April, 1976 appeared in the *Gazette* later that year. On the other hand, while Robert Sibson, like Goodstein a year earlier, was a Wrangler in the Mathematical Tripos at Magdalene College, Cambridge in 1934, he pursued a different career, going into school teaching, and thence joining HM Inspectorate of Schools in 1947, retiring in 1973. His final appointment was as Joint Secretary of the Schools Council.

Although Jacob Bronowski's name is most remembered in association with the BBC television documentary series *The Ascent of Man* he made at the end of his life — it inspired discussion [17, 18] in the *Gazette* of a tessellation found at the Alhambra — he read mathematics at the University of Cambridge, where he went on to take a doctorate with a thesis in geometry and topology. It would seem that he distilled something of his own experience in the *gleaning* [19], culled from *The Observer* for 22 April, 1951, that was picked up by the *Gazette* in 1953, but which retains a certain quotable resonance today:

> *The scientist, pure or applied, is still often treated as an uncouth Philistine. I find this a perverse charge, coming from men whose cultural interests fit primly into the clues to* The Times *crossword.*

However, he does not appear to have published anything in the *Gazette* until 1963, when he was at the National Coal Board. But appropriately enough, his note [20] was on an elementary test for divisibility by 7. Based on the fact that 7 is a divisor of 1001, it provides a simple way to compute remainders on division by 7. As might be expected, Bronowski's contribution was one of many items on division and divisibility in the *Gazette*, too numerous to detail here.

Other observations on digits have been presented in the *Gazette* from time to time. One example, in [21], concerns the cycles of integers that can be formed on repeatedly taking certain sums of consecutive digits. There can be something contrived or artificial about such problems. But cycling of digits appears naturally — and runs deeper — in the representation of $k/7, 1 \leq k < 7$ as recurring decimals (all of period 6). In terms of *primitive roots* discussed in [22], this reflects the fact that 10 is a primitive root modulo 7. Already Johann Carl Friedrich Gauss (1777–1855), in *Disquisitiones Arithmeticae*, raised the question of determining the primes p for which 10 is a primitive root modulo p. Emil Artin (1898–1962) distilled the ensuing investigations of special cases in a general conjecture in 1927: any integer m, other than 0 or -1, and not divisible by a square, is the primitive root of infinitely many primes; and such primes have positive density in the set of primes independent of the choice of m. Although much progress has been made on this conjecture, it has been of a conditional or non-constructive kind, and, as yet, no m is known which is a primitive root for infinitely many primes.

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