Sliding mode control for vision based leader following

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***Abstract*— In this paper a discrete-time sliding mode approach for the control of nonholonomic robots performing a leader following task is presented. The task can be seen as a service task for bringing robots to a working area. Our aim is to design, develop and test a new software and hardware framework for formation and coalition problems in a mobile multi robot scenario. Given a desired robot coalition formation, a visual servoing approach is used for tracking. The proposed robust control strategy is also used together with the visual system to recover from failures (when the tracker fails). Real robots experiments are reported, showing the performances of the proposed vision-based follow-the-leader control strategies in an indoor environment.**

***Index Terms*— Multi-Robot Systems, Motion Planning, Navigation and Vision.**

I. Introduction

The last few years have seen active research in the control and coordination of multiple mobile robots, with applications including exploration [4], surveillance [3], search and rescue [7], mapping of unknown or partially known environments, distributed manipulation [11], [9], and transportation of large objects [12], [13]. The advantages of using multiple agents to solve problems, either by a simple division of the labour or trying to economize on the cost for developing agents by dividing up specialized skills, are well known. Many approaches have been developed for decomposing problems, allocating tasks within a group, and combining results, using expertise from a wide array of fields, from game theory to sociology.

In this paper we address the problem of leader following in the case of a heterogeneous multi-robot team. This task requires robots to move in a pattern, each following the preceding robot, with the first robot teleoperated, following a human operator or, as in our experiments, going through a predetermined path. The task can be seen as a service task for bringing robots to a working area. Of course the team is required to perform in a robust way, so that the team is able to correctly operate even if some external, undesired or unforeseen event occurs. Pose estimation is based on the tracking of the color regions that characterize the robot, without any special landmark. Thanks to recent advances in computer vision, one can now address formation control without using explicit communication. For example, Vidal et al. [15] consider a formation control scenario for omnidirectional image based visual servoing [6, 1, 8] in which motion segmentation techniques enable each follower to estimate the image plane position and velocity of other robots in the formation. Determination of the position and orientation of the follower robot can be achieved by estimating its distance and relative orientation with regard to the leader robot. Only the position (x, y) and the orientation θ need to be estimated. In the proposed setup, cameras are fixed on the platform, and their pan angle can be read from the platform. Thus, the position of the observed robot is obtained from its estimated distance and the measured angle. Besides, the orientation of the observer robot and of its camera does not need to be the same. The trajectory tracking control system for a wheeled mobile base considers the presence of disturbances that violate the nonholonomic constraint and uses an approximated discrete-time Sliding Mode Control (SMC) for the vehicle [14, 10]. The proposed solution is based on a discrete-time SMC to ensure that the controller is both robust and feasible. Further details about the trajectory tracking control system can be found in [2].

Experimental results with two robots confirm the validity of the presented approach. Starting from position $(0, 0)$, the leader robot moves along a pre-programmed path. The follower robot starts half meter away from the leader, searches for the leader turning around and starts following the leader. The task programmed into the robot is to keep the distance to the target constant. The robot can maneuver in flat surfaces only and the position of the robot is determined by dead reckoning only. Although it is not necessary to keep a record of the distances moved by the robot in this application.

Main contributions of this paper are in the overall control strategy, in the failure recovery system, in the use of a simple and very fast vision based servoing approach and in the validation of the proposed methods on a real indoor scenario.

The paper is organized as follows: the next section introduces the general multirobot system that uses the proposed leader following sub-system. Section III introduces the control system and Section IV the visual servoing approach. Experimental results with real robots are presented in Section V, before conclusions and future works.

II. System overview

As multi-robot systems evolve, multi-robot tasks are becoming more complex. The increase in task complexity results in situations where a task cannot be completed by a single robot and it becomes necessary to assign a team of robots to an individual task. Multi-robot coalition formation deals with the issue of how to organize multiple robots into subgroups to accomplish tasks collectively. The motivation behind coalition formation is to enable the team members to work together as a group to accomplish tasks that cannot be handled by individual robots.

Here we are not interested in high level problems, such as when it is appropriate to form a coalition and how the robots should cooperate within a coalition, but our aim is to design, develop and test a new software and hardware framework for formation and coalition problems in a mobile multi robot scenario, useful for indoor and outdoor applications. The coalition can involve different kinds of robots equipped with different sensors: vision, sonars, lasers and GPS. In particular only few robots have all the sensors and serve as leader of the whole team, providing higher level information, such as mapping or exploration. All the robots are equipped with vision sensors, which ensure a great quantity of information at low cost.

In this paper we present results of the study and experimental validation of a vision based tracking control approach to a fundamental task of a robot coalition: leader following.

While this is sufficient for column formations, as future developments we intend to solve the same problem for more coalition formations, such as wedge formations (Fig.1). Besides, we planned to develop and test a leader capable of localizing itself in a fully or partial explored environments. Then, a set of followers using a visual feedback will localize themselves with respect to the leader, eventually using the emergent methodology Cooperative Simultaneous Localization And Mapping (CSLAM), thanks to the visual feedback of each robot.

Following sections give a detailed description of the methodological approach and the results of the actual experiments: a follower using robust control techniques and vision tracking.

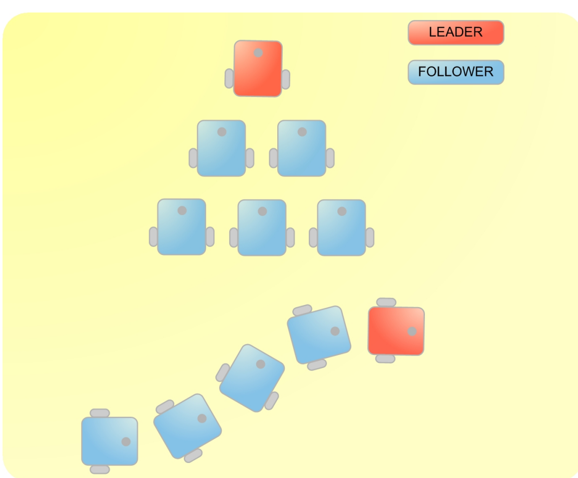

Fig. 1. Typical formations of leader and follower robots; wedge (top of figure) and column.

III. Control Problem

In this section we present a set of control solutions; the main aim is not the generation of an optimal path (which can be done using optimal path planning algorithms), but the tracking of a pre-determined trajectory. An abstraction of a common unicycle mobile robot is represented in figure 2. In the case

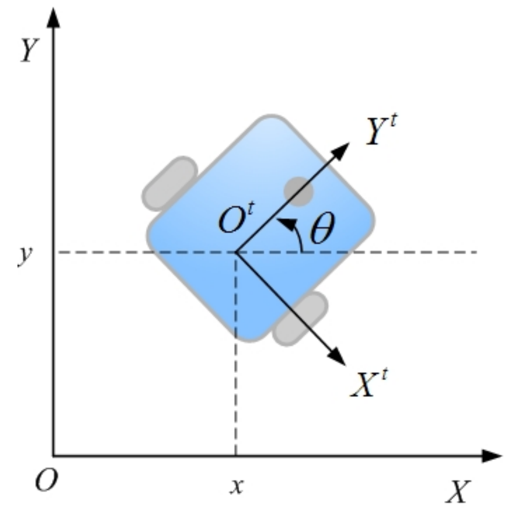

Fig. 2. A graphical representation of an unicycle mobile robot.

of motion on a plane, the robot is fully described by a three-dimensional state constituted by x and y and θ, respectively the position of midpoint (defined as the median point on the axis that links the two drive wheels) and heading respect to a fixed frame. We define $v(t)$ and $\omega(t)$ as longitudinal and angular velocity. The well known kinematic model is represented by (1).

$$\dot{X} = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & 0 \\ \sin(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \tag{1}$$

When skidding and slipping effects are absent the robot is subjected to a nonholonomic constraint (2) that reduces the degrees of freedom from three to two.

$$\dot{x}(t)\sin(\theta(t)) - \dot{y}(t)\cos(\theta(t)) = 0 \tag{2}$$

When the skidding and slipping effects are present the nonholonomic constraint is violated and, under some assumptions [2], (2) becomes:

$$\dot{x}(t)\sin(\theta(t)) - \dot{y}(t)\cos(\theta(t)) = \mu(t) \tag{3}$$

where $\mu(t)$, which measures the constraint violation, is bounded by a known constant ρ so that $|\mu(t)| \leq \rho$.

The proposed model is in the continuous time domain; model must be discretized because the domain of real test is discrete. The sampling time T_s must be chosen small enough so that linearization errors of nonlinear terms are small in the period from kT_s to $(k+1)T_s$. Under this assumption and considering a zero order hold for the control input vector $[v(t), \omega(t)]^T$ the result of discretization is shown below:

$$\begin{cases} x_{k+1} = x_k + T_s v_k \cos(\theta_k) + T_s \eta_k \sin(\theta_k) \\ y_{k+1} = y_k + T_s v_k \sin(\theta_k) - T_s \eta_k \cos(\theta_k) \\ \theta_{k+1} = \theta_k + T_s \omega_k \end{cases} \tag{4}$$

where the term k means that the variable is evaluated at time $t = k \cdot T_s$.

Problem: *Given a trajectory, the problem is to find a discrete-time feedback controller that ensures a robust trajectory tracking and a bounded position error.*

In the following subsection we introduce two different solutions to solve the tracking problem. The first technique is based on the inversion of the kinematic model; the second one instead uses a sliding mode control approach with robustness properties.

A. Inverse Kinematic Control

The tracking of a trajectory accomplished by the inversion of the kinematic model is a simple methodology. For the unicycle robot the inverse model, after some algebraic manipulations, can be expressed as:

$$\begin{cases} v_d(t) = \dot{x}_d(t)\cos(\theta(t)) + \dot{y}_d(t)\sin(\theta(t)) \\ \omega_d(t) = \dot{\theta}_d(t) \end{cases} \tag{5}$$

where $X_d = \begin{bmatrix} x_d(t) & y_d(t) & \theta_d(t) \end{bmatrix}^T$ is the assigned trajectory. The weak performances of this open loop controller are overtaken designing a feedback controller that considers the position error. This can be decomposed into two main components; tangential error and normal error. The controller is reflected through:

$$\begin{cases} v_d(t) = \dot{x}_d(t)\cdot\cos\theta(t) + \dot{y}_d(t)\sin\theta(t) + K_{et}e_t \\ \omega_d(t) = \dot{\theta}_d(t) + K_\theta(\theta_d(t) - \theta(t)) + K_{en}e_n \end{cases} \tag{6}$$

where K_θ, K_{et} and K_{et} are obtained experimentally by simulation and real tests. The results achieved by this approach are influenced by the high sensibility to errors; tracking fails also if the errors are "small". To improve the robustness of the control, a discrete-time SMC was developed.

B. Sliding Mode Control

Sliding Mode Control (SMC) has been widely used to control systems with parameter uncertainties and external disturbances [14, 10]. Before introducing the solution adopted, we define:

$$\begin{aligned} \Delta x_k^d &= x_{k+1}^d - x_k^d \\ \Delta y_k^d &= y_{k+1}^d - y_k^d \\ \Delta\theta_k^d &= \theta_{k+1}^d - \theta_k^d \end{aligned}$$

Tracking errors are defined as:

$$e_k^x = x_k - x_k^d \tag{7}$$

$$e_k^y = y_k - y_k^d \tag{8}$$

$$e_k^\theta = \theta_k - \theta_k^d \tag{9}$$

The dynamics of tracking errors is expressed by the following equations:

$$\begin{cases} e_{k+1}^x = e_k^x - \Delta x_k^d + T_s \mathrm{v}_k\cos(\theta_k) + T_s\eta_k\sin(\theta_k) \\ e_{k+1}^y = e_k^y - \Delta y_k^d + T_s \mathrm{v}_k\sin(\theta_k) - T_s\eta_k\cos(\theta_k) \\ e_{k+1}^\theta = e_k^\theta - \Delta\theta_k^d + T_s\omega_k \end{cases} \tag{10}$$

Let us introduce sliding surfaces σ_x and σ_y defined as:

$$\sigma_x = \left(e_{k+1}^x - e_k^x + \gamma_x e_{k-1}^x\right),\ \gamma_x \in (0, 0.25] \tag{11a}$$

$$\sigma_y = \left(e_{k+1}^y - e_k^y + \gamma_y e_{k-1}^y\right),\ \gamma_y \in (0, 0.25] \tag{11b}$$

Position error tracking is asymptotically bounded thanks to quasi-sliding motion on surfaces (11) and the properly choice of γ_x and γ_y. Quasi-sliding motion is accomplished using variable $v_k^{(x)}$ and $v_k^{(y)}$ that are not independent:

$$\begin{aligned} v_k^{(x)} &= v_k\cos(\theta_k) \\ \mathrm{v}_k^{(y)} &= v_k\sin(\theta_k) \end{aligned}$$

Then, another condition must be satisfied:

$$\theta_k = \arctan\left(\frac{v_k^{(y)}}{v_k^{(x)}}\right) \tag{12}$$

A sliding surface (13), used to impose the fulfilment of (12), is defined as:

$$s_k = \arctan\left(\frac{v_k^{(y)}}{v_k^{(x)}}\right) - \theta_k \tag{13}$$

So the angular tracking error e_k^θ is bounded thanks to a properly choice of $v_k^{(x)}$ and $v_k^{(y)}$.

Let us introduce the control law that assure the quasi-sliding motion on $\sigma_x = 0$, $\sigma_y = 0$ and $s_k = 0$ using $v_k^{(x)}$ and $v_k^{(y)}$.

Theorem 1: Given the tracking error system (7), the achievement of quasi-sliding motions on surfaces (11) and (13) is guaranteed by the following control law:

$$v_k = \tfrac{1}{T_s}\left(v_k^{eq} + \nu_k\right) \tag{14a}$$

$$\omega_k = \tfrac{1}{T_s}\left[\arctan\left(\frac{\Delta y_{k+1}^{(d)} - \gamma_y e_k^{(y)}}{\Delta x_{k+1}^{(d)} - \gamma_x e_k^{(x)}}\right) - \theta_k\right] \tag{14b}$$

with

$$p_k^x = \Delta x_k^d - \gamma_x e_{k-1}^x \tag{15a}$$

$$p_k^y = \Delta y_k^d - \gamma_y e_{k-1}^y \tag{15b}$$

$$\mathrm{v}_k^{eq} = \sqrt{\left(p_k^x\right)^2 + \left(p_k^y\right)^2} \tag{15c}$$

where

$$\nu_k = \begin{cases} \alpha\sqrt{\sigma_k - T_s^2\rho^2} & if \quad \sigma_k \geq (\rho T_s)^2 \\ \nu_{k-1} & if \quad \sigma_k < (\rho T_s)^2 \end{cases} \tag{16a, 16b}$$

where $\alpha \in (1-, 1)$. The proof of this theorem can be found in [2].

IV. Visual Feedback

"Leader presumes follower and follower presumes choice". This sentence expresses the logic that is at the base of our leader-follower approach.

In our case, we have two robots as shown in Fig.3; the tasks of each robot are different and based on their role:

- **Leader** has as primary task to track a pre-defined trajectory using the robust controller introduced in the previous section.
- **Follower** using a visual feedback provided by a webcam, tries to identify and track the trajectory of the leader.

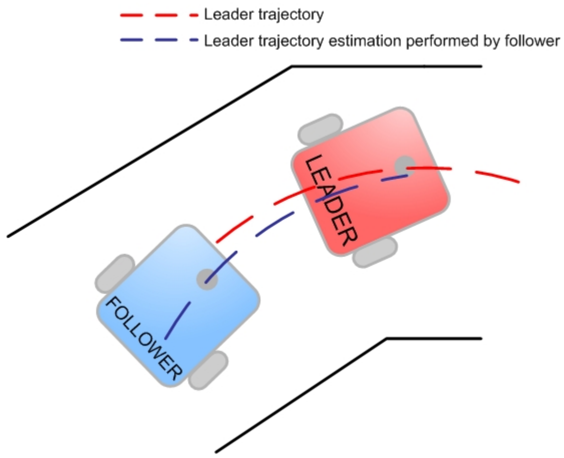

Fig. 3. A graphical abstraction of our leader following approach with robust controller.

Common aspects between the two robots are the use of the robust controller introduced in the previous section and the capability to avoid an obstacle with active sensors as sonars.

The follower uses a webcam to identify the position of the leader and makes an estimation of the trajectory.

The leader is identified in the image as an object with a known color (in our case red) as shown in Fig.4. To reduce the computational complexity, the real time image processing is very simple.

Fig. 4. A snapshot of webcam before color segmentation.

The ”target” is extracted from the image in two phases:

- color segmentation (on red channel);
- calculation of center of mass.

In Fig.5 it is shown an example of color segmentation based on an image sampled by the webcam mounted on board of the follower robot.

The estimation of distance is obtained using an hash-table; the values contained into this table are determined by an off-line calibration; at given distance d the height of center of mass of leader is constant. This property is true also for different positions of the leader in the scene, i.e. robot on the left or right. The calibration curve is reported in Fig.6.

When the leader is out of scene, then the follower sets immediately translational speed at zero and a search of leader is started using self rotation; this situation happens when the follower fails in the leader estimation/tracking or when it's kidnapped.

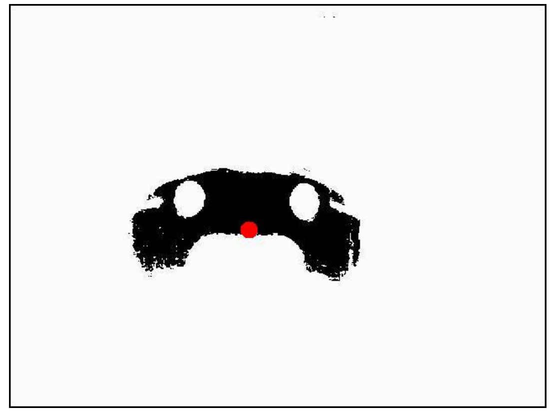

Fig. 5. The circle represents the center of mass calculated by the algorithm; the robot is in the center of the scene.

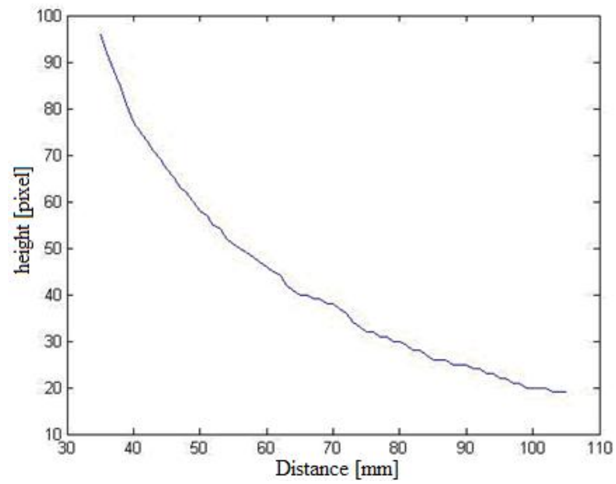

Fig. 6. Calibration curve used for estimation of leader position.

The proposed approach has a low computational weight because color segmentation and calculation of the center of mass can be quickly performed. A high resolution image is not tightly necessary and after some test was set to 160x120.

V. Tests and Results

A set of experimental tests was performed to validate and identify the limits of proposed control approaches based on inversion of kinematic model and sliding mode control. The experimental configuration is characterized by two robots as AmigoBot-SH and Pioneer P3-DX produced by MobileRobots.

Simulations were performed using Mobilesim simulator especially to validate the control laws; special care has been reserved for the calibration of webcam to improve the performance of visual feedback process.

Robots are remotely controlled using a WiFi link; the control software runs on matlab. To reduce time to simulation, we have developed a framework for simulations and tests of mobile robotics tasks; for further details see [5]. A typical test configuration is represented in Fig.7 .

The set of parameters used for simulations and real experiments in the case of inverse kinematic and sliding mode

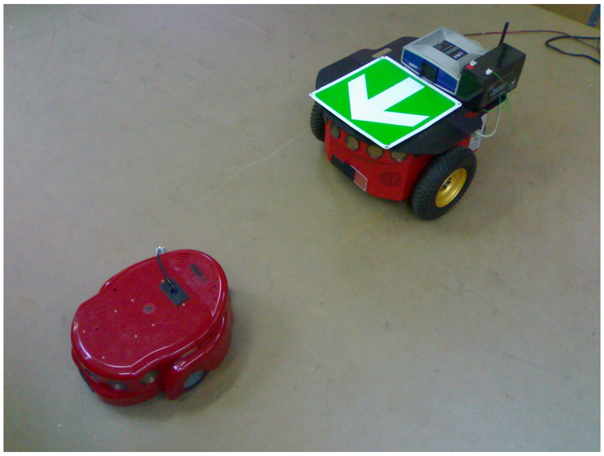

Fig. 7. Pioneer DX (upper right corner) is the follower and the Amigobot the leader.

control are reported in Table I.

Parameter	Value
Inverse Kinematic Controller	
K_{et}	1
K_{en}	1
$K_{e\theta}$	1
T_s	0.1s
Sliding Mode Control	
γ_x	0.05
γ_y	0.05
ρ	0.1
α	0.5
T_s	0.1s

TABLE I

LIST OF PARAMETERS ADOPTED FOR EACH CONTROLLER.

In Fig.8 and Fig.9 are plotted the results obtained. The performances of SMC controller are quite evident if we consider the norm of error as shown in Fig.9.

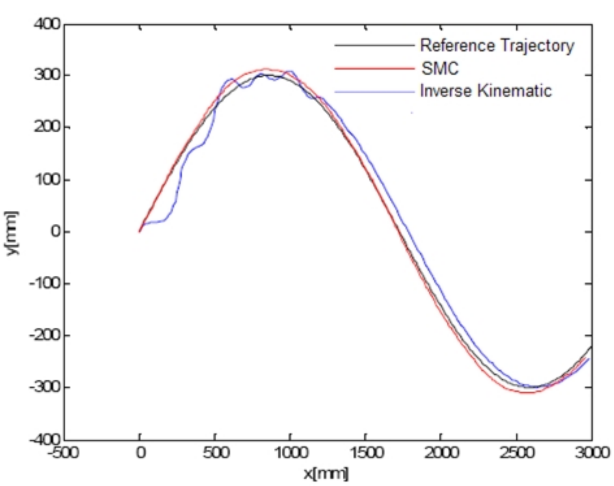

Fig. 8. SMC controller ensures a good performance during trajectory tracking.

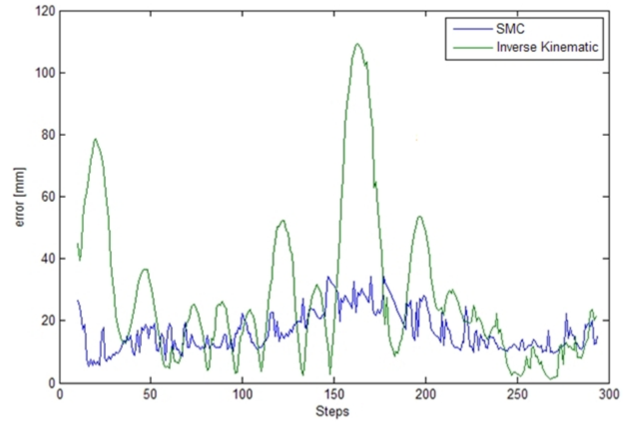

Fig. 9. Position error $e_k = \sqrt{\left(e_k^x\right)^2 + \left(e_k^y\right)^2}$ obtained by inverse kinematic controller is greater than SMC.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper we presented an approach to leader following using a robust controller with visual feedback.

A comparison with a classical inverse kinematic controller was performed highlighting the goodness of sliding mode control approach.

We used a visual feedback provided by a low cost vision sensor (standard webcam) to estimate the leader's trajectory and track it with good results.

All the control and image processing software was quickly developed using Matlab; adopting the developed framework [5], time to simulation was drastically reduced.

Future works will focus on the development of more sophisticated image processing algorithms; an omnidirectional camera will be installed to improve the performance of leader search into the scene. The use of conventional vision system for mobile robot control requires the camera to be positioned towards a center of attention in order to keep a subject in the field of view so that the robot can be visually servo controlled. This could be done by a two servo mechanisms to move the camera up and down and sideways independent of the robot's movement or the robot could move the camera towards the subject. In any of these cases some effort should be expended to the position of the camera besides the movement of the robot to accomplish its task. By the other hand, a vision system that can provide a 360° field of view can avoid camera movements and keep a subject on the field of view all the times independent of the subject's position around the robot. This type of vision system is known as omnidirectional vision system. This system can be achieved by the use of a fish eye lens and a CCD camera or by the use of a convex mirror and a conventional lens and CCD camera. The second type is going to be used to control a mobile robot in a closed loop visual guided control system to accomplish the task of tracking a select target in real time.

More complicated behaviours and the increase of the number of robots will be investigated and tested.

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