Firefly Synchronization with Asynchronous Wake-Up

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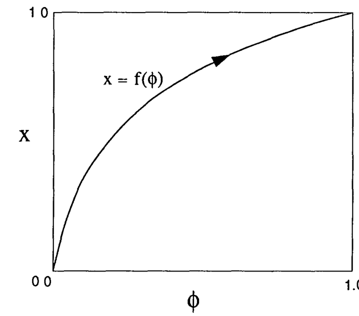
Fireflies synchronize

“The Trials of Life,” © BBC Documentaries

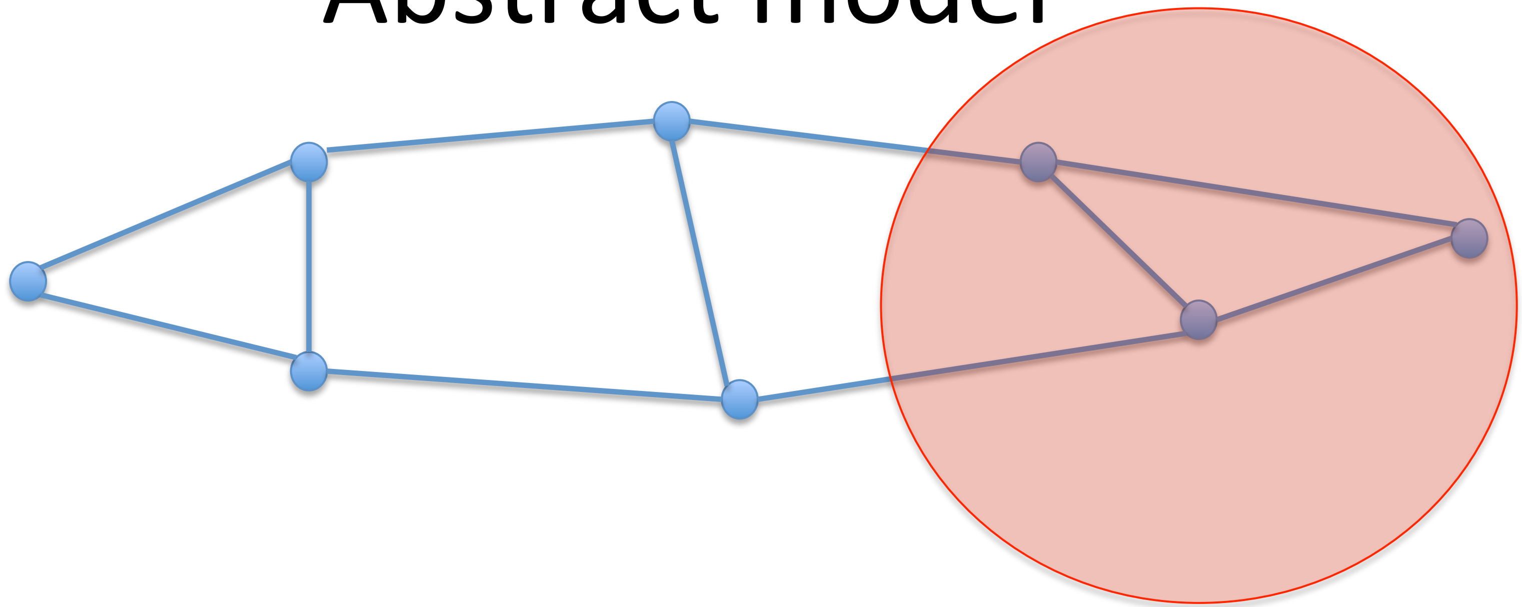
Research on Firefly Synch



[MS] Integrate-and-fire model

- Early research
  - E.g., [Smith35], [Buck88]
  - [Peskin73], [KuramotoN87]
- Mirollo-Strogatz [MS90]
  - Dynamical system model for the phenomenon, explaining synchronization in a clique
- Sparked considerable research on applications
  - Clock synchronization in computer systems [LucarelliWang05, Gopal06, SimeoneS08, etc.]

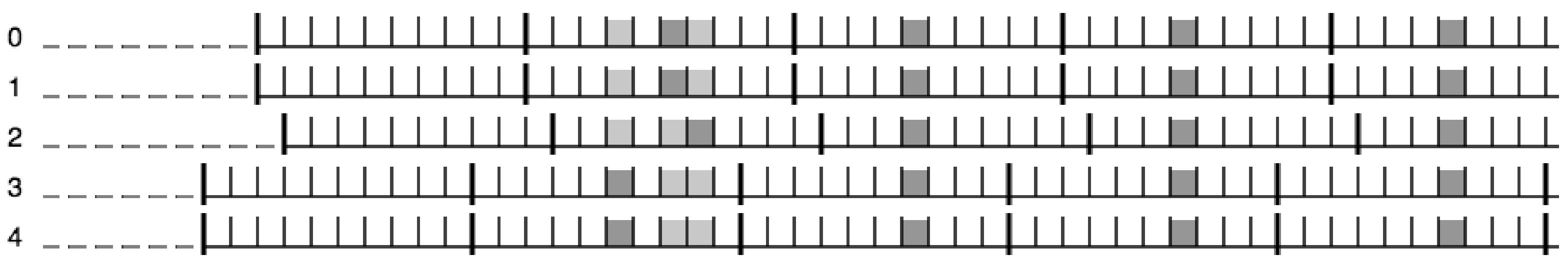
Abstract model



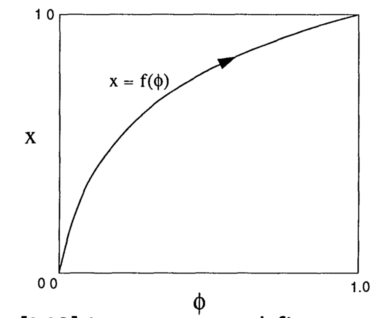
- N nodes (fireflies) in a *connected* topology
  - wake up at arbitrary times
- Communicate through *beeps (pulses)*
  - *Binary* information
  - Only neighbors can "see" pulse

Synchronization

- *Synch*: exists ***Global Synch Time (GST)***, period ***T > 1***, and ***offset o*** such that, after GST:
  - Nodes beep at global time t if (t – o) mod T = 0
  - Don't beep otherwise



Time models



[MS] Integrate-and-fire model

- Nodes:
  - share a period *T*
  - beep once per period
- Node dynamics
  - either *continuous (integrate-and-fire)*
  - or *discrete (averaging)*
- Continuous time:
  - Characterized by a *dynamical system*
  - Fixed point: all nodes beep synchronously
- Discrete
  - Characterized by a *system of equations*
  - Sync: all nodes beep in the same time slot

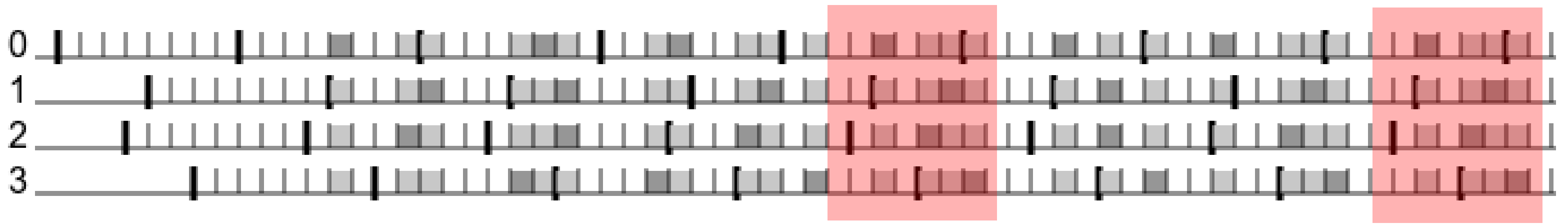
Discrete time

- Time divided into **discrete, *aligned*[*]** slots
- Each node *i*:
  - Wakes up at (global) time w_i
  - Beeps once in every period
    $t_i(k) = w_i + kT + \tau_i(k)$
  - Averages over its neighbors: $\tau_i(k) = (1/\Delta_i)\sum_j (t_j(k-1) - t_i(k-1))$
  - *System: t(k) =* ***A*** *t(k–1)*, where A is a Laplacian

All this is well known, and seems to work fine.

However…

The problem

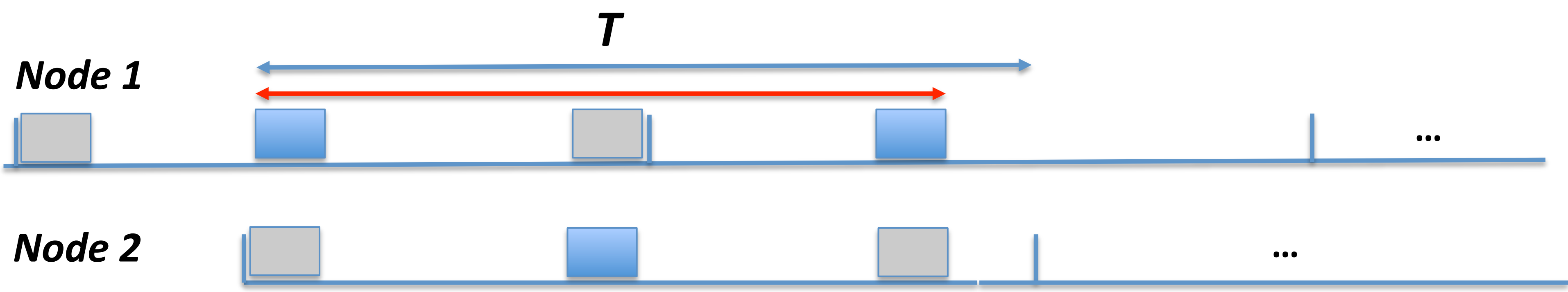


- The algorithm *does not always converge!*

$$t_i(k) = w_i + (k-1)\cdot T + (1/\Delta)\sum_j (t_j(k-1) - t_i(k-1))$$

The problem:
The *round structure* is *not respected*
under *asynchronous wakeup*!

The problem (2)



$$t_i(k) = w_i + (k-1)\cdot T + \sum_j (t_j(k-1) - t_i(k-1))$$

The problem:
The system equation no longer holds!
...and in fact, the system does not converge.

Assuming *synchronous wake-up* not a solution, since then nodes are *already synchronized.*

Our project

There is a problem with "averaging" algorithms (even if initial offsets are less than *T*)

- Hint of a solution:
  - We give an averaging algorithm, under assumptions on system parameters
  - Simple non-averaging algorithm
- Interesting open questions

The Algorithm

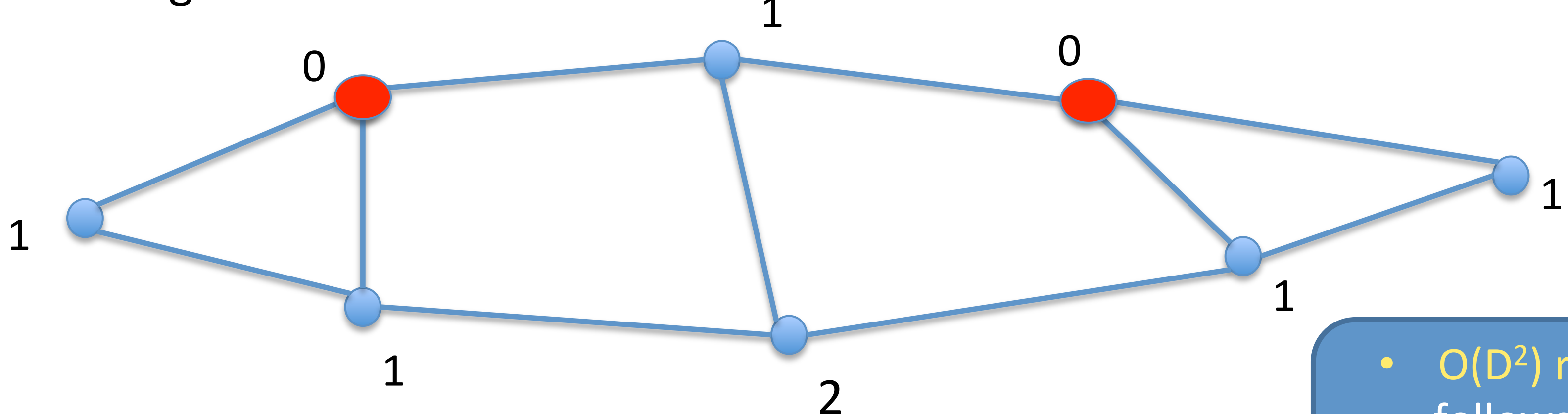
Assumptions:
1. Each node wakes up its neighbors by beeping
2. T ≥ 4n

Wake-up phase

- the adversary wakes up a subset of the nodes
- a node beeps as soon as it wakes up
- sets its next beep T / 2 slots later

Convergence phase

- nodes then start ***averaging*** in each “round”
- average rounded ***down***



- $O(D^2)$ rounds follows easily
- O(D) rounds the right answer

O(D) Round Analysis

- Claim 1: Rounds are communication-closed.
- Claim 2: Neighbors *are always* at most one slot apart.
- For node ***v***, round ***k***, diameter ***D***, define

$$F(v,k) = (1 + 1/D) \cdot offset(v, root) - dist(v, root) + k - 1$$

- Claim 2: For any v, either
  offset (v, root) = 0, or ***F(v, k) ≤ D + 1***.
- For k > 2D + 2, ***offset (v, root) = 0***, so nodes are in sync.

Averaging works in O(DT) time units,
given "gradient property" and T ≥ 4n.

A simpler algorithm

- Under the same assumptions, consider the trivial move-to-the-left (if you see something to your left) algorithm
- It also converges in O(TD) time

The "gradient property" + $T \geq 4n$ trivialize the problem to some extent.

Open questions

- We gave a working averaging algorithm
- Two strong assumptions:
  - Nodes wake up on neighbor beep (gradient)
  - $T \geq 4n$ (consistent rounds)

How about asynchronous wakeup?

Lower bounds?

How do they do it?