Magnetostatics of superconductors without an inversion center

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The penetration of a magnetic field into a London superconductor without an inversion center is analyzed. The magnetization produced in the Meissner layer corresponds to a magnetic-induction jump at the superconductor surface.

Thermodynamic-equilibrium phenomena, which are related to magnetoelectric phenomena, can occur in superconductors lacking an inversion center, since the phase of the order parameter changes sign under time reversal.

Let us discuss this topic using as an example a London superconductor, in which these phenomena are seen most clearly, although quantitatively this example, as we will see below, is not suitable. Working from a linear relationship between the current density **j** and the electromagnetic-field potential **A**

$$j_\mu(x) = \int_V d^3x' Q_{\mu\nu}(\mathbf{R},\vec{\rho}) A_\nu(x'),$$

where $\mathbf{R} + (\mathbf{x}+\mathbf{x}')/2$, $\boldsymbol{\rho} = (\mathbf{x}-\mathbf{x}')$, and V is the conductor volume and expanding $\mathbf{A}_\nu(\mathbf{x}') = \mathbf{A}_\nu(\mathbf{x}-\boldsymbol{\rho})$ in $\boldsymbol{\rho}$ up to the linear term, we find the following expression for j_μ:

$$j_\mu = a_{\mu\nu} A_\nu - b^\lambda_{\mu\nu} \frac{\partial A_\nu}{\partial x_\lambda} . \tag{1}$$

Here

$$a_{\mu\nu} = \tilde{Q}_{\mu\nu}(0), \quad b^\lambda_{\mu\nu} = i \frac{\partial \tilde{Q}_{\mu\nu}(\mathbf{k})}{\partial k_\lambda}\Big|_{\mathbf{k}=0} ,$$
$$\tilde{Q}_{\mu\nu}(\mathbf{k}) = \int d^3\rho \, e^{-i\mathbf{k}\vec{\rho}} Q_{\mu\nu}(\vec{\rho}). \tag{2}$$

This expression is vaild far from the boundary, where the kernel Q does not depend on **R**, i.e., at distances greater than d (d is the correlation length which determines the nonlocalizability of the kernel). Since $Q_{\mu\nu}(\mathbf{R},\boldsymbol{\rho}) = Q_{\nu\mu}(\mathbf{R},-\boldsymbol{\rho})$, we have $a_{\mu\nu} = a_{\nu\mu}$ and $b^\lambda_{\mu\nu} = -b^\lambda_{\nu\mu}$. The tensor $b^\lambda_{\mu\nu}$, which arises in the absence of an inversion center, doubles as a second-rank pseudotensor, which we can write, in the general form, as

$$b^\lambda_{\mu\nu} = e_{\mu\nu\rho} b_{\rho\lambda} ; \quad b_{\rho\lambda} = e_{\rho\lambda\eta} p_\eta + \beta\delta_{\rho\lambda} + \tilde{b}_{\rho\lambda} ; \quad \tilde{b}_{\rho\lambda} = \tilde{b}_{\lambda\rho} ; \quad \sum_\lambda b_{\lambda\lambda} = 0.$$

In cubic crystals, $b_{\rho\lambda}$ reduces to a β pseudoscalar, which is nonvanishing in the presence of enantiomorphism. The vector **p** arises in the case of a symmetry which is compatible with pyroelectricity.[1] Superconductors without an inversion center, we might note, have been described in the literature.[2]

In the case of Maxwell's equation without a current curl $\mathbf{B} = (4\pi/c)\mathbf{j}$, the boundary condition, in contrast with the condition under which $\mathbf{B}$ is constant at the boundary, must be changed. This fundamental case can be clearly explained by using a conductor with a diffuse boundary as an example, in which the kernel Q increases as a function of $\mathbf{R}$, from zero to a constant value in a layer of thickness h within the conductor, such that $d \ll h \ll \delta$ (δ is the London penetration depth). Taking into account that $\mathbf{R} = \mathbf{x} - \boldsymbol{\rho}/2$ and expanding in $\boldsymbol{\rho}$, we find an expression for the current which is localized in a layer near the surface

$$j_\mu^{\mathrm{surf}}(\mathbf{x}) = -\frac{1}{2}\frac{\partial b_{\mu\nu}^{\lambda}}{\partial x_\lambda} A_\nu(\mathbf{x}).$$

This surface current corresponds to the appearance of a "bulk" magnetization in the Meissner layer. In a very simple case of a cubic mirror isomer, we find

$$\mathbf{j} = -\frac{c}{4\pi\delta^2}\left[\mathbf{A} - \frac{c}{2e}\vec{\nabla}\varphi - \lambda\mathbf{B}\right] + c\,\mathrm{curl}\,\mathbf{M}\,,$$

$$4\pi\mathbf{M} = \frac{\lambda}{\delta^2}\left(\mathbf{A} - \frac{c}{2e}\vec{\nabla}\varphi\right), \qquad \mathbf{B} = \mathrm{curl}\,\mathbf{A},\ \lambda = \frac{2\pi}{c}\delta^2\beta. \tag{3}$$

In the case of a sharp boundary, the expression for surface currents cannot be found by using such a simple method. Even a London superconductor requires the solution of an integral equation in this case, since these currents are distributed in a $\sim d$-thick layer. Expression (3) for $\mathbf{M}$ remains valid, however, in the bulk of the Meissner layer. Using this expression, we can solve the standard boundary-value problem. In the case of a two-dimensional boundary (x, y) and an external field B_{0x}, we find

$$B_+ = B_x + iB_y = B_{0x}\frac{e^{i\nu}}{\cos\nu}\exp\left\{-\frac{z}{\delta}e^{i\nu}\right\}, \quad \sin\nu = \frac{\lambda}{\delta}. \tag{4}$$

As we can see, a spasmodic crossing of the boundary (over a length d) gives rise to $B_y = B_{0x}\tan\nu$ and then to a damping of the field accompanied by a rotation. The damping depth increases without restriction as $\lambda \to \delta$, and the spatially homogeneous state finally becomes thermodynamically unstable. In the model under consideration, however, $\lambda \lesssim \delta$ and the stability condition holds. Nabutovskiĭ and Shapiro[3] pointed out that the field rotates in this case, putting aside the problem involving the boundary condition. In the current, the term linear in $\mathbf{B}$ was used in some studies (see Ref. 4 and the bibliography cited there) to interpret the data on the critical currents. A justification given for this approach is not convincing.

We should point out that in the equilibrium state a normal conductor does not have that part of the current which is linear in the induction: As we can infer from (3), the presence of this current component would involve the use of a gauge-noninvariant expression for $\mathbf{M}$. The result obtained previously by one of the authors[5] is therefore incorrect.

We used the same method to evaluate this effect as the one we used in the previous study.[6] The main contribution comes from the scattering of electrons by impurities, whose potential has an asymmetric lattice-induced distortion. From an analysis of the diagrams similar to those analyzed in Ref. 6 we estimate the value of λ in (3) to be

$$\lambda \sim \xi_0\left(\frac{\xi_0}{l}\right)^2, \quad l \gg \xi_0; \quad \lambda \sim l, \quad l \ll \xi_0. \tag{5}$$

This estimate is correct for the entire temperature interval. The temperature dependence of the coefficient ν that determines the magnitude of the effect is therefore linked, as we can see from (4), primarily with the temperature dependence of δ. Far from T_c we have $\lambda/\delta \sim \xi_0/\delta_0$ even with $l \sim \xi_0$ ($\xi_0 = \hbar v_F/T_c$, and δ_0 is the penetration depth at $T=0$). The discussion above pertains to a London limit in which $\delta_0 > \xi_0$.

We should point out that the effect is influenced by the spin polarization due to spin-orbit scattering by the impurities and by the asymmetric scattering by phonons. Furthermore, allowance for the interband matrix elements of the velocity operator also gives rise to a certain contribution. All these contributions are important only when there are a few impurities and the effect is small.

We will finally consider two typical linear effects. In the first effect the critical current flowing in one direction in a fine superconducting wire (of thickness less than δ), along the axis of which a magnetic field is applied, is different from the current flowing in the opposite direction if the material of the superconductor is characterized by a nonvanishing pseudoscalar β. In the second effect, which is not related to isomerism, the critical field H_{c3} of the surface superconductivity[7] is different at the opposite parallel crystal faces if the field direction is the same. (This circumstance also pertains to the field-penetration depth.) To estimate these effects near T_c, we must use odd-power terms for the field and the gradients Δ in the Ginzburg-Landau equation. The relative magnitude of the effects such as $(j_c^+ - j_c^-)/j_c$ is therefore proportional to $(T_c - T)^{1/2}$, but far from T_c and for $H \sim H_c$, these effects, like the coefficient ν in (4) which was estimated above, are not literally small. We emphasize in this connection that in all the phenomena mentioned above the numerical factor depends on the degree of asymmetry of the scattering potential. Specifically, the required symmetry can be achieved through an appropriate deformation. The magnitude of the effects in this case would be proportional to the degree to which the crystal is deformed.

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