Reusable PVS Proof Strategies for Proving Abstraction Properties of I/O Automata *

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Abstract

Recent modifications to PVS support a new technique for defining abstraction properties relating automata in a clean and uniform way. This definition technique employs specification templates that can support development of generic high level PVS strategies that set up the standard subgoals of these abstraction proofs and then execute the standard initial proof steps for these subgoals. In this paper, we describe an abstraction specification technique and associated abstraction proof strategies we are developing for I/O automata. The new strategies can be used together with existing strategies in the TAME (Timed Automata Modeling Environment) interface to PVS; thus, our new templates and strategies provide an extension to TAME for proofs of abstraction. We illustrate how the extended set of TAME templates and strategies can be used to prove example I/O automata abstraction properties taken from the literature.

1 Introduction

One approach to supporting strategies in tactic-based provers such as PVS is to adhere to specification templates that provide a uniform organization for specifications and properties upon which strategies can rely. This approach has been used in the TAME (Timed Automata Modeling Environment) interface to PVS [1, 2].

Until now, TAME proof support has been aimed at properties of a single automaton—mainly state and transition invariants for (both timed and untimed) I/O automata, though TAME does include minimal strategy support for proofs of properties of execution sequences of I/O automata. All of TAME's proof support is aimed at supplying "natural" proof steps that users can employ in checking high level hand proofs of properties of automata that are specified following the TAME automaton template.

One long standing goal for TAME has been to extend its proof support to include proofs of abstraction properties, such as refinement and simulation relations, involving two automata. This goal includes the ability to reuse established specifications and invariants of two automata in defining and proving an abstraction relation between them. A second goal is that the new proof support for abstraction properties should be generic in the same way as TAME support for invariant proofs: that is, there should

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be a fixed set of TAME proof steps, supported by PVS strategies, that can be applied to proofs of abstraction properties without being tailored to a specific pair of automata.

The theory interpretation feature [10] in the latest version of PVS (PVS Version 3), combined with some recent enhancements to PVS, makes it possible to accomplish these goals. In previous work [8], we outlined our plan for taking advantage of these new PVS features in specifying abstraction properties and developing uniform PVS strategies for proofs of these properties. In this paper, we describe how specification and proofs of abstraction relations between two I/O automata can now in fact be accomplished in TAME, and illustrate these new capabilities on examples.

This paper is organized as follows. Section 2 reviews TAME's support for invariant proofs and utility of abstraction in verification of I/O automata. Section 3 discusses the past problem with designing TAME support for abstraction proofs, and shows how with PVS 3.2, methods similar to those used in TAME support for invariant proofs can now be used to provide TAME support for abstraction proofs. Section 4 discusses some verification examples from the literature along with the formalization of the relevant abstraction properties in TAME. Section 5 presents a new TAME strategy for proving weak refinement, and shows its usage with examples. Finally, Section 6 discusses some related work, and Section 7 presents our conclusions and plans for future work.

2 Background

I/O Automata model. The formal model underlying TAME is the MMT timed automaton [7], which subsumes the class of untimed I/O automata. In this paper, we refer to MMT timed automata simply as (timed) I/O automata. The main components of an I/O automaton are its set of states, determined by the values of a set of state variables; its set of (usually parameterized) actions that trigger transitions; and its set of start states. The actions are partitioned into visible and invisible actions. The visible actions, in turn, partition into input and output actions. For systems involving timing, a special time passage action records passage of time.

An execution of an I/O automaton A is an alternating sequence of states and actions of A in which the first state is an initial state of A and each action in the sequence transforms its predecessor state into its successor state. The trace, or externally visible behavior of A, corresponding to a given execution α is the sequence of visible actions in α . For timed I/O automata, there are analogous notions of timed executions and timed traces. Further details can be found in [5, 6].

TAME support for invariant proofs. State (or transition) invariants of an I/O automaton are properties that hold for all of its reachable states (or reachable transitions). To support proofs of invariants of an I/O automaton, TAME provides a template for specifying a (timed or untimed) I/O automaton, a set of standard PVS theories, and a set of strategies that embody the natural high-level steps typically needed in hand proofs of invariants. The standard PVS theories include generic theories such as machine that establishes the principle of induction over reachable states, and special-purpose theories that can be generated from the DATATYPE declarations in an instantiation of the TAME automaton template. A sample of typical TAME steps for invariant proofs is shown in Figure 1.

Proof Step	TAME Strategy	Use
Get base and induction cases and do standard initial steps	AUTO_INDUCT	Start an induction proof
Appeal to precondition of an action	APPLY_SPECIFIC_PRECOND	Demonstrate need to use precondition
Apply the inductive hypothesis to non-default argument(s)	APPLY_IND_HYP	Supplement AUTO_INDUCT's use of default arguments
Apply an auxiliary invariant lemma	APPLY_INV_LEMMA	Needed in proving "non-inductive" invariants
Break down into cases based on a predicate	SUPPOSE	Add proof comments and labels to PVS' CASE
Apply "obvious" reasoning, e.g., propositional, equational, datatype	TRY_SIMP	Finish proof branch once facts have been introduced
Use a fact from the mathematical theory for a state variable type	APPLY_LEMMA	Perform special mathematical reasoning
Instantiate embedded quantifier	INST_IN	Instantiate but don't split first
Skolemize embedded quantifier	SKOLEM_IN	Skolemize but don't split first

Figure 1: A sample of TAME steps for I/O automata invariant proofs

Abstraction properties and trace inclusion for I/O automata. Quite often it is not natural to formulate and prove a desired property P of an automaton A as an invariant property, but is instead natural to think of P as being represented by the behavior of another, more abstract automaton B. In this case, one can show that A satisfies P by showing that every trace of A is a possible trace of B. Since abstraction relations imply trace inclusion, by the careful choice of a specification automaton B for P, the verification that P holds for A can be reduced to proving an abstraction relation between A and B.

Possible abstraction relations between two automata include homomorphism, refinement, weak refinement, forward simulation, backward simulation, and so on. Forward-and-backward simulation relations are both sound and complete with respect to trace inclusion of I/O automata [6], and therefore they constitute a powerful set of tools for automata-based verification.

3 Formalizing abstraction properties for I/O automata

In this section, we describe the hurdles we encountered earlier in developing TAME support for abstraction proofs and how we take advantage of theory interpretations and other new PVS features in adding abstraction proof support to TAME.

Previous barriers to TAME support for abstraction proofs. Abstraction properties involve a pair of automata, and hence to express them generally, one needs a way to represent abstract automaton objects in PVS.

The most convenient way to represent abstract automaton objects would be to make them instances of a type automaton. But, there are barriers to doing this in PVS. An I/O automaton in TAME is determined by instantiations of two types (actions and states), a set of start states, and a transition relation. Abstractly, these elements can be thought of as fields in a record, and an abstract automaton object can be thought of as an instance of the corresponding record type. However, record fields in PVS are not permitted to have type "type". An alternative way to express a type of automata is to

```
automaton: THEORY

BEGIN

actions : TYPE+;

visible (a:actions) : bool;

states : TYPE+;

start (s:states) : bool;

enabled (a:actions, s:states) : bool;

trans (a:actions, s:states) : states;

reachable (s:states) : bool;

converts (s1, s2: states) : bool;

END automaton
```

Figure 2: The new TAME supporting theory automaton

use parametric polymorphism, as in [9]. However, unlike Isabelle/HOL, which was used in [9], PVS does not support parametric polymorphism.

Because no general automaton type can be defined in PVS, I/O automaton objects have been defined in TAME as theories obtained by instantiating the TAME automaton template. Invariants for I/O automata are based on the definitions in these theories.

One can define an abstraction property between two automata defined by instantiating the TAME template theory by creating a new template that imports the template instantiations (together with their associated invariants), and then tailoring the details of a definition of the abstraction property to match the details of the template instantiations. However, this approach is very awkward for the user, who must tailor fine points of complex definitions to specific cases and be particularly careful about PVS naming conventions. It is also awkward for the strategy-writer, whose strategies would need to make multiple probes in a standard definition structure to find specific names. Further, this scheme relies on following a property template to permit a strategy to be reused in different instantiations of the property.

A new design for defining and proving abstraction in TAME. With the theory instantiation feature of PVS, together with other new PVS features, we have been able to design support for defining abstraction relations between two I/O automata that is both straightforward for a TAME user and clean from the point of view of the strategy developer. This support relies on (1) a new TAME supporting theory automaton, (2) a library of property theories, and (3) new TAME templates for stating abstraction properties as theorems.

Figure 2 shows the theory automaton, which can be instantiated by an automaton declaration by concretely defining the components actions, visible, states, etc. A new PVS feature allows the use of syntax matching to automatically extract the concrete definitions, thus simplifying the instantiation of automaton from a TAME automaton specification. Because states and actions are both declared as TYPE+, i.e., nonempty types, instantiating automaton results in two TCCs (type correctness conditions) requiring these types to be nonempty.

Examples of property theories for weak refinement and forward simulation are shown in Figures 3 and 4. We are building a library of property theories which include other commonly used abstraction relations such as refinement, backward simulation, etc.

```
weak_refinement[ A, C : THEORY automaton,
                 actmap: [C.actions -> A.actions],
                 r: [C.states -> A.states] ] : THEORY
 BEGIN
   weak_refinement_base: bool =
     FORALL(s_C:C.states): (C.start(s_C) => A.start(r(s_C)));
   weak_refinement_step : bool =
     FORALL(s_C:C.states, a_C:C.actions):
             C.reachable(s_C) AND C.enabled(a_C,s_C) =>
               (C.visible(a C) =>
                 (A.enabled(actmap(a_C),r(s_C)) AND
                  r(C.trans(a_C,s_C)) = A.trans(actmap(a_C),r(s_C))) AND
               (NOT C.visible(a_C) =>
                 ((r(s_C) = r(C.trans(a_C,s_C)))
                  OR (r(C.trans(a_C,s_C)) = A.trans(actmap(a_C),r(s_C))))
   weak_refinement: bool = weak_refinement_base & weak_refinement_step;
 END weak refinement
```

Figure 3: The new TAME property theory weak_refinement

```
forward_simulation[C, A : THEORY timed_automaton,
                   amap: [C.actions-> A.actions],
                   r: [C.states, A.states -> bool]] : THEORY
BEGIN
  f_simulation_base:bool = FORALL (s_C: C.states):
    (C.start(s_C) => EXISTS(s_A: A.states): A.start(s_A) AND r(s_C,s_A));
  f_simulation_step:bool =
    FORALL (s_C,s1_C: C.states1, s_A: A.states, a_C: A.states):
     A.reachable(s_C) AND reachable(s_A) AND r(s_C,s_A) AND
     A.enabled(a_C,s_C) AND s1_C = A.trans(a_C,s_C) =>
          (A.visible(a_C) AND (NOT A.nu?(a_C)) =>
           EXISTS (s1_A,s2_A,s3_A: A.states):
            converts(s_A,s1_A) AND converts(s2_A,s3_A) AND r(s1_C,s3_A) AND
            A.enabled(amap(a_C),s1_A) AND A.trans(amap(a_C),s1_A) = s2_A)
      AND (A.nu?(a_C) \Rightarrow EXISTS (s3_A: A.states):
                          A.t_converts(s_A,s3_A,A.timeof(a_C)) AND r(s1_C,s3_A))
      AND (NOT A.visible(a_C) => EXISTS (s3_A: A.states):
                                  converts(s_A,s3_A) AND r(s1_C,s3_A));
  forward_simulation: bool = f_simulation_base & f_simulation_step;
  END forward_simulation
```

Figure 4: The new TAME property theory forward_simulation

A particular instance of the TAME template for stating the abstraction properties as theorems is shown in Figure 5. This theory instantiates two copies—one for each of the abstract and the concrete automata—of the automaton theory, defines the action and state mappings between the two automata, and imports the relevant property theory with all the above as parameters.

4 Examples

In this section, we illustrate with three examples how to use the theories and template introduced in the previous section to state an abstraction property between a pair of I/O automata as a theorem (to be proved).

```
tip_abstraction: THEORY
 REGIN
    IMPORTING automaton
   IMPORTING tip_invariants
   IMPORTING tip_spec_invariants
       : THEORY = automaton :-> tip_decls
       : THEORY = automaton :-> tip_spec_decls
    amap(a_C: MC.actions): MA.actions
            CASES a_C OF
               nu(t): nu(t),
               add_child(e): noop,
               children_known(c): noop,
               ack(a): noop,
               resolve_contention(r): noop,
               root(v): root(v),
               noop:noop
            ENDCASES
   ref(s_C: MC.states): MA.states =
       (# basic := (# done := EXISTS (v:Vertices): root(v,s_C) #),
          now := now(s_C),
          first := (LAMBDA(a:MA.actions): zero),
          last := (LAMBDA(a:MA.actions): infinity) #)
    IMPORTING weak_refinement[MA, MC, amap, ref]
    tip_refinement_thm: THEOREM refinement
 END tip_abstraction
```

Figure 5: Instantiating the weak_refinement template for TIP

Leader Election Protocol. Our first exercise in using the extended templates and strategies of TAME was to formalize a weak-refinement proof for a simple spanning-tree based leader election protocol. Figure 5 shows our template instantiated with the automata TIP (representing the leader election protocol) and SPEC from [3]. The TAME specifications of these two automata are the theories tip_decls and tip_spec_decls. A set of invariants proved for TIP (both by the authors of [3] and in TAME [2]) establishes that at any given point in the execution of the algorithm, at most one leader has been chosen. The automaton SPEC has only one visible action (excluding the time passage action nu): namely, root. A weak refinement from TIP to SPEC is used to establish that all traces of TIP are included in the set of traces of SPEC, thus ensuring that the choice of a leader—the root action—occurs at most once in any execution of TIP.

The tip_abstraction theory in Figure 5 imports the library theory weak_refinement (Figure 3) with four parameters. The parameters MA and MC are instantiations of the automaton theory corresponding to the TIP and the SPEC automata; amap is a map from the actions of TIP to the actions of SPEC, and ref is the refinement function from the states of TIP to the states of SPEC. As a result of this importing the weak_refinement relation between TIP and SPEC is defined, and hence the corresponding refinement theorem tip_refinement_thm can be stated.

Failure Prone Memory Component. Our second case study concerns the specification and implementation of the memory component of a remote procedure call (RPC)

```
;;; Base case
    |-----
{1}    FORALL (s_C: tip_decls.states):
        (tip_decls.start(s_C) => tip_spec_decls.start(ref(s_C)))
```

Figure 6: Initial base case sequent for tip_abstraction.

module taken from [11]. A failure prone memory component MEM and a reliable memory component REL_MEM are modeled as I/O automata, and the requirement is to show that every trace of REL_MEM is a trace of MEM. The MEM and REL_MEM automata are almost identical, except that the failure action in MEM is absent in REL_MEM . Owing to this similarity, the refinement function ref is a bijection and the action map amap is an injection. As noted in [11], once again, a weak refinement from REL_MEM to MEM, suffices to establish trace inclusion, and we state this weak refinement property in the same way as in the previous example.

Periodic Send-Timeout Process. The final example concerns the composition of a periodically sending process P, a timed channel C, and a timeout process T, taken from [4]. The process P periodically sends messages, every u_1 time until an externally controlled **failure** action occurs. C enqueues each message with a deadline for its delivery, which is at most b time from its sending time. An enqueued message is received by T sometime before its delivery deadline. If no message is received by T over an interval longer than u_2 , then it performs a timeout action and suspects P (to be failed). If $u_2 > u_1 + b$, then T suspects P only if P has really failed. The external behavior of the composed automaton PCT is captured by a simple abstract automaton ABS in which a failure action is always followed by a timeout action, within $u_2 + b$ time.

Taking failure, timeout, and time passage actions to be visible, we have proved a forward simulation relation from PCT to ABS in PVS, thus establishing that every trace of PCT is a trace of ABS. The forward simulation property is stated using a template similar to that in Figure 5. The only differences are that ref is now a relation instead of a function and the property theory imported is forward_simulation from Figure 4.

5 Strategies for abstraction proofs

So far, we have used PVS to prove two weak_refinement examples and one for-ward_simulation example. For weak_refinement, we have developed an initial strategy called PROVE_REFINEMENT. PROVE_REFINEMENT is designed to be invoked on a theorem which, like tip_refinement_thm in Figure 5, asserts weak_refinement. PROVE_REFINEMENT undertakes to prove the weak-refinement theorem inductively by exploiting its known structure. First, it splits a theorem into a base case and an induction step. Then, the induction step is further subdivided into cases for each individual action type of the concrete automaton.

For the *TIP* example the base case yields the sequent in Figure 6, in which tip_decls.start and tip_spec_decls.start are the start predicates of *TIP* and *SPEC*, respectively. The base case is handled by skolemizing, applying PVS's EXPAND to the definitions of start and ref, and then performing some minor simplifications. In both our refinement case studies, this sufficed to make the base case "trivial" (see Figure 9).

The induction step of the refinement proof is handled by the substrategy **REFINE-MENT_INDUCTION**. This substrategy splits up the induction step into individual subgoals for each of the action types in the actions datatype. Then each subgoal is skolemized, and the definition of visible is expanded. After simplification, this gives different sets of subgoals for visible and invisible actions. For each invisible action, a single subgoal is generated from the condition in lines 8-9 in the weak_refinement_step definition in Figure 3. For each visible action, two new subgoals are generated from lines 5 and 6 in weak_refinement_step. For example, Figures 7 and 8 show the two subgoals generated for the (visible) nu branch in *TIP*.

The (enablement) subgoal in Figure 7 is further split into subgoals for the general (timeliness) precondition and the specific precondition of the action, respectively handled by **APPLY_GENERAL_PRECOND** and **APPLY_SPECIFIC_PRECOND**, followed by simplification. The second subgoal (congruence) is handled by expanding the transition definition and repeatedly simplifying. This sequence of operations resolves many simple action cases of refinement proofs. For the remaining action cases, the user must interact with PVS, using steps such as TAME's **APPLY_INV_LEMMA**, **INST_IN** and **SKOLEM_IN** (see Figure 1), as in Figure 9 below.

Using PROVE_REFINEMENT, we have established the weak-refinement relations in our leader election and failure-prone memory case studies. Figure 9 shows the saved proofs, which will be better structured once PROVE_REFINEMENT is polished. In the leader election (TIP/SPEC) example, all but the base case and the inductive goals for the root action were resolved by the strategy automatically. The base case can be discharged with TRY_SIMP. Proving the root enablement goal required using two invariant properties (invariants 13 and 15 from [3]) TIP, proved earlier with TAME. The root congruence goal required INST_IN. In the failure-prone memory (REL_MEM/MEM) example, because of the simple relationship between REL_MEM and MEM, all but the base case and the return action subgoals were resolved automatically by PROVE_RE-FINEMENT, and these remaining goals were easily discharged with TRY_SIMP.

We have also proved in PVS the forward simulation relation for the periodic send-timeout process described in the previous section, and we are currently in the process of developing a generic TAME strategy PROVE_FWD_SIMULATION for proving forward simulation. The initial steps of this strategy are similar to those in PROVE_RE-FINEMENT: splitting the simulation theorem into base and induction cases, and then into subcases for the individual actions. The greater complexity of the definition of forward_simulation means that our ultimate PROVE_FWD_SIMULATION will be more complex than PROVE_REFINEMENT, and that we may need to design additional TAME proof steps to be used in completing interactive proofs of forward simulation.

6 Related work

A metatheory for I/O automata, based on which generic definitions of invariant and abstraction properties are possible, has been developed in Isabelle by Müller [9], who also developed an associated verification framework. Example proofs of (e.g.) forward simulation have been done for at least simple examples using this framework; it is not clear to what extent uniform Isabelle tactics are employed. PVS has been used by others to do abstraction proofs, and in fact a refinement proof for *TIP* and *SPEC* was

Figure 7: Initial enablement sequent for the action **nu** in *TIP*.

```
;;; Induction cases
[-1,(reachable C.prestate)]
      reachable(sC_theorem)
[-2,(enabled C.action)]
      enabled(nu(timeofC_action), sC_theorem)
{1,(congruence)}
      ((# basic :=
           (# done:= EXISTS (v: Vertices):
                            root(v, trans(nu(timeofC_acton), sC_theorem)) #),
          now := now(trans(nu(timeofC_action), sC_theorem)),
          first := (LAMBDA (a: MA.actions): zero),
          last := (LAMBDA (a: MA.actions): infinity) #) =
      tip_spec_decls.trans(nu(timeofC_action),
                           (# basic :=
                               (# done:= EXISTS (v: Vertices):
                                                root(v, sC_theorem) #),
                             now := now(sC_theorem),
                             first := (LAMBDA (a: MA.actions): zero),
                             last := (LAMBDA (a: MA.actions): infinity) #)))
```

Figure 8: Initial congruence sequent for the action **nu** in *TIP*.

Figure 9: TAME refinement proofs for TIP/SPEC (left) and REL_MEM/MEM (right).

mechanized by Devillers et al. [3]. However, to our knowledge, no one has developed "generic" PVS strategies to support proving abstraction properties with PVS.

7 Conclusions and future work

Currently, we have developed a reusable PVS weak refinement strategy and supporting PVS theories, and added them to TAME. In the example weak refinement proofs we have done so far, existing TAME strategies provide sufficient proof steps for interactively completing the refinement proofs. We have begun the development of a reusable forward simulation strategy.

We plan to complete the development of the forward simulation strategy and add similar support for proving other abstraction relations using TAME. We also expect to add further TAME steps, if needed, for (interactively) completing proofs of action cases. Eventually, we expect to add our enhanced version of TAME to the set of tools being developed to support specification and verification of timed I/O automata (TIOA) [4].

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