Distributed Quantum Computation via Optical Fibers

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We investigate the possibility of realizing effective quantum gates between two atoms in distant cavities coupled by an optical fiber. We show that highly reliable swap and entangling gates are achievable. We exactly study the stability of these gates in the presence of imperfections in coupling strengths and interaction times and prove them to be robust. Moreover, we analyze the effect of spontaneous emission and losses and show that such gates are very promising in view of the high level of coherent control currently achievable in optical cavities.

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The study of the possibilities allowed by coherent evolutions of quantum systems is central to quantum information science. Most notably, exploiting suitable coherent dynamics to implement deterministic quantum gates between separate subsystems is a basic aim for quantum computation. Several proposals have been suggested to engineer entanglement or quantum communication between atoms trapped in distant optical cavities, either through direct linking of the cavities [1–4], or through detection of leaking photons [5,6]. The realization of quantum gates between distant qubits in quantum optical settings has also been recently envisaged [7,8]. Such proposals are very promising and highly inventive. However, they are either probabilistic or relying on accurately tailored sequences of pulses (thereby requiring a considerable degree of control). In this Letter, an alternative to such schemes is proposed, with a particular focus on the implementation of distributed quantum computation. To this aim, we investigate the possibility of realizing deterministic gates between two-level atoms in separate optical cavities, through a coherent resonant coupling mediated by an optical fiber. The only control required would be the synchronized switching on and off of the atom-field interactions in the distant cavities, achievable through simple control pulses. The study of such a system (which would constitute the basic cell of scalable optical networks) is crucial in view of the outstanding improvements currently achieved in the control of single atoms trapped in optical cavities [9] and of the recent realization of microfabricated cavity-fiber systems [10].

In the considered system the interaction between the qubits is mediated by the bosonic light field. It has been shown that, in principle, an exact deterministic gate may be realized if the interaction between two qubits is mediated by another two-level system through XY nearest neighbor interactions [11]. If the central system is a bosonic field, though, interacting with the two qubits through a rotating wave Hamiltonian, a perfect gate is not possible, as the Rabi frequencies in the two- and single- excitation subspaces are no longer commensurate and the mediating field does not exactly decouple from the qubits at short enough times. However, as we will show, times do exist for which the qubits are decoupled from the field at a high degree of accuracy. The resulting effective dynamics of the two qubits can then be described in terms of quantum operations which approximate unitary gates with a high fidelity. The discrepancy between such approximate gates and the desired unitary ones would be negligible with respect to the errors involved by an experimental implementation of the scheme.

We consider two two-level atoms in distant optical cavities, interacting with the local cavity fields through dipole interactions in rotating wave approximation. The two cavities will be henceforth labeled by the indices 1 and 2. We will allow for a detuning Δ of the transition of atom 2 from the resonance frequency ω of the cavities (whereas atom 1 will be assumed to be at resonance). The cavities are connected by an optical fiber, whose coupling to the modes of the cavities may be modeled by the interaction Hamiltonian $H_{If} = \sum_{j=1}^{\infty} \nu_j [b_j (a_1^\dagger + (-1)^j e^{i\varphi} a_2^\dagger) + \text{H.c.}]$ [2], where b_i are the modes of the fiber, a_1 and a_2 are the cavities' modes, ν_i is the coupling strength with the fiber mode i, and the phase φ is due to the propagation of the field through the fiber of length l: $\varphi = 2\pi\omega l/c$ [12].

Now, let $\bar{\nu}$ be the decay rate of the cavities' fields into a *continuum* of fiber modes. Taking into account a finite length l of the fiber implies a quantization of the modes of the fiber with frequency spacing given by $2\pi c/l$. One has then that the number of modes which would significantly interact with the cavities' modes is of the order of $n = (l\bar{\nu})/(2\pi c)$ [2]. We will focus here on the case $n \lesssim 1$, for which essentially only one (resonant) mode of the fiber will interact with the cavity modes ("short fiber limit") [13]. Notice that such a regime applies in most realistic experimental situations: for instance, $l \lesssim 1$ m and $\bar{\nu} \simeq 1$ GHz (natural units are adopted with $\hbar = 1$) are in the proper range. We recall that the coupling ν to the modes of a fiber of finite length can be estimated as $\nu \simeq \sqrt{4\pi\bar{\nu}c/l}$. Let us also notice that the coupling strength ν can be increased by decreasing the reflectivity of the cavity mirror connected to the fiber. In the specified limit, the Hamiltonian H_{If} reduces to H_f

$$H_f = \nu[b(a_1^\dagger + e^{i\varphi}a_2^\dagger) + \text{H.c.}], \quad (1)$$

where b is the resonant mode of the fiber. The total Hamiltonian of the composite system can be written, in a frame rotating at frequency ω, as

$$H = \Delta|1_2\rangle\langle 1_2| + \sum_{j=1}^{2}(g_j|0_j\rangle\langle 1_j|a_j^\dagger + \text{H.c.}) + H_f, \quad (2)$$

where $|1_j\rangle$ and $|0_j\rangle$ are the excited and ground states of atom j, g_j is the dipole coupling between atom and field in cavity j (generally complex, as local coupling phases, depending on the positions of the atoms in the cavities, might be present), and Δ is the detuning of the transition of atom 2. The addressed system is thus equivalent to two qubits connected by a chain of three harmonic oscillators. For ease of notation, let us also define $g \equiv |g_1|$, $\delta \equiv |g_2| - |g_1|$ and $\sigma_j^- = |0_j\rangle\langle 1_j|$ for $j = 1, 2$.

Before proceeding, let us remark on an interesting feature of the Hamiltonian H, which unveils some significant insight about the dynamics we intend to study. Let us consider the normal modes c and $c_\mp$ of the three interacting bosonic modes. One has $c = (a_1 - e^{-i\varphi}a_2)/\sqrt{2}$, with frequency ω, and $c_\mp = (a_1 + e^{-i\varphi}a_2 \mp \sqrt{2}b)/2$, with frequencies $\omega \mp \sqrt{2}\nu$. The three normal modes are not coupled with each other but interact with the atoms because of the contributions of the cavity fields. However, for $\nu \gg |g_j|$, the interaction of the atoms with the nonresonant modes is highly suppressed (it is essentially limited to the second order in the Dyson series) and the system reduces to two qubits resonantly coupled through a single harmonic oscillator. Remarkably, as the dominant interacting mode c has no contribution from the fiber mode b, the system gets in this instance insensitive to fiber losses. On the other hand, note that fulfilling the condition $\nu \gg |g_j|$ might require weak couplings, thus implying larger operating times.

Let us now discuss the computational possibilities allowed by the coherent evolution described by the Hamiltonian (2). To this aim, we will be interested in the reduced dynamics of the two distant atoms. We will assume that the system can be "initialized" bringing all the field modes in the vacuum state and allowing for any initial state of the qubits. The Hamiltonian H clearly conserves the number of global excitations and, for our aims, one can restrict to the zero-, single-, and two-excitation subspaces. The quantum operation describing the effective dynamics of the atoms can thus be exactly worked out determining its Kraus operators for any values of ν, g_j, and Δ. Denoting by $|ijk\rangle$ the state of the field given by the number state i in the mode of cavity 1, k in the mode of cavity 2, and j in the fiber mode, one has $E_{ijk}(t) = \langle ijk|\exp(-iHt)|000\rangle$ for $i, j, k = 0, 1, 2$ and the state of the atoms $\varrho(t)$ is given by $\varrho(t) = \sum_{i,j,k=0}^{2} E_{ijk}(t)\varrho(0)E_{ijk}^\dagger(t)$. In particular, we are interested in singling out "decoupling times" at which the state of the atoms will be highly decoupled from the light field so that their evolution will be approximately unitary. At such times the field has a very high probability of being in the vacuum state in both the single- and two-excitation subspaces (the global vacuum is a trivial eigenvector of H). This condition is fulfilled when the Kraus operators $E_{ijk} \simeq 0$ for $i, j, k \neq 0$, so that the Kraus operator E_{000} approximates a unitary evolution. More precisely, the fidelity of a Kraus operation $\{E_{ijk}\}$ emulating a unitary gate U can be properly estimated as follows. Suppose a pure two-qubit state $|\psi\rangle$ enters the operation as input: a measure of the reliability of the gate is given by the overlap

$$f(|\psi\rangle) = \langle\psi|U^\dagger\Big(\sum_{i,j,k=0}^{2} E_{ijk}|\psi\rangle\langle\psi|E_{ijk}^\dagger\Big)U|\psi\rangle.$$

The fidelity F of the gate may then be obtained by averaging over all pure input states: $F \equiv \langle f(|\psi\rangle)\rangle_{|\psi\rangle}$.

Setting $\Delta = 0$, $\delta = 0$, and $g \simeq \nu$ yields a highly reliable swap gate at the decoupling time $t \simeq \pi/g$. The fidelity of the proposed swap operation is shown in Fig. 1. As apparent, such a fidelity can exceed the value 0.99 and is remarkably stable with respect to possible imperfections in the coupling strengths and in the temporal resolution needed to switch off the interaction once the desired evolution is achieved. Let us remark that the values $g \simeq \nu \simeq 1$ GHz (at hand with present technology in optical cavities) would grant an operating time $\tau \simeq 1$ Ns. We also report that, after a time $t \simeq 3.4/g$, a swap gate with fidelity $F \simeq 0.98$ can be obtained for $\nu \simeq 100g$ (and $\Delta = \delta = 0$), i.e., in the range of parameters for which the system gets insensitive to fiber losses. This agreeable advantage is thus achieved by allowing a longer operating time (due

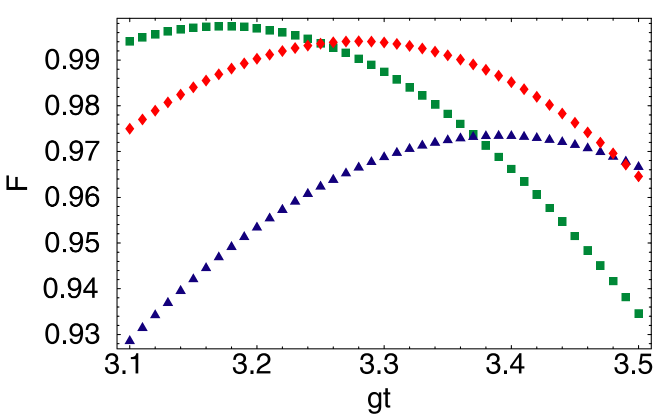

FIG. 1 (color online). Fidelities of an emulated swap gate as a function of time. The gate is obtained for $|g_1| = |g_2| = g$ and $\Delta = 0$; the diamonds refer to $\nu/g = 1.1$, the squares refer to $\nu/g = 1.2$, while the triangles refer to $\nu/g = 1$. All the quantities plotted are dimensionless.

to the condition on g) and a slightly lower (but still almost perfect) fidelity.

Moreover, this model allows for a reliable emulation of an entangling gate. To fix ideas, we focus on a ‘‘controlled-phase’’ (CPHASE) gate between the two qubits, described by the unitary matrix U_ϑ in the computational basis: $U_\vartheta = \mathrm{Diag}(1, 1, 1, e^{i\vartheta})$. This gate is equivalent, up to local unitaries, to the gates $\mathrm{Diag}(1, e^{i\vartheta_1}, e^{i\vartheta_2}, e^{i\vartheta+\vartheta_1+\vartheta_2})$ for any ϑ_1, $\vartheta_2 \in [0, 2\pi]$, since the phases ϑ_1 and ϑ_2 can be canceled out by local phase gates. We will thus henceforth refer to all such gates as ‘‘CPHASE’’ gates. The entangling power of such gates increases as the phase ϑ increases between 0 and π (for which a controlled-Z gate is achieved). Let us also recall that any of these entangling gates, together with local unitary operations, make up a universal set of gates (as *any* two-mode gate can be recovered as a proper combination of the entangling gate and of local gates [14]). The symmetry of the Hamiltonian (crucial in realizing a swap gate), must be broken here because it prevents a phase ϑ from appearing at decoupling times. In point of fact, if the transition of atom 2 is detuned (e.g., by Stark or Zeeman effect), a phase does arise, thus allowing for an effective entangling gate. Reliable decouplings allowing us to emulate such a gate are achieved for $\nu \gg |g_j|$, for which the fiber is ‘‘bypassed’’ and fiber losses do not affect the performance of the gate. For $\nu \simeq 100g \simeq 200\delta \simeq 10\Delta$ a sequence of CPHASE gates—separated by a period of about $4.4g^{-1}$—with increasing ϑ (ranging from $\vartheta \simeq 0.15\pi$ to $\vartheta \simeq 0.93\pi$) is emulated. The most entangling CPHASE gate ($U_{0.93\pi}$) is achieved after six ‘‘Rabi-like’’ oscillations in the two-excitation subspace. The fidelity F of the emulated gate exceeds the value 0.99. Its stability is demonstrated in Fig. 2. The operating time of the gates would range, for $\nu \simeq$ 10 GHz, from 3 to 0.3 μs, according to the desired entangling power. Figure 3 shows the entanglement of formation between the two atoms generated for an initial state $(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)/2$ (which gets maximally entangled if processed by a controlled-Z gate) with several choices of parameters. As apparent, a speedup in the creation of entanglement is achieved by increasing the relative difference δ/g. However, too large differences ($\delta/g \gtrsim 0.5$) affect the fidelity and stability of the emulated gate and thus, while advantageous for building up entanglement, are not convenient to perform actual computation.

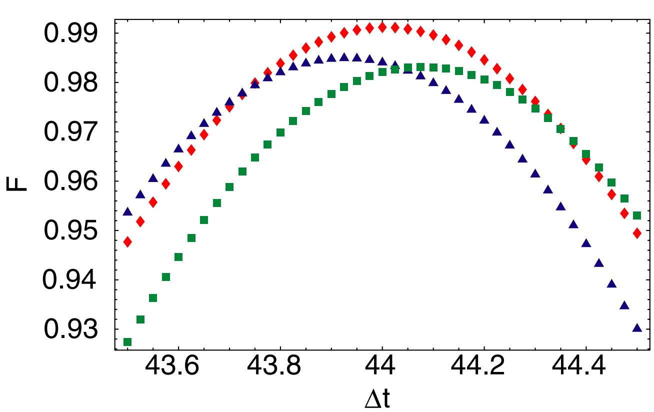

FIG. 2 (color online). Fidelities of an emulated CPHASE gate ($U_{0.15\pi}$) as a function of time. The diamonds refer to $\nu/\Delta = 10$, $|g_1|/\Delta = 0.1$, and $|g_2|/\Delta = 0.15$; the squares and the triangles refer, respectively, to a relative variation of -5% and $+5\%$ in $|g_1|$, $|g_2|$, and ν. The fidelities of the successive (more entangling) CPHASE gates are similar. All the quantities plotted are dimensionless.

We now take into account dissipation due to spontaneous emission of the atoms and to cavity and fiber losses. The global system is then governed, in the Schrödinger picture, by the following master equation

$$\dot{\varrho} = -i[H, \varrho] + \frac{\gamma}{2}\sum_{j=1}^{2} L[a_j]\varrho + \frac{\kappa}{2}\sum_{j=1}^{2} L[\sigma_j^-]\varrho + \frac{\beta}{2}L[b]\varrho, \tag{3}$$

where the superoperator $L[\hat{o}]$ is defined as $L[\hat{o}] = 2\hat{o}\varrho\hat{o}^\dagger - \hat{o}^\dagger\hat{o}\varrho - \varrho\hat{o}^\dagger\hat{o}$ for operator $\hat{o}$ and κ, and γ and β stand, respectively, for the spontaneous emission rate and for the cavity and fiber decay rates (assumed for simplicity to be equal in the two cavities). The thermal contributions of the bath have been neglected, as is possible at optical frequencies. Considering decoherence analytically for one excitation and numerically for two excitations [by integrating Eq. (3)], the operator tomography of the process encompassing decoherence has been reconstructed in the cases interesting for emulating gates.

In the regime $\nu \gg |g_j|$ the fidelities of the gates have been consistently found to be essentially unaffected by fiber losses. In general, moreover, the ‘‘direct’’ effect of spontaneous emission proves to be more relevant than the ‘‘indirect’’ effect of cavity losses. For the swap gate with $\nu \simeq 1.2g$ (with maximum fidelity $F \simeq 0.997$ without dis-

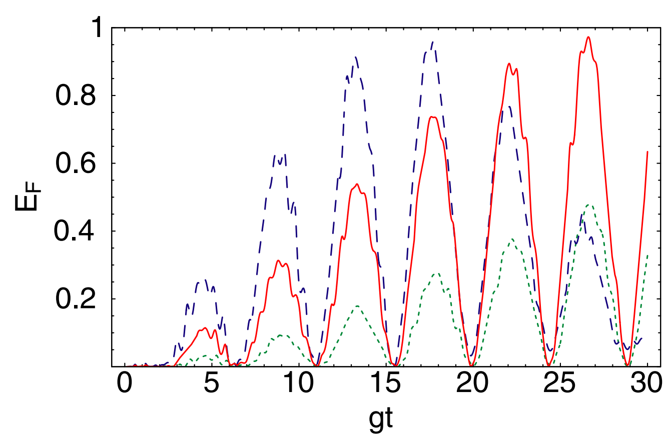

FIG. 3 (color online). Entanglement of formation in ebits as a function of time (in units g^{-1}) for $\nu = 100g = 10\Delta$ and $\delta = g$ (dashed line), $\delta = 0.5g$ (solid line) and $\delta = 0$ (dotted line). At the peaks, CPHASE gates are emulated.

sipation), the maximum fidelity drops to $F \simeq 0.956$ for $\kappa = 10^{-2}g$, thus allowing for a still relatively reliable gate, while a fidelity $F \simeq 0.989$ is maintained for $\kappa = \gamma = \beta = 10^{-3}g$. Lower decay rates leave the gate virtually unaffected, while higher rates completely spoil it. Notice that values permitting an effective swap would already be at hand for rubidium atoms in integrated fiber-cavity systems (see data from Ref. [10], with length of the cavity $L \simeq 100\ \mu$m). The case $\nu = 100g = 200\delta = 10\Delta$, selected to demonstrate the possibility of a CPHASE gate, proved to be slightly more sensitive to spontaneous emission and cavity losses. Let us focus on the first gate (after one Rabi-like oscillation): for $\kappa = 10^{-2}g$, the fidelity of the gate falls to $F \simeq 0.93$ (in which case the fidelity of the optimal most entangling gate, achieved after six oscillations, is completely spoiled), while for $\kappa = \gamma = 10^{-3}g$ (recall that this regime is insensitive to fiber losses), the fidelity of the first gate is still $F \simeq 0.97$. Generally, decay rates as low as $10^{-4}g$ have a negligible effect on the performances of the gates, but also decay rates of the order of $10^{-2}g$ would allow for remarkable experimental demonstrations of swap and entangling gates. In view of the quality attained in the fabrication of high-finesse optical cavities, the main technical issue left seems to be limiting the spontaneous emission rates. Hyperfine ground levels (with negligible ''intrinsic'' spontaneous emission rates) of effective two-level lambda systems could thus be good candidates for the implementation of such computational schemes. In fact, let us consider a lambda system (refer to Ref. [2] for details), where one transition is driven by a laser of strength h with detuning d and the other is mediated by a mode of the field with resonant coupling h (assumed for simplicity to be real and equal to the laser strength). Let ξ stand for the spontaneous emission rate of the excited level, which will be adiabatically eliminated under the condition $d \gg h$. Let us suppose to exploit such a two-level system for the proposed scheme. In our previous notation, one would have [2] $g \simeq dh^2/(d^2 + \xi^2)$ and $\kappa \simeq \xi h^2/(d^2 + \xi^2)$, with $g/\kappa \simeq d/\xi$: a large enough detuning would thus allow to coherently implement the scheme with these effective two-level systems.

We have investigated the implementation of quantum computation and entangling schemes for atoms trapped in distant cavities coupled by an optical fiber. Imperfections and dissipation have been considered showing that, in the short fiber regime, reliable gates with promising operating times could be at hand with present technology. Let us also mention that, in the considered system, not only entangling and swap gates, but also perfect quantum state transfer is possible. Besides, the proposed setup would also allow for the unitary generation of cluster states between distributed atoms or ions [8], and could thus find application not only in gate-based but also in ''one-way'' quantum computation. More generally, our results strongly emphasize the potentialities of quantum optical systems towards the realization of effective quantum networking schemes.

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[1] J. I. Cirac *et al.*, Phys. Rev. Lett. **78**, 3221 (1997); S. J. van Enk *et al.*, *ibid.* **79**, 5178 (1997).
[2] T. Pellizzari, Phys. Rev. Lett. **79**, 5242 (1997).
[3] S. J. van Enk *et al.*, Phys. Rev. A **59**, 2659 (1999).
[4] S. Clark *et al.*, Phys. Rev. Lett. **91**, 177901 (2003).
[5] S. Bose *et al.*, Phys. Rev. Lett. **83**, 5158 (1999); S. Mancini and S. Bose, Phys. Rev. A **64**, 032308 (2001); D. E. Browne *et al.*, Phys. Rev. Lett. **91**, 067901 (2003).
[6] L.-M. Duan and H. J. Kimble, Phys. Rev. Lett. **90**, 253601 (2003).
[7] Y. L. Lim *et al.*, Phys. Rev. Lett. **95**, 030505 (2005); S. D. Barrett and P. Kok, Phys. Rev. A **71**, 060310(R) (2005); X.-F. Zhou *et al.*, *ibid.* **71**, 064302 (2005); Y.-F. Xiao *et al.*, *ibid.* **70**, 042314 (2004); Y.-F. Xiao *et al.*, Phys. Lett. A **330**, 137 (2004).
[8] J. Cho and H.-W. Lee, Phys. Rev. Lett. **95**, 160501 (2005).
[9] A. Boca *et al.*, Phys. Rev. Lett. **93**, 233603 (2004); P. Maunz *et al.*, *ibid.* **94**, 033002 (2005); S. Nußmann *et al.*, *ibid.* **95**, 173602 (2005).
[10] M. Trupke *et al.*, quant-ph/0506234.
[11] M.-H. Yung *et al.*, Quantum Inf. Comput. **4**, 174 (2004).
[12] Actually, the description of the system in terms of distinct cavity and fiber modes—interacting through the Hamiltonian H_{If}—holds for high-finesse cavities and near resonant operations over time scales long with respect to the fiber's round-trip time (see Ref. [3]). All these assumptions are matched by the system at hand in the operating conditions we will consider (in particular in the short fiber limit; see next paragraph).
[13] Let us remark that the fiber could be also filtered in frequency to ensure the selection of a single interacting mode [M. Trupke (private communication)].
[14] M. J. Bremner *et al.*, Phys. Rev. Lett. **89**, 247902 (2002).