Brief Announcement: Broadcast in Radio Networks Time vs. Energy Tradeoffs.

Extended Abstract*

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ABSTRACT

In wireless networks, consisting of battery-powered devices, energy is a costly resource and most of it is spent on transmitting messages. Broadcast is a problem where a message needs to be transmitted from one node to all other nodes of the network. We study algorithms that can work under limited energy measured as the maximum number of transmissions among all the stations. The goal of the paper is to study tradeoffs between time and energy complexity of broadcast problem in unknown multi-hop radio networks with no collision detection.

We propose and analyse two new randomized energy-efficient algorithms. Our first algorithm works in time $O((D + \varphi) \cdot n^{1/\varphi} \cdot \varphi)$ with high probability and uses $O(\varphi)$ energy per station for any $\varphi \leq \log n/(2 \log \log n)$ for any graph with n nodes and diameter D. Our second algorithm works in time $O((D + \log n) \log n)$ with high probability and uses $O(\log n/\log \log n)$ energy.

We prove that our algorithms are almost time-optimal for given energy limits for graphs with constant diameters by constructing lower bound on time of $\Omega(n^{1/\varphi} \cdot \varphi)$. The lower bound shows also that any algorithm working in polylogaritmic time in n for all graphs needs energy $\Omega(\log n/\log \log n)$.

CCS CONCEPTS

• **Theory of computation → Distributed computing models**;
• **Networks → Ad hoc networks**;

KEYWORDS

broadcast; radio networks; energy efficiency; algorithm

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1 INTRODUCTION

The problem of broadcast consists in delivering a single message from a source to all the nodes of a communication network. In multi-hop networks, neighboring stations can send messages to each other but when two neighbors of one station are sending at the same time, then these transmissions interfere and the messages are not delivered. A station can locally distinguish only the case where exactly one neighbor transmits from all the other situations (no collision detection).

Two most important parameters of an algorithm in radio networks is the time complexity, measured as the number of steps necessary to complete the execution, and energy complexity, which is the maximum number of rounds in which a station is transmitting. We want to also show a tradeoff between time and energy for broadcasting protocols. This will allow greater flexibility when designing algorithms by decreasing the maximum energy expenditure of a station at a cost of the runtime of the algorithm. Clearly, minimizing the energy cost can sometimes be a critical aspect of real-life systems as they are often composed of small, cheap, battery-powered devices whose batteries cannot be easily recharged or replaced. For such systems it may be reasonable to sacrifice time and save some energy of the stations.

1.1 Model and Problem Statement

In this paper we consider a radio network represented as an undirected, connected graph $G = (V, E)$, where the nodes symbolize stations and the edges are bidirectional communication links between them. By n we denote the number of nodes of the graph and by D its diameter. We assume that the nodes are given value of n and the energy limit. The stations do not know the topology of the network, are identical and do not have any labels. Symmetry between the stations can be in our model broken as the stations have access to independent sources of random bits.

Time is divided into discrete *rounds* and all the stations know the number of the current round. The model is synchronous and in each round each node either transmits a message or listens. Each station receives a packet from its neighbors only if it listens in a given round and **exactly one** of its neighbors is transmitting. We say that Single occurs in such a round. If zero or more than one neighbor of v transmits, then v receives no message.

The single-message broadcast problem is defined as follows. Initially some node, called *originator* has a message and the goal is to deliver this message to all the nodes in the network. We assume that the nodes that have not received the message are not allowed to make any transmissions.

Energy metrics. We define e_v, an *energetic effort* of a station $v \in V$, as the number of rounds when v transmitted. Note that both successful as well as unsuccessful (due to collisions) transmissions count. We will say that algorithm uses energy at most E if $\max_{v \in V} e_v \leq E$. In other words we aim at limiting the energetic expenditure of **all** stations that are present in the network since we need all stations working. Broadcast problem with the goal of minimizing the time and with a bounded number of transmissions per station is sometimes called k-shot broadcasting problem.

1.2 Related work

Energy aspects of broadcast in known and unknown networks has been investigated by Kantor and Peleg in [3] using the same energy-efficiency metric as in our paper. For the model with unknown topology they presented a protocol that uses k energy and is completed in $O\left((D + \min\{D \cdot k, \log n\}) \cdot n^{1/(k-1)} \log n\right)$ rounds for $k > 1$ and $O\left(D \cdot n^2 \log n\right)$ for $k = 1$ w.h.p. Moreover they demonstrated $\Theta(\log n)$-shot broadcasting protocol that terminates in $O\left(D \log n + \log^2 n\right)$ rounds. Berenbrink *et al.* [2] presented a broadcast algorithm with optimal time $O\left(D \log(n/D) + \log^2 n\right)$ with expected $O\left(\log^2 n / \log(n/D)\right)$ transmissions per node.

Since authors of [2, 3] use the same model, the results can be compared with ours. First, let us note that their protocols are based on a different constructions. For time $O\left(D \log n + \log^2 n\right)$ our algorithm Green-Decay uses $\log \log n$ times less energy than [3]. We also show our energy bound is tight and energy $\Theta(\log n / \log \log n)$ is the always required for any algorithm has time polylogarithmic in n. We can also compare our results that use any energy. Algorithm of Kantor and Peleg [3] works with probability $1 - n^{-1/(k-1)}$. If we use the same probability of success then our algorithm BB-Broadcast will have time complexity $O\left((D + k) n^{1/(k-1)} k\right)$ which is by a factor of $\log n / k$ faster. Algorithm by Berenbrink *et al.* [2] works in a different regime of energies – our algorithms work with energy $o(\log n)$ and time $\Omega(D \log n + \log^2 n)$.

2 ENERGY-EFFICIENT ALGORITHMS

In this section we present two new algorithms. BB-Broadcast ("Balls-into-Bins Broadcast") can work with arbitrarily small (also constant) available energy. Of course smaller energy leads to higher running time. The second algorithm GD-Broadcast ("Green Decay Broadcast") is built based on classical Broadcast by Bar-Yehuda et al. [1] that has energy complexity $O(\log n \log \log n)$. Our GD-Broadcast algorithm works in the same asymptotic time as the original one, but has reduced energy complexity $O(\log n / \log \log n)$.

2.1 Balls into Bins

Here we introduce a subprocedure Balls-into-Bins(k) that will be used in both our algorithms. In this subprocedure each participating station transmitts in one, randomly chosen out of k time slots. Balls into Bins is a classical concept in probability theory with applications in load balancing where the studied value is usually the maximum number of balls in any bin. In our setting, balls correspond to transmissions by the stations and bins are the time slots, hence we are interested whether some bin contains exactly one ball as this corresponds to a successful transmission.

Let t be the first time slot of the subprocedure
Choose a random number $i \in [0, 1, .., k-1]$
`Transmit` in slot $t + i$

Algorithm 1: Balls-into-Bins(k)

In the following lemma we bound the probability that in a procedure Balls-into-Bins(k) there is a slot that is chosen by exactly one station.

LEMMA 2.1. *If m neighbors of any fixed node v are performing procedure* Balls-into-Bins(k) *with $k = 24\lceil n^{1/\varphi} \rceil + 1$ if $1 \leq \varphi < \frac{\log n}{\log \log n}$ and if $1 \leq m \leq 12/\varphi \cdot n^{1/\varphi} \ln n$ then v receives the message with probability at least $1 - \frac{1}{2n^{1/\varphi}}$ for sufficiently large n.*

2.2 Balls into Bins Broadcast

The goal of this section is to design an algorithm that uses at most $O(\varphi)$ transmissions and has time complexity roughly $O\left(D \cdot n^{1/\varphi} \cdot \varphi\right)$ for any $\varphi > 0$. The idea of the algorithm is as follows. We consider the algorithm from the perspective of a fixed node u at the first step when at least one of its neighbors v has the message. The goal is to deliver the message to u with high probability in time $O\left(n^{1/\varphi}\right)$. Each participating station (neighbor of u that has the message) chooses a phase being a number chosen according to geometric distribution with parameter $\varphi / n^{1/\varphi}$. We will show that regardless of how many participants are there, some value will be chosen by at most $O\left(n^{1/\varphi} / \varphi \cdot \ln n\right)$ stations. Each phase is a Balls-into-Bins($n^{1/\varphi}$) procedure. We already know that if the number of participants is $O\left(n^{1/\varphi} / \varphi \cdot \ln n\right)$, then the failure probability of such procedure (i.e. no Single transmission occurs) is at most $n^{-1/\varphi}$. Then, repeating it $\left\lceil \varphi \cdot \left(1 + \frac{\log \frac{2}{\epsilon}}{\log n}\right) \right\rceil$ times will give us the desired probability. This intuition is further specified in the following pseudocode and formalized in the theorem.

$a \leftarrow \left\lceil \frac{\varphi \log n}{\log n - \varphi \log \varphi} \right\rceil$
$k \leftarrow 24\lceil n^{1/\varphi} \rceil + 1$
$t_{ph} \leftarrow ak$
Wait until receiving the message;
repeat
    Wait until ($Time \bmod t_{ph}$) = 0
    Choose a number $x \sim \text{Geo}(\varphi / \text{n}^{1/\varphi})$
    Skip ($\min\{x, a\} - 1$) $\cdot\, k$ rounds
    Balls-into-Bins(k)
until $\left\lceil \varphi \cdot \left(1 + \frac{\log \frac{2}{\epsilon}}{\log n}\right) \right\rceil$ *times*;

Algorithm 2: BB-Broadcast(φ, ϵ)

Theorem 2.2. *For any* $1 \leq \varphi < \frac{\log n}{\log \log n}$ *and* $\epsilon > 2n^{-3}$, *if* $n \geq 4$, *then Algorithm* BB-Broadcast(φ, ϵ) *completes broadcast in time* $O\Big((D+\varphi) \cdot n^{1/\varphi} \cdot \frac{\varphi \log n}{\log n - \varphi \log \varphi}\Big)$ *using at most* $\Big\lceil \varphi \left(1 + \frac{\log(2/\epsilon)}{\log n}\right) \Big\rceil$ *energy per station, with probability at least* $1 - \epsilon$.

2.3 Green-Decay Broadcast

Algorithm BB-Broadcast can operate under a wide range of energy limits, however for the optimal energy $\log n/(\log\log n + 1)$ it has time $O\Big(\Big(D + \frac{\log n}{\log \log n}\Big) \cdot \frac{\log^2 n}{\log\log\log n}\Big)$, which is slower than the algorithms from the literature. In this section we want to develop an algorithm that is less universal (i.e. does not offer parametrized energy vs. time trade-off) but achieves an almost-optimal time $O((D + \log n) \log n)$ using optimal energy $O(\log n/\log\log n)$.

We will first present Green-Decay which is a simple modification of classical procedure Decay introduced in [1]. It will serve as a subprocedure to our energy-efficient algorithm GD-Broadcast. In original Decay, each station transmits for a number of rounds being a geometric random variable. We note that instead of broadcasting in each round of the procedure it is sufficient to broadcast only in the last one. With this we save energy whilst the probability of success remains the same.

```
Transmit;
repeat
    x ← 0 or 1 with equal probability;
    if x = 1 then
        Transmit;
until x = 1 but at most k times;
```

Algorithm 3: Green-Decay(k)

The high-level idea of GD-Broadcast is as follows. In Broadcast algorithm from [1] each station participates in $\Theta(\log n)$ Decay procedures and the expected maximum energy is $\Theta(\log n \log\log n)$. By simply replacing it with Green-Decay (that takes always at most a constant energy per participant) we can reduce the energy complexity from $\log n \log\log n$ to $\log n$. In order to reduce the energy complexity further we observe that a station does not necessarily need to participate in all the $\log n$ procedures Decay. If some number x of neighbors of v have the message and want to transmit it to v it is sufficient that in at least a constant fractionof the $\Theta(\log n)$ procedures Decay at least one among the x stations participate. In our algorithm each station participates in $\Theta(\log n/\log\log n)$ procedures Decay chosen at random. If x is sufficiently large, at least a constant fraction of procedures Decay will have at least one participant and the algorithm will work correctly with high probability. On the other hand if x is small we can use procedure Balls-into-Bins which gives a probability of success of order $1 - 1/\log n$, for $\varphi = \log n/\log\log n$. Hence for small x, $O(\log n/\log\log n)$ procedures Balls-into-Bins is sufficient to obtain the high probability of successful transmission. Our algorithm combines Green-Decay and Balls-into-Bins to cover both cases of small and large number of neighbors trying to deliver the message. One more difficulty we need to overcome is that the number of participating (i.e., holding the message) neighbors of v might increase over time.

```
ll ← ⌈log log n⌉
k ← 24⌈log n⌉ + 1
state ← new
Wait until receiving the message;
repeat
    Wait until (Time mod 3·k) = 0
    phase ← Time/(3·k) mod ll
    if phase = 0 and state = new then
        state ← normal
    endif
    if state = new then
        Balls-into-Bins(k)
        Skip k rounds
    else if phase = 0 then
        Skip k rounds
        Balls-into-Bins(k)
    endif
    if phase = 0 or state = new then
        if state = new then state ← normal ;
        myPhase ← Random([0, 1, ..., ll − 1])
    endif
    if phase = myPhase then
        Green-Decay(k)
    endif
until 2⌈log n⌉ + 2 times;
```

Algorithm 4: GD-Broadcast

Theorem 2.3. *If* $n \geq 32$ *then algorithm* GD-Broadcast *completes broadcast in time* $O((D + \log n) \log n)$ *using* $O(\log n/\log\log n)$ *energy per station with probability at least* $1 - 2/n$.

3 LOWER BOUND

Our algorithm BB-Broadcast has a surprisingly large multiplicative factor $n^{1/\varphi}$. In this section we want to prove that such a factor is sometimes necessary by showing that time $\Omega(n^{1/\varphi} \cdot \varphi)$ is needed for any algorithm using energy φ. Our lower bound also shows that energy $\Omega(\log n/\log\log n)$ is required for any algorithm that works in time polylogarithmic in n which shows energy-optimality of GD-Broadcast in this class of algorithms.

Theorem 3.1. *For any randomized broadcast algorithm* $\mathcal{A}$ *successful with probability at least* $(1 - e^{-1})/2$ *in all multi-hop radio networks there exists a graph* G *with constant diameter and* n *nodes, such that if* T *is the runtime and* E *is the energy used by the algorithm then the expected value of* $E \cdot \log(T/E)$ *is* $\Omega(\log n)$.

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