Josephson junction with a ferromagnetic layer

A. I. Buzdin and M. Yu. Kupriyanov
M. V. Lomonosov Moscow State University

(Submitted 21 February 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 6, 308–312 (25 March 1991)

A superconductor-ferromagnet-superconductor (*SFS*) contact is examined in the "dirty" limit. It is demonstrated that the critical current oscillates and vanishes with changing thickness of the *F* layer and the exchange field in the ferromagnet.

Critical current oscillations are possible as the parameters of the *F* layer material change in clean *SFS* contacts, as noted in Ref. 1. A "dirty" limit, in which the oscillations are suppressed, as assumed in Ref. 1, are almost always realized, however, in practice in an experiment. In our study we have used a method, proposed in Ref. 2, to analyze the Josephson properties of an *SFS* contact in the "dirty" limit and to prove that the change in the critical current I_c as a function of d_n behavior in an oscillatory manner as does the exchange field I of the layer material.

We assume that "dirty" limit conditions hold for the materials comprising the *SFS* sandwich (see Fig. 1), along with a zero critical temperature of the ferromagnet

 0021-3640/91/060321-06$01.00

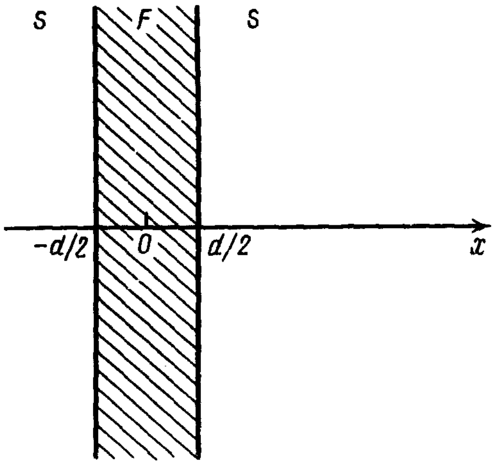

FIG. 1. Geometry of the *SFS* sandwich.

and an exchange field $I > T_c$, where T_c is the critical temperature of the bulk superconductor. A typical value of I in ferromagnets lies in the range 10^2–10^3 K. Hence, the latter of the two conditions nearly always occurs in practice, at least for ordinary superconductors.

1. In the near-critical temperature range, following Ref. 1, we introduce the functions

$$F^{\pm}_{s,n} = F_{s,n}(\omega) \pm F_{s,n}(-\omega), \tag{1}$$

where $F_{s,n}$ are the Usadel functions in the S and F regions, and $\omega = \pi T(2n+1)$ are the Mössbauer frequencies. We place the coordinate origin at the center of the F layer and the X axis perpendicular to the interface surfaces. Taking into account the assumptions made above, we can write the Usadel equations describing the properties of the *SFS* sandwich in the form

$$\pi T_c(\xi^*_n)^2 \frac{d^2}{dx^2} F^{\pm}_n \mp iIF^{\mp}_n = 0, \quad |x| \le d_n, \tag{2a}$$

$$\pi T_{c0}(\xi^*_s)^2 \frac{d^2}{dx^2} F^{\pm}_s - |\omega| F^{\pm}_s = 2\Delta\delta^{\pm}, \quad |x| \ge d_n. \tag{2b}$$

Here $\delta^+ = 1$, $\delta^- = 0$, $\xi^*_{n,s} = (D_{n,s}/2\pi T_c)^{1/2}$ and $D_{n,s}$ are the coherence length and the diffraction coefficients of the F and S metals. The system of equations (1) must be supplemented by the boundary conditions at the SF boundaries[4] (at the points $x = \pm d_n/2$):

$$F^{\pm}_s = F^{\pm}_n, \quad \sigma_s \frac{d}{dx} F^{\pm}_s = \sigma_n \frac{d}{dx} F^{\pm}_n, \tag{3}$$

where $\sigma_{s,n}$ are the conductivities of the S and F layers in the normal state. The solutions of boundary problems (2), (3) are given in the form

$$F_n^{\pm} = A_1 \cosh \beta_n x \pm A_2 \cosh \beta_n^* x + B_1 \cosh \beta_n x \pm B_2 \cosh \beta_n^* x,$$

$$\begin{aligned} &\beta_n = (1+i)(I/D_n)^{1/2}, \\ &F_s^- = C \exp(-\beta_s(x - d_n/2)), \quad \beta_s = (2|\omega|/D_s)^{1/2}, \qquad x > d_n/2, \\ &F_s^- = \tilde{C} \exp(\beta_s(x + d_n/2)), \qquad x < -d_n/2, \end{aligned} \tag{4}$$

where $A_{1,2}$, $B_{1,2}$, C, and $\tilde{C}$ are the constants to be determined. Substituting Eqs. (4) into boundary conditions (3), we find that for a layer material of sufficiently high resistance

$$\gamma = \frac{\sigma_n \xi_s^*}{\sigma_s \xi_n^*} \ll \begin{matrix} (\pi T_c/I)^{1/2}, & d_n > \xi_n^* (\pi T_c/I)^{1/2}, \\ min\{(d_n/\xi_n^*), \ (\xi_n^*/d_n)^3 (\pi T_c/I)^2\} & d_n < \xi_n^* (\pi T_c/I)^{1/2}, \end{matrix}$$

the boundary conditions on the functions $F_{s,n}^{+}$ are, in fact, independent of ω. This makes it possible to reduce the problem to a solution of the Ginzburg–Landau (GL) equations in superconducting electrodes with a boundary condition for $x = \pm d_n/2$ of the type

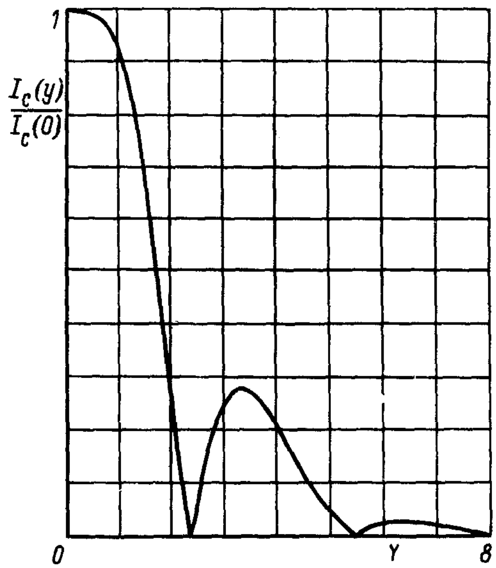

FIG. 2. Dependence of the critical current on the parameter $y = d_n/\xi_n^* \sqrt{(2I/\pi T_c)}$.

$$\frac{\Delta'(d_n/2)+\Delta'(-d_n/2)}{\Delta(d_n/2)-\Delta(-d_n/2)}=\frac{\sigma_n}{2\sigma_s}(\beta_n\coth(\beta_n d_n/2)+\text{c.c.}),$$

$$\frac{\Delta'(d_n/2)-\Delta'(-d_n/2)}{\Delta(d_n/2)+\Delta(-d_n/2)}=\frac{\sigma_n}{2\sigma_s}(\beta_n\tanh(\beta_n d_n/2)+\text{c.c.}), \tag{5}$$

Here the prime denotes differentiation with respect to the coordinate x.

Using the first integral of the GL equations and proceeding in accordance with Ref. 5, we obtain a sinusoidal dependence of the supercurrent on the phase difference of the order parameters φ with the critical current

$$I_cR_n=V_0y\frac{|\sinh(y)\cos(y)+\cosh(y)\sin(y)|}{\sinh^2(y)\cos^2(y)+\cosh^2(y)\sin^2(y)},\quad y=\frac{d_n}{\xi_n^*}\left[\frac{2I}{\pi T_c}\right]^{1/2}, \tag{6}$$

where R_n is the contact resistance, and $V_0=\pi\Delta^2(d_n/2)/4eT_c$. It should be pointed out that the negative values of the expression in the modulus in Eq. (6) correspond to a so-called π-contact,[6] in which the phase difference π corresponds to minimum energy, i.e., the opposite sign of the order parameter at the contact electrodes.

The relation $I_c(y)$ calculated from Eq. (6) is shown in Fig. 2. Clearly, the critical current oscillates with increasing y and vanishes at $3\pi/4+\pi k$, where the value of I_cR_n at the second maximum amounts to approximately 30% of $I_cR_n(y=0)$. The nonmonotonic behavior of the relation $I_c(y)$ can thus be observed experimentally.

2. It is important to remember in calculating I_c at finite temperatures that the anomalous Usedel functions F and $\tilde{F}$ are related in an F metal by the relation $\tilde{F}(\omega)=F^*(\omega\to-\omega)$, and that the normalization condition reduces to $G^2+F\tilde{F}=1$. In contrast with the ordinary case [where $F(\omega)=F^*(-\omega)=F^*(\omega)$], we have $\tilde{F}(I)=F^*(-I)$ in the presence of an exchange field.[1)]

In the limit of large F-layer thicknesses $y>1$ and in the case of satisfaction of inequality (4), which guarantees the applicability of the rigid boundary conditions on the SF boundaries, we can write the solution of the Usadel equations in the form[7,8]

$$F=\exp(-i\varphi/2)\sin\alpha^-+\exp(i\varphi/2)\sin\alpha^+,$$

$$\alpha^\pm=4\arctan\{A\exp[\pm(1+i\,\mathrm{sign}\omega)(I/D_n)^{1/2}(x\mp d_n/2)]\}, \tag{7}$$

$$A=|\Delta|/\{\Omega+|\omega|+(2\Omega(\Omega+|\omega|)^{1/2}\},\quad \Omega=(\omega^2+|\Delta|^2)^{1/2}.$$

Substituting Eq. (7) into the expression for the supercurrent

$$j(\varphi)=i\pi N(0)D_nT\sum_{\omega=-\infty}^{\infty}\left(F\frac{d}{dx}\tilde{F}-\tilde{F}\frac{d}{dx}F\right) \tag{8}$$

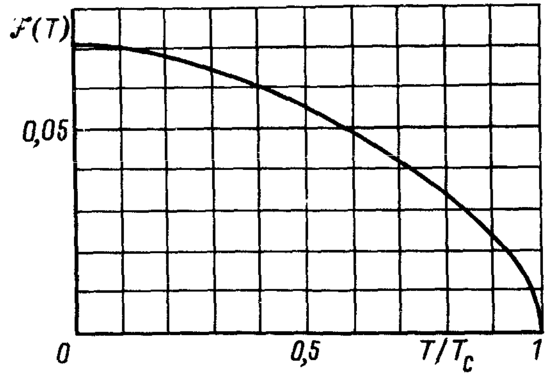

FIG. 3. Temperature dependence of the function $\mathcal{F}(T)$.

and taking into account the symmetry properties of the functions F upon substitution of ω by $-\omega$, we obtain the following expression for the critical current I_c:

$$I_c R_n = 32\sqrt{2}(\Delta/e)\mathcal{F}(\Delta/T)y\exp(-y)\sin(y+\pi/4),$$

$$\mathcal{F}(\Delta/T) = \pi T \sum_{\omega=0}^{\infty} \frac{\Delta}{(\Omega+\omega)[(2\Omega)^{1/2}+(\Omega+\omega)^{1/2}]} = \begin{cases} \frac{\pi}{128}(\Delta/T_c), & T \approx T_c \\ 0,071, & T \ll T_c \end{cases}. \quad (9)$$

The temperature dependence of the function $\mathcal{F}(T)$ is shown in Fig. 3. Near the critical temperature expression (9) becomes Eq. (6) near $y \gg 1$. This leads us to conclude that the curve of $I_c(y)$, shown in Fig. 2, is valid in the entire temperature interval $T \leqslant T_c$ and is a phenomenon common to *SFS* structures.

From the experimental point of view, it is most interesting to investigate oscillatory effects of I_c in structures whose layer has a Curie temperature θ near T_c. The variation in the exchange field in the F region with temperature in this case will cause oscillations of the critical current as a function of temperature. Such behavior may be of interest from the viewpoint of controlling the critical temperature of Josephson junctions.

It should be pointed out that the results obtained here are also valid for bridge-type structures of variable thickness, although in this case they will have substantially less rigid [compared to Eq. (4)] constraints on the parameters of the weak-link material.

[1)] Disregard of this circumstance resulted in an incorrect conclusion in Ref. 1 that critical current oscillations vanish in the dirty limit.

[1]A. I. Buzdin, L. N. Bulaevskiĭ, and S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 147 (1982) [JETP Lett. **35**, 178 (1982)].

[2]A.I. Buzdin and M. Yu. Kupriyanov, Pis'ma Zh. Eksp. Teor. Fiz. **52**, 1089 (1990) [JETP Lett. **52**, 487 (1990)].
[3]K. Usadel, Phys. Rev. Lett. **52**, 507 (1970).
[4]M. Yu. Kupriyanov and V. F. Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
[5]Z. G. Ivanov *et al.*, Fiz. Nizk. Temp. **7**, 560 (1981) [Sov. J. Low Temp. Phys. **7**, 274 (1981)].
[6]L. N. Bulaevskiĭ, V. V. Kuzii, and A. A. Sobyanin, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 314 (1977) [JETP Lett. **25**, 290 (1977)].
[7]K. K. Likharev, Usp. Fiz. Nauk. **127**, 184 (1979) [Sov. Phys. Usp. **127**, 386 (1979)].
[8]A. D. Zaikin and G. F. Zharkov, Fiz. Nizk. Temp. **7**, 375 (1981) [Sov. J. Low Temp. Phys. **7**, 184 (1981)].

Translated by Kevin S. Hendzel