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Truong-Hong, L, Laefer, DF, Hinks, T et al. (1 more author) (2013) Combining an angle criterion with voxelization and the flying voxel method in reconstructing building models from LiDAR data. Computer-Aided Civil and Infrastructure Engineering, 28 (2). 112 - 129. ISSN 1093-9687

https://doi.org/10.1111/j.1467-8667.2012.00761.x

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COMBINING AN ANGLE CRITERION WITH VOXELIZATION AND THE FLYING VOXEL METHOD IN RECONSTRUCTING BUILDING MODELS FROM LIDAR DATA

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Abstract: Traditional documentation capabilities of laser scanning technology can be further exploited for urban modelling through the transformation of resulting point clouds into solid models compatible for computational analysis. This paper introduces such a technique through the combination of an angle criterion and voxelization. As part of that, a k-nearest neighbor (kNN) searching algorithm is implemented using a predefined number of kNN points combined with a maximum radius of the neighborhood, something not previously implemented. From this sample points are categorized as boundary or interior points. Façade features are determined based on underlying vertical and horizontal grid voxels of the feature boundaries by a grid clustering technique. The complete building model involving all full voxels is generated by employing the Flying Voxel method in order to relabel voxels inside openings or outside the facade as empty voxels. Experimental results on 3 different buildings, using 4 distinct sampling densities show successful detection of all openings, reconstruction of all building façades, and automatic filling of all improper holes. The maximum nodal displacement divergence was 1.6% compared to manually generated meshes from measured drawings. This fully automated approach rivals processing times of other techniques with the distinct advantage of extracting more boundary points, especially in less dense data sets (<175pts/m²), which may enable its more rapid exploitation of aerial laser scanning data and ultimately preclude needing a priori knowledge.

1 INTRODUCTION

Point clouds from laser scanning technology, known as Light Detection and Ranging (LiDAR), can collect object surface data quickly and accurately. These pointclouds have been used for reconstructing object surfaces in applications from medicine (Weyrich et al., 2004) to product design (Várady et al., 2007). LiDAR is being used in Civil Engineering applications most significantly in transportation for road modelling (Cai and Rasdorf, 2008; Tsai, et al. 2009), sign inventorying (Wang et al. 2010), road defect identifycation (Zhang and Elaksher, 2011), and disaster planning (Laefer and Pradhan 2006). Increasingly, it is also being applied for structural health monitoring (Park et al. 2006 and

Lee and Park, 2011), texture mapping (Zalama et al. 2010), historic documentation (Böhm et al. 2007) and Building Information Model generation (Huber et al., 2011). Most recently, pointclouds are employed for populating complex computational models for climate modelling (Wenisch et al., 2007) and subsidence prediction (Laefer et al., 2010), for which highly accurate geometries are needed.

Many methods have been developed to extract geometries from LIDAR data [both airborne and terrestrial] and photogrammetry, but most concentrate on reconstructing models for visualization. To date, the conversion of these models for computational analysis has required significant manual intervention to obtain high geometric accuracy (Laefer et al., 2011a). With Aerial Laser Scanning (ALS) data, cityscale, polyhedral building models are typically generated from boundaries of roof segments (Dorninger & Pfeifer, 2008; Elberink, 2009; Zhou & Neumann, 2009). In such cases, resulting building outlines are of low accuracy, as roof outlines are normally slightly larger than the buildings. Also, ALS's traditionally low sampling density (<100 pts/m² horizontally projected and <35 pts/m² vertically projected) creates difficulties in generating detailed vertical surfaces. Greater success has been achieved using the substantially denser Terrestrial Laser Scanning (TLS) datasets. A good approach proposed by Pu and Vosselman (2009) involves segmenting potential façade features (e.g. windows, doors) and the subsequent fitting of polygons. Many alternative approaches are based on façade grammars (Becker & Haala, 2007 and 2009; Hohmann et al., 2009; Wonka et al., 2003); see Laefer et al. (2011a,b) for an extensive review of related literature and commercial applications for terrestrial and aerial options, respectively.

As most existing approaches have concentrated on visualization, significant problems tend to arise in either mesh convergence or geometric accuracy when the resulting models are used as the basis for computational modelling. Solid models of existing buildings generated through such means have had several drawbacks: (1) re-quire significant user experience; (2) are of relatively low geometric accuracy of the façade and its openings; (3) cannot overcome unrealistic openings caused by sparse or missing data; and/or

(4) may produce degenerate shapes causing difficulty in generating Finite Element Method (FEM) meshes. To surmount these shortcomings, a feature detection approach entitled the FacadeAngle (FA) algorithm is proposed to create highly accurate boundaries of façade features through the combination of an angle criterion, voxelization, and a recently introduced voxel location detection approach entitled the Flying Voxel method (Truong-Hong et al. 2012).

2 RELATED WORKS

To identify relevant sample points for detecting building features various algorithms and criteria have been proposed, many of which employ (1) an angle criterion or a half-disc criterion (Becker & Haala, 2007), (2) a Delaunay triangulation [e.g. (Pu & Vosselman, 2007)] and/or (3) proximity-based alternatives.

2.1 Angle and Half-disk Criteria

The general idea of boundary point detection by an angle criterion has been described by several researchers (Bendels et al., 2006; Gumhold et al., 2001; Linsen, & Prautzsch, 2001, 2002). The criterion is based on the distribution of neighboring points consisting of the k-nearest neighbor points (kNNs) to a given point in Euclidean space (Samet, 2008) around a given point. To select appropriate kNNs for a small portion of an object's surface, Linsen and Prautzsch (2001) implemented an angle criterion to establish the neighborhood of a given point, where the maximum angular gap between two consecutive points within the k-neighbors projected onto a fitting plane was less than a threshold angle (e.g. $\pi/2$). Subsequent work allowed rapid generation of locally triangulated meshes suited for object representation and real-time rendering of three-dimensional (3D) scenes (Linsen, & Prautzsch, 2002). Similarly, an angle criterion was used to enhance a standard kNN during selection of a given point's neighborhood, where no angles between two consecutive neighboring points were larger than a pre-specified threshold (Moenning & Dodgson, 2004). The given point was classified either as a boundary point or as one lying in an under-sampled region, if no points were detected within a spherical neighborhood (as defined by a user-controlled radius).

Elsewhere, Gumhold et al. (2001) applied an angle criterion to raw point data to extract the points on an object's surface. There, sample points were classified as surface, crease, corner, or border points based on a penalty function dependent upon the maximum angle between neighboring points on a tangent-fitting plane (Hope et al., 1992). In related work, Bendels et al. (2006) presented a boundary probability by combining various criteria for automatic hole detection. In this, boundary probability was computed from the maximum gap between two consecutive projected neighbor points for an angle criterion. The distance between a given point and the average of its neighbors for the shape criterion was calculated. Then the eigenvector of the given points calculated from its neighbors were compared to the eigenvector of a sample point belonging to standard objects.

2.2 Delaunay Triangulation and Related Approaches

In FEM meshing, Delaunay triangulation is a common approach, in which a circumcircle of any triangle may not contain any sample points of the set (Berg et al., 2000). For feature detection this is useful, as there are no sample points inside openings. The resulting triangles in those areas have longer sides. Using Delaunay triangulation meshes, sample points on the boundaries of a façade and its openings can be identified. These are the end points of the triangle sides that are longer than a predefined length threshold (Boulaassal et al., 2009; Pu & Vosselman 2007). Those points have been used for generating polygons as representations of complete building models by a least-squares fitting approach (Pu & Vosselman 2009) or by transforming them into parametric models (Boulaassal et al., 2010). Some drawbacks involve incomplete window generation, low accuracy of wall outlines, and dependence upon a predefined length threshold (Tang et al., 2009). In related work, Ali et al. (2008) introduced adaptive thresholds to detect contours of a rectangular, bounding window. This was based on the high variability of absolute differences of adjacent laser measured distances that occur in window regions where part of the laser beam falling on a window surface is reflected back when it hits an internal object's surface. In such cases, windows were segmented by implementing closing morphological operations, from which window positions and global shapes were detected and subsequently retrieved by using contour analysis.

2.3 Proximity-based Alternatives

Based on the observation that glass reflectivity is low for normal light incidence, Wang et al. (2011) proposed detecting boundary points by examining neighboring spaces to low intensity ones; defined as voxels along vertical and horizontal directions. A voxel contains boundary points, if at least one empty voxel appears in an interval width of the considered voxel. The methods can efficiently detect windows but with relatively low geometric accuracy and cannot distinguish windows from occlusions. Similarly, in 2008, Hinks et al. (2008) proposed a point-based voxelization method to generate solid models of building façades from LiDAR data for simulation. In this, a voxel grid divides a bounded 3D region (containing whole point clouds of the façade) into a set of cells along vertical and horizontal grids parallel to axial directions of a Cartesian coordinate system. The voxel is assigned a value of 0 or 1 corresponding to the voxel containing a number of sample points less than a user-specified threshold value or vice versa, of which the voxel is called "active" or "inactive". While the resulting building models are fully compatible with FEM mesh generation, they require a fairly dense data set for accurate feature detection, thereby precluding applicability to aerial data. The long range processing of city wide data also must address the challenges of working with very large data sets, so although the current TLS data capture capability is nearly limitless, further data density may not be the optimal solution, although more recently available mobile mapping may be an alternative (Alshawa et al. 2009).

Furthermore, to date these approaches have not provided sufficiently reliable and accurate boundary and feature detection for solid model reconstruction for computational modeling for Civil Engineering. The next section describes a new approach towards integration of these technologies for improved boundary and feature detection.

3 PROPOSED FACADEANGLE ALGORITHMS

While the point-based voxelization technique patented by Hinks et al. (2008) can by itself generate a solid model quickly, it insufficiently defines boundaries of a façade and its openings for computational purposes. Furthermore, other attempts to exploit that technique such as the FacadeDelaunay (FD) algorithm recently proposed by the same research group (Truong-Hong et al. 2012) require data densities that will not be available through aerial LiDAR capture for many years as they are two orders of magnitude greater than current data capture abilities (Hinks et al. 2009). Therefore, the FacadeAngle (FA) algorithm is proposed for detection of façade and building features with less dense data sets, as it harvests greater numbers of boundary points for facades and building features, which provide a wider range of subsequent processing opportunities. Herein, the FA algorithm is applied to TLS data of various densities, as these are not yet achievable with ALS.

The proposed workflow can be divided (Figure 1): (i) initial feature detection, in which sample points (called boundary

points) lying on the façade and its openings are extracted by using the angle criterion and then unrealistic holes are eliminated by comparing characteristics of detected holes to standard building openings and (ii) geometric model reconstruction, in which a geometric model is produced.

Since the solid models herein are used for structural analysis, some non-structural elements (e.g. balconies and window ledges) are not included. Further, this work assumes that buildings are quadrilateral in shape, with structural elements residing within a planar façade, with openings comprised of primarily rectangular windows and glass-plated doors, and currently only reconstructs two-dimensional (2D) façades but could be extended to 3D models.

3.1 Boundary detection (Step 1)

As mentioned above, a pre-processing step classifies input sample points into boundary and interior classifications and discards unrealistic holes (Figure 2). There, each sample point is examined as to whether or not it lies on a boundary using an angle criterion. A boundary coherence technique (as will be discussed subsequently) is then applied to improve robustness. Holes are then assessed by comparing their characteristics to those of standard building openings. Their boundary points are finally re-classified as interior points, if they fail to meet the criteria (Figure 2).

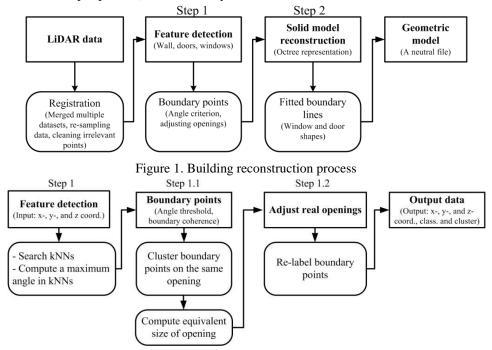


Figure 2. Feature detection processes

3.1.1 Angle criterion in boundary point detection (step 1.1)

Point, p_i, is an "interior point" if the neighboring points are distributed around an entity (Figure 3a), or a "boundary point", if the neighboring points form a partial ball (Figure 3b). Thus, the maximum angle between two consecutive

neighbor points is critical for classification. Ideally, sample points on boundaries of a façade and its openings would be straight lines, for which the maximum angle between consecutive neighbors would be equal to π except at interior window corners, which should not exceed $\pi/2$. Thus, the critical angle is chosen as $\pi/2$, so the given point is a

boundary point, if the maximum angular gap exceeds an angular threshold. Otherwise, it is an interior point.

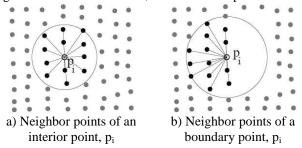
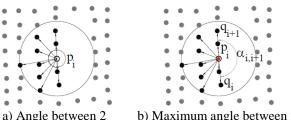


Figure 3. Distribution of the number of nearest neighbor points of a given point

The FA algorithm starts with a randomly selected sample point and searches for a set of neighboring points, \mathbf{q} (discussed below). The neighboring points, \mathbf{q} , are projected onto a target-fitting plane (Figure 4). Cartesian coordinates of neighboring points are then transformed into relative cylindrical coordinates, with the local origin set at a given point p_i . An angle between two consecutive neighboring points, $\alpha_{i,i+1} = \angle q_i q_{i+1}$, is computed as the difference between their azimuths (Figures 4a). The given point is labeled as a boundary one, if the angle $(\alpha_{i,i+1})$ exceeds a given threshold (Figure 4b), and an interior point, otherwise.



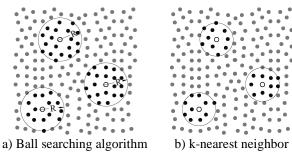
a) Angle between 2 b) Maximum angle between 2 consecutive neighbor points of a given point p_i points of a given point p_i

Figure 4. Computing a maximum angle between two consecutive neighbor points of a given point p_i

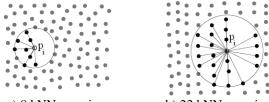
For selecting neighboring points of a given point, various methods have been used, such as a ball neighborhood (Figure 5a) and kNN (Figure 5b). Herein, a binary search k-dimension (k-d) tree was implemented into the FA algorithm for searching kNN points, where each leaf node contains a number of predefined target points (Bentley, 1975) – e.g. the 20 points (as discussed subsequently) adopted in this k-d tree. A number of kNNs, $\mathbf{q}\{q_1, q_2,...,q_n\}$, the nearest points of the given point, p, in the Euclidean distance, were extracted by the use of a k-d tree; [see Moore (1990) for more details of the searching].

Selecting am optimal number of kNN points is an important task, because this affects the running time and feature detection quality. For example, if the data have a high level of noise or are relatively sparse, selecting a small number of kNN points will lead to large errors in classification (e.g. a sample point in the sparse Figure 6a data set was classified as a boundary point instead of an interior one). Similarly, boundary points are overlooked with excessively large

neighborhoods (Figure 6b). Additionally, as point density varies within a data set, with low densities normally occurring in boundary areas, a neighborhood with a predefined number of kNN points can unintentionally contain sample points belonging to surface patches along two boundaries of the same opening (Figure 6b), which causes improper classification. Herein it is proposed, that the problem can be minimized by implementing a radius threshold to constrain the ball of the neighborhood. In this, the ball's radius must be less than or equal to the selected threshold. This threshold is set to a normal minimum window opening size (i.e. 0.4m) based on empirical work by (Pu & Vosselman 2007). The kNN searching algorithm implemented in this study is defined by an input number of kNN points combined with the neighborhood's maximum radius (something not previously done). Results with and without a radius threshold are illustrated in Figures 7a and 7b, respectively.



a) Ball searching algorithm b) k-nearest neighbor with a pre-defined radius 0.5m searching for 10 kNNs Figure 5. Searching nearest neighbor point algorithm



a) 8 kNNs causing
misclassification of point misclassification of point p_i due
p_i in a sparse dataset to overly large neighborhood
Figure 6. Misclassification of sample points due to either an
overly sparse data or an excessively large neighborhood.

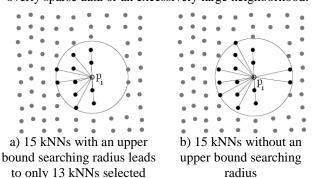


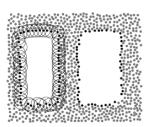
Figure 7. Selecting 15 kNNs of the given point p_i with and without an upper bound searching radius

To improve robustness of boundary point detection, the boundary coherence technique proposed by Bendels et al. (2006) was implemented to eliminate incorrect boundary points due to noisy data. The technique defines a point as being on a boundary, if and only if, two neighbor points of

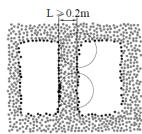
the given point forming the maximum angle are also boundary points. Otherwise, the point is classified as an interior one. In this study, the process is used to check through all boundary points detected in the previous step.

3.1.2 Adjusting openings

To distinguish real from apparent openings, any detected hole is compared to a standard façade opening. A detected hole is presumed false, if its characteristics fail to meet predefined opening criteria (as will be discussed later). To make this check, firstly all points on the boundary of a single hole are clustered using a clustering technique with a predefined searching radius (Guha et al., 1999). The radius value must ensure that none of the boundary points of adjacent openings are unintentionally included, while still capturing all the relevant boundary points (Figure 8a). In real buildings, the distance between adjacent rows or columns of openings or an opening and the façade edge is normally larger than 0.2m (Ripperda, 2008) [Figure 8b]. Thus, a predefined searching radius equal to 0.2m is adopted.



Window 1 Window 2 a) Clustering all boundary points of a selected opening



Window 1 Window 2 b) Unintentional selection boundary points from adjacent openings due to an overly large searching radius

* Black points are respectively boundary points of windows 1 and 2

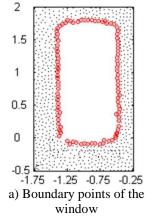
Next, characteristics of each detected hole (height, length, and height/length ratio) are computed using a histogram along the horizontal and vertical directions (Figure 9), where the hole is assumed to be rectangular. The possible horizontal and vertical boundary lines of a hole can be determined from two peaks of the histogram along the x- and y-directions, respectively (Figure 9b and c), in which a number of bins in each direction is adopted equal to 10. However, for holes (such as doors) at the ground surface, only one peak on the upper side appears on the histogram along the y-direction. Based on this concept, equivalent dimensions (height and length) of each hole are consequently determined in accordance with equations 1 and 2.

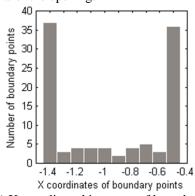
$$H = \begin{vmatrix} \frac{\sum_{i=1}^{n} y_{i}^{up}}{n} - \frac{\sum_{j=1}^{m} y_{j}^{low}}{m} \end{vmatrix}$$
 (1)

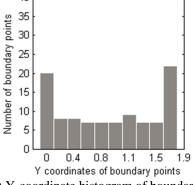
$$L = \begin{vmatrix} \sum_{i=1}^{k} x_i^{\text{left}} & \sum_{j=1}^{l} x_j^{\text{right}} \\ k & 1 \end{vmatrix}$$
 (2)

where n and m, and k and l are a number of boundary points belonging to 2 peaks ("up" and "down") along the yhistogram, and to 2 peaks ("left" and "right") along the xhistogram. Notably, for a hole along the ground surface, the down peak is the lowest bin in the y-histogram.

Figure 8. Clustering boundary points of the openings







b) X-coordinate histogram of boundary points along the x-direction

c) Y-coordinate histogram of boundary points along the y-direction

Figure 9. Using histograms to determine height and length of a window

Finally, holes are categorized as occlusions, if their characteristics differ from a predefined minimum opening dimensions greater or equal to 0.4m (as established by Pu & Vosselman 2007) and a height (H_o) to length (L_o) ratio greater than 0.25 and less than 5.0 (as established by Mayer & Reznik 2005; Ripperda 2008) (Equation 3). Thereafter,

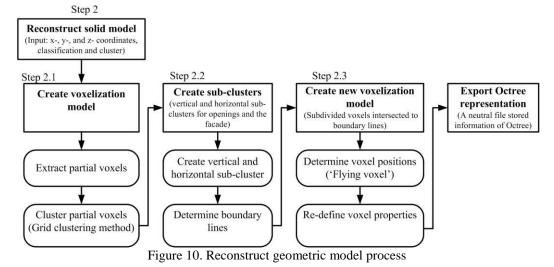
boundary points of occlusions are reclassified as interior points. In order to eliminate unrealistic holes, characteristics of detected holes are compared to ones of openings in building structures, in which the opening is assumed as rectangular; height and length are used post identification.

$$f(H_{o}, L_{o}, H_{o}/L_{o}) = \begin{cases} i fH_{o} \ge 0.4m; L_{o} \ge 0.4m; \\ 0.25 \le H_{o}/L_{o} \le 5.0 : \text{Opening (3)} \\ O \text{ her wise : Non - opening} \end{cases}$$

3. 2 Solid model reconstruction (Step 2)

The aim of Step 2 (Figure 10) is to reconstruct a solid model of the building façade from raw data points adding a

point classification of either "interior" or "boundary" as defined in Step 1. In surface reconstruction from a set of sample point, an object's surface-based triangulation consists of curved surfaces, instead of the generally planar surface of building façades. Surface-based triangles may cause difficulty in generating FEM meshes or result in distorted FEM meshes leading to unstable numerical solution. For these reasons, voxelization is implemented into the FA algorithm to generate a complete building model.



Initially, a bounding box enclosing the building's entire façade is established. Often, a bounding box has equal edge lengths but is defined herein by equations (4) and (5) thereby implying the possibility of non-cubic voxels.

$$L_{BB} = X_{m \ a \ \overline{x}} - X_{m \ i \ i} \tag{4}$$

$$H_{BB} = y_{m \ a \ x} - y_{m \ i} \tag{5}$$

where $x_{\text{max},}\ y_{\text{max},}\ x_{\text{min},}\ y_{\text{min}}$ are minimum and maximum coordinates of input sample points.

In the proposed FA algorithm, a voxel's size along its short side (either longitudinal or vertical) of the façade is less than half of the minimum feature size (e.g. if the minimum opening dimension is 0.4m, voxel size must be less than 0.2m.). Thus, the required octree depth along x- an y-directions can be given:

$$dept h_x = \left[log_2 \frac{L_{BB}}{Mi \, nVoxel \, Si \, ze}\right]$$
 (6)

$$dept h_{y} = \left[log_{2} \frac{H_{BB}}{Mi \, \text{Noxel Si } ze} \right]$$
 (7)

in which MinVoxelSize = 0.2 m and the sign [] means that the value is rounded up toward the nearest integer. As the octree depth is unique, the maximum octree depth for recursively subdividing in this proposed approach can expressed as:

$$max_Octree_depth = max(depth_x, depth_y)$$
 (8)

Subsequently, an initial voxel is subdivided along the xand y-directions into four smaller voxels. Herein, the voxel is then categorized as "Empty", "Full" or "Partial". The voxel is "Empty", if it contains no data points, or "Full" if it contains exclusively interior points. The last category is "Partial", if the voxel contains interior and boundary points or only boundary points. In the three sample buildings to be presented in the experimental section a depth of 8 was found to be ideal for the smaller structures and a depth of 9 for the larger.

Boundary lines of the building façade and its openings are determined based on boundary points underlying partial voxels. To achieve this, a group of empty voxels is assumed to be inside an opening, and then the partial voxels around the opening are clustered using a flood-filling algorithm (Agoston, 2005), in which 8 voxels connected to the selected voxel are checked.

The assumed quadrilateral shape of the façade and its openings implies that boundary points of each side can be extracted from vertical and horizontal grids of partial voxels by using a grid clustering technique. A new grid cluster is generated, if the boundary points within the partial voxel of the grid satisfy the following conditions: (1) a maximum distance between two boundary points in the grid is not less than the minimum opening size, and (2) a minimum distance between two boundary points belonging to two adjacent partial voxels is not greater than half of the opening size, in which an opening size of 0.4 m is adopted. By im-

plementing a constrained condition in the grid clustering technique, incorrect boundary points due to noise in the data can be eliminated, because those boundary points may be stored as partial voxels on fragment girds. Often, the boundary points for reconstructing boundary lines lie in the same grid voxels, which implies a low noise level for these points.

Furthermore, these grid clusters are classified as sub-cluster voxels containing boundary points representing boundary lines for each opening's side by comparing coordinates of the grid cluster voxels to coordinates of the opening's centre (Figure 11a). For example, a window has four subcluster voxels corresponding to the four boundary lines (two vertical, one bottom, and one top), whereas a groundfloor door normally includes only three representing to three boundary lines (no bottom boundary lines). For generating two vertical lines and a bottom boundary line, these line segments were determined based on coordinates of all boundary points contained within the sub-cluster's voxels by using a least median of squares approach (Pighin & Lewis, 2007, Fleishman et al., 2005). To obtain realistic vertical and horizontal boundary lines, fitting lines were determined from a set of boundary points in each sub-cluster $S = \{p_i = (x_i, y_i, z_i) | 1 \le i \le n\}$ as shown in equation 9 and 10.

$$x = a$$
 for vertical lines (9)

$$y = b$$
 for horizontal line (10)

where the parameters a and b were determined by minimizing the median of the residuals:

$$\underset{a}{\operatorname{arg\,m\,i\,n\,medi\,an}} \left| a - x_{i} \right| \text{ for vertical lines}$$
 (11)

$$\underset{b}{\operatorname{argminmed i anb}} - y_i | \text{ for horizontal lines}$$
 (12)

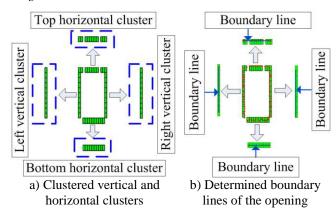


Figure 11. A modified grid clustering technique is employed to cluster vertical and horizontal voxels and to determine boundary lines of the opening

For the detailed process for parameter determination see Pighin & Lewis (2007) and Fleishman et al. (2005). This technique can eliminate 50% of all outlier points (Fleishman et al., 2005). Additionally, as the top boundary lines of real openings may be straight (as in rectangular openings,) bi-linear (as in wedge openings) or curved (as in arched openings), the y-coordinates of the end points of the top line were determined from average y-coordinates of these boundary points, while x-coordinates of the end points were set equal to two outlier boundary points along horizontal direction. A similar process was applied for determining the façade's boundary lines.

The full voxels were then stored in a database to describe the geometric model of the solid wall for computational modeling. Thus, the properties of the voxels must be reclassified based on their positions. Namely, the empty voxels are either "inside" openings (interior to a set of an opening's boundary lines) or outside the façade (exterior to a set of the façade's boundary lines). To achieve this, a further voxelization is created by dividing the initial voxelization model (Figure 12a) by the boundary lines (Figure 12b). For this, the number of child voxels depends on the number of boundary lines intersecting a parent voxel. For example, four child voxels would be created, if two boundary lines intersected the parent voxel. Conversely, no subdivision would occur, if no boundary line intersected the parent voxel, or if boundary lines(s) coincided with surface plane(s) of the parent voxel.

In re-voxelization (Figure 12c), voxels inside of openings or outside of the façade are now labeled as empty and all others as full. For that, the "Flying Voxel" method (Truong-Hong et al., 2012) is employed to quickly determine one of three possible positions with respect to various boundary lines: Case 1 - voxel outside of the façade; Case 2 - voxel inside the façade and inside an opening; or Case 3 - voxel in-side the façade but not inside any opening. This approach is summarized in Truong-Hong et al. (2012).

Finally, all full voxels in the final voxelization model are converted into a neutral file for computational modeling into the commercial FEM code such as ANSYS Mechanical APDL Product (ANSYS Academic Research Release 13.0). Topology and geometry of the full voxels are converted to a Boundary Representation (B-Rep) scheme that defines a solid model, similar to that done by Hinks (2011) but with the voxel criteria described herein.

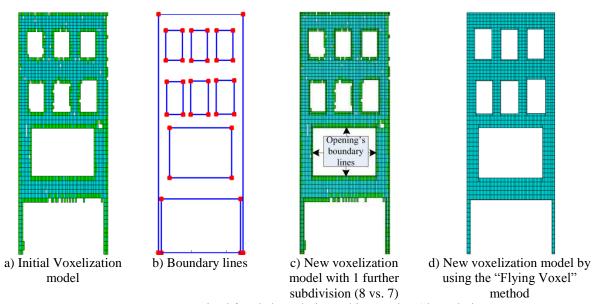


Figure 12. Determined façade boundaries and its openings' boundaries

4 EXPERIMENTAL RESULTS AND DISCUSSION

To validate the algorithms, four datasets were created for each of three brick buildings in Dublin, Ireland (Figures 13a-15a), which were selected for their proximity to an upcoming metro project and the availability of independent survey measurements. TLS point cloud data were collected using a Trimble GS200 device. Point clouds of each building acquired from multiple scanner stations because of traffic, terrain limitations, and footpath space were registered by using RealWork Survey (RWS) V6.3, proprietary software associated with the Trimble GS200 scanner. In this survey, two scanner stations were set up for each building. A minimum of three reference regions for each scanner station was acquired at 2 mm resolutions in order to merge pointclouds of the façade, in which the reference regions were chosen at well-defined positions such as window corners or window ledges. A trial and error merge process was manually undertaken by selecting a pair of points from the source and target stations until the average error of each target point expressed in term of distance errors was less than 5mm. Additionally, sampling point clouds were obtained by using a random sampling function built on RWS, in which a required distance between two adjacent sample points was defined.

The first set was the original scans (NS00) after being coregistered and having points +/-20 cm behind the expected building façade removed. The other three were subsets using random re-samplings with expected distances between sampling points of 20mm (S20-2500pts/m²), 50mm (S50-400pts/m²) and 75mm (S75-175pts/m²) [Table 1]. The densities were selected to test algorithm sensitivity.

For extracting only sample points of considered facades, a MATLAB subroutine incorporated with MATLAB libraries

was developed. Point clouds of adjoining building were removed. Similar to work of Adan and Huber (2011), the façade surface was determined from a histogram peak along depth direction. For that, points ± 20 cm from the facade were considered as sample points of objects behind the façade (e.g. floors, internal walls and objects) or non-functional structures in front of the façades and were manually removed. Finally, the ground level was also determined as a peak of a histogram along vertical direction.

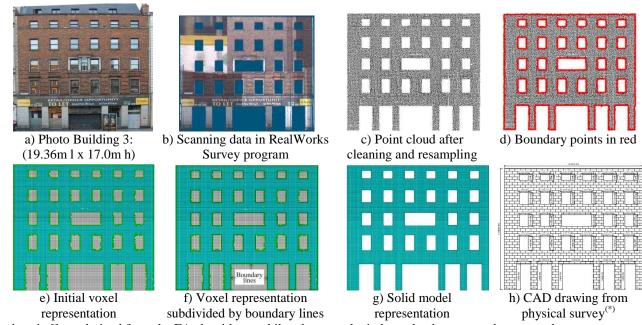
Table 1. Dataset Sizes

Building	Sampling dataset				
Dunding	NS00	S20	S50	S75	
B1: 2 Anne St. South	264,931	51,171	9,909	4,643	
B2: 5 Anne St. South	190,865	51,884	11,119	5,366	
B3: 2 Westmoreland St.	650,306	353,848	71,155	35,468	

The FA algorithm was implemented in MatLab program (MathWorks, 2007), using TLS data in ASCII format for point cloud coordinates. Experimental tests were run on a Dell Precision Workstation T5400 with Intel (R) Pentium (R) Xeon (8CPU) CPU speed 2GHz with 24 Gb RAM. For the boundary point detection, 20 kNN points and 0.4 m of a radius threshold were preselected for searching kNN, and an angle threshold of $\pi/2$ was preselected for the angle criterion. Each building had four geometric models: B1FANS00 describes the geometric models of Building 1 (B1) reconstructed by using the FacadeAngle algorithm on the original dataset (NS00), while B2FAS20 describes the same applied to the sampling point S20 (distance of 20mm between points) dataset for Building 2 (B2). Due to space limitations, only one set of solid models is graphically presented herein for each building.



Figure 14. Facade reconstruction for Building 2 based on a dataset of 400 pts/m² (S50-distance between adjacent sample points no less than 50mm)



(*) values in [] are derived from the FA algorithms, while others are the independently measured survey values Figure 15. Facade reconstruction for Building 3 based on a dataset of 175 pts/m²
(S75-distance between adjacent sample points no less than 75mm)

5 GEOMETRIC VALIDATION

Geometric accuracy of building façades in figures 13a, 14a, and 15a derived from the FA approach was compared to those processed in the commercial program, Kubit (1999). In this comparison, the building facades from on-site survey

are considered for benchmarking quantities. In Kubit, the solid models were created within an AutoCAD program by manually indentifying openings and building boundaries based on a building's photograph. Building façades generated from the Kubit program based on input datasets in Figure 13c, 14c and 15c are shown in Figure 16.

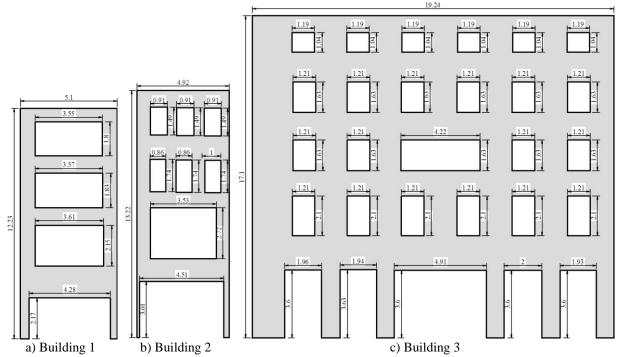


Figure 16. Building façades created by Kubit program based on input datasets in Figure 13c, 14c and 15c

To evaluate accuracy, the geometries of the derived building models were compared to measured drawings from independently produced on-site surveys (Figure 13h, 14h and 15h). In general, length of building models derived from the FA algorithm were slightly underestimated while ones from the Kubit algorithm were overestimated and to a greater extent, when compared to the real buildings underlying CAD drawings (Figure 17a). The maximum relative error in length by the FA algorithm underestimated Building 2 by 1%, while the maximum relative error by the

Kubit program overestimated Building 1 by 3% (Figue 17a). Similarly, the FA algorithm also tended to underestimate the building height, whereas the Kubit program tended to overestimate it. While, the relative errors of the building height from the FA algorithm were slightly

higher than ones from the Kubit program, the maximum relative errors were no more than 1.2% (Building 3) for the FA-based models versus 0.6% for the Kubit-based models (Figure 17b), with the benefit of the FA algorithm being fully automated.

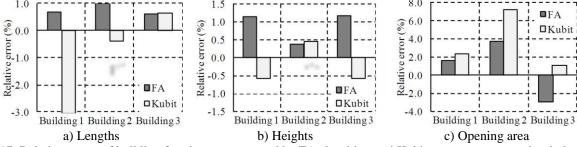


Figure 17. Relative errors of building facades reconstructed by FA algorithm and Kubit program compared to independently measured drawings

In terms of opening areas, for the small buildings (Building 1 and 2), the FA algorithm was more accurate (maximum relative error of 3.7% for Building 2) than the Kubit program generated ones (maximum relative error of -7.2% for Building 2), as shown in Figure 17c. However, for the larger Building 3, the opposite trend was found with the FA algorithm overestimating the opening area by 3% and the Kubit program underestimating by 1% (Figure 17c). As the resulting geometry of the building components (e.g. openings) may or may not affect building response depending upon the application, the geometric accuracy of the components was also evaluated.

By considering the most important building components in masonry buildings (e.g. openings), the approach was also shown to be comparable. The average absolute errors of the opening dimensions in the FA-based solid models were generally less than the Kubit-based ones (Table 2), in which the average error was 12.7mm [Standard deviation (SD)=121.1mm] in Building 2or the FA-based models, while 75.6mm (SD=102.7mm) for the Kubit-based ones. However, for the larger Building 3, opening dimensions in the Kubit-based model were more accurate than those from the FA-based models. For that, the average errors were -30.3mm (SD=54.1mm) and 4.5mm (SD=58.6mm) for the FA- and Kubit-based models. In conclusion, the FA algorithm more commonly automatically generated building models of superior accuracy than those created by the operator-assisted Kubit ones. This was likely caused by the fact that the Kubit program depends on visual interpolation by the user, while the FA algorithm generates the boundaries of the façade and its openings strictly from sample points.

Table 2. Geometric differences between CAD drawings against the FA and Kubit-based solid models

Aspects	Building 1		Building 2		Building 3	
(mm)	FA-S20	Kubit	FA-S50	Kubit	FA-S75	Kubit
Average	8.6	28.8	12.7	75.6	-30.3	4.5
Min. error	-109.0	-110.0	-165.0	-40.0	-177.0	-140.0
Max. error	179.0	250.0	359.0	370.0	45.0	70.0
Stand. dev.	111.4	110.0	121.1	102.7	54.1	58.6

5.1 Quality of boundary point detection

The proposed FA algorithm consistently detected all openings for each of the three building facades (Figure 13-15). Using 20 kNNs, sufficient boundary points were found on the façade boundaries and its openings to reconstruct realistic boundary lines (Figure 13c, 14c and 15c), with improved detection for denser datasets (Figures 18a-b vs. Figures 18c-d). Notably, the FA algorithm can detect all boundary points around openings corners, a shortcoming that occurs in the Delaunay triangulation based FD algorithm (Truong-Hong et. al, 2011) [Figure 19] or with use of a shape criterion (Becker & Haala, 2007). Additionally, the FA algorithm also detected approximately twice the number of boundary points on building's features than the FD algorithm (Figure 18 vs. Figure 19), even in the presence of occlusions (Figure 18a black circle). This opens the way for further processing options or further density reductions, which may soon enable use of existing ALS data for solid model generation of building facades.

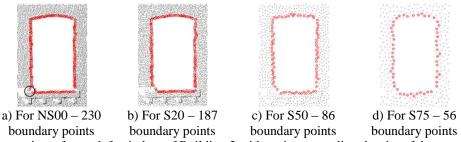


Figure 18. Boundary points of a top left window of Building 2 with various sampling density of datasets to show parameter sensitivity

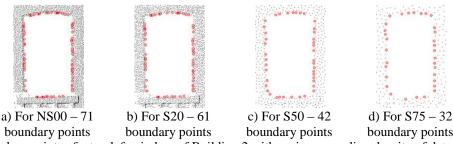


Figure 19. Boundary points of a top left window of Building 2 with various sampling density of datasets from the FD algorithm (adapted from Truong-Hong et al. 2012)

5.2 Processing time

The majority of the total processing time was devoted to feature detection (Figure 20). With a dataset of 190,865 points [as shown as 5.28 (log190,865) on x-axis of Figure 20], the feature detection took 102.3 minutes while voxelization required only 1.8 minutes (Figure 20). This is because the feature detection algorithm must pass through the entire dataset, whereas the voxelization process is mainly searching through partial voxels to reconstruct boundary lines of building's features. The voxelization process depends on not only depth of octree representation but also the number of openings needing to reconstructed. For example, the process took 0.8 minutes for Building 2 (51,884 sample points, 8 openings and depth of octree representation by 8) but 6.28 minutes for Building 3 (71,155 sample points, 28 openings and depth of octree representation by 9; the greater size of Building 3 required an additional level of division).

While, the FA and FD algorithms (Truong-Hong et al., 2012) were nearly equivalent in speed, particularly for datasets less than 350k sample points, the FA algorithm may be further optimized, by having the feature detection portion of the algorithm search only on the sample points around openings instead through all sample points, as currently reflects its implementation.

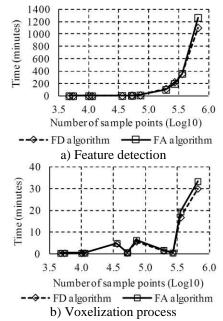


Figure 20. Running time of the algorithms.

To investigate efficiency of the proposed FA algorithm and Kubit program in reconstructing building models, the dataset of S75 was selected and the results of the building models was illustrated in Figure 13-16. Processing time for manual cleaning irrelevant points with the RealWork Survey is 5 minutes for all datasets while ones for reconstructing the building models is shown in Figure 21 (below). In general, the FA algorithm is faster than the Kubit program, particularly for small buildings (Building 1 and 2) having less number of openings.

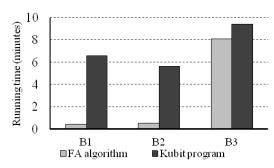
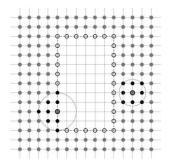


Figure 21. Comparing processing time between FA algorithm and Kubit program based on dataset of S75

5.3 Influence of critical input parameters

In the angle criterion, the vital parameters were identifying the critical angle and selecting a suitable neighborhood. In this study, the critical angle was set equal to $\pi/2$, which worked well, even in the corners, because the data points were fairly regularly distributed. False results occur, if the number of kNNs is set either too low or too high. As such, as discussed previously, for cases of 8 or 22 kNNs selected, detection of either interior or boundary points would be incomplete (Figure 6). By assuming a perfectly distributed rectangular data set, 8 kNN points would be sufficient for boundary point detection (Figure 22 with sample points in circles), but real data are not distributed as such. Thus, 20 kNN points were selected for all datasets, corresponding to 0.2m of the ball's radius of the kNN points for 75mm sampling dataset. This selection provides rapid boundary point detection while avoiding accidental sample point collection on adjacent openings. Additionally, the 0.2m radius enabled robust clustering of all boundary points on the boundaries of the façade's features for each of the 12 datasets.



Note:

- * Dark gray points: sample points
- * Black gray points: given points
- * Unfilled circle points: boundary points
- * Black points: neighbor points
- * Circles: selecting 8 kNN points
- * Dash lines: an expected window boundary lines

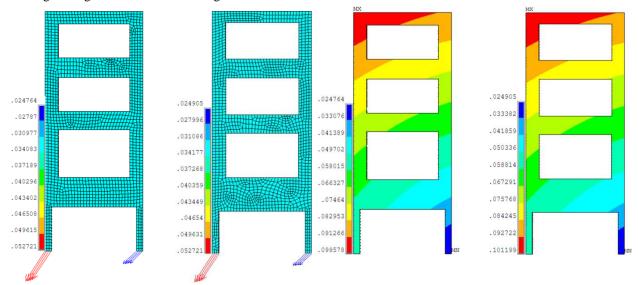
Figure 22. Selecting neighboring points in rectangular distribution of sample points

6 NUMERICAL ASSESSMENT

As the main goal of the FA algorithm was to reconstruct solid models of existing building façades for computational modeling, compatibility of the solid models and the impact of the aforementioned geometric discrepancies on numerical results must be discussed. To evaluate the usability of these models for a relevant case, the responses of the FEM models derived solid models from the FA algorithm were compared to ones based on CAD drawings from on-site drawings submitted for planning permission. In this section, the solid models of Building 1 respectively shown in Figure 13g and 13h for the FA algorithm and the

CAD drawing were selected for further investigation.

Non-linear analysis was adopted for analyzing the solid model of Building 1 by using ANSYS Mechanical APDL product (ANSYS Academic Research Release 13.0), where a macro modeling strategy was employed to model the building facade by using a SOLID65 element. Additionally, a William Warnke (WW) failure criterion and Drucker-Prager (DP) yield criterion built into the ANSYS program are respectively to model masonry behavior in tension and compression. Thus, the WW failure criterion provides a tension cut-off for the DP yield criterion (Truong-Hong and Laefer, 2008). Material properties were selected from existing experimental reports and the peer-reviewed literature to represent medium-strength masonry properties used for this analysis. These were as follows: for elastic behavior Young's modulus of 3,480 MPa and Poisson's ratio 0.16 and for plastic behavior for 26.15/1.15 MPa of compressive/tensile strength, 6.81 MPa internal cohesion, 35° internal friction angle and 10° dilatancy angle. The analysis was conducted under self-weight and imposed displacements due to excavation-induced foundation settlements (Truong-Hong 2011), in which an element size of 0.15 m was predefined for FEM mesh generation, and displacements were directly applied to nodes on the bottom of the model (Figure 23a and b). As discrepancy of building length (Figure 17a) and the building edge is set at 2 m behind excavation face (Truong-Hong, 2011), there are small different displacements imposed on FEM models based on CAD drawing and the FA algorithm, where the minimum displacements were respectively 24.764 mm and 24.905 mm (Figure 23a and b).



a) FEM mesh & imposed displacement – CAD based displacement – FA based model – CAD based FA based FA based

Figure 23. Finite element models and numerical results from CAD and FA based models

Graphically, the numerical analysis showed a consistency of nodal displacements between the FEM models based on two sources of the solid models (Figure 23c and d). The maximum nodal displacement differed by no more 1.6%, with an absolute difference of only 1.6 mm (Figure 23). In terms of an engineering perspective, this difference in FEM

results was well below accepted uncertainty levels within structural design [e.g. the Load and Resistance Factor Design specification allows a nominal force effect increase of 5% to consider ductility, redundancy, and operational importance (Hoffman et al., 1996)]. Thus, the algorithm proposed herein can be used for auto-generating

computational models from TLS data.

7 CONCLUSIONS AND FUTURE WORK

The newly proposed FacadeAngle algorithm combines an angle criterion, voxelization, and the positional determining Flying Voxel method to automatically extract building façades and their major features from LiDAR point cloud data for further computational modeling. By use of an angle threshold combined with a pre-specified searching radius and a pre-selected number of nearest neighbor candidates, the algorithm performed within acceptable limits in a wide variety of tests against a semi-automatic, commercial alternative on three urban buildings at 4 data densities. Favorable results were also obtained for geometric variance of the overall form and individual elements compared to solid models derived from measured drawings, as well as for resulting displacements and stresses in an adjacent excavation scenario (within 1.6% or 1.6 mm). Furthermore, even without full optimization, the processing time was comparable to another voxel-based automated approach and was also able to automatically overcome holes caused by data occlusions. In addition to these favorable aspects, the strategic advantages that the FacadeAngle potentially possesses over other approaches are as follows:

- Ability to precisely detect all boundary points around the corners of openings, except where occlusions exist
- 2. Capability to harvest twice the number of boundary points over other automated methods
- 3. Item 2 offers the potential for further processing for more exact results and/or for the abandonment of any a priori knowledge (e.g. general window sizes); something not yet obtainable in any approach
- 4. Item 2 also offer the potential use of less dense data sets, such as aerial ones in the near term as there is a difference of two orders of magnitude between current high level aerial data when vertically projected and the typical terrestrial scan.

However, the proposed algorithm needs to be extended to non-rectangular forms, optimized for use with large and sparse datasets, and needs to be freed from reliance on prior knowledge. Finally, increasing automation and applicability of this method will require its extension to fully 3D models and its integration with a procedure appropriate to eliminate irrelevant sample points prior to the initial processing.

ACKNOWLEDGMENTS

This work was made possible through the generous funding of Science Foundation Ireland (SFI/PICA/I850). Thanks to Donal Lennon of Urban Institute Ireland for assistance with data acquisition.

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