# An MILP formulation for the optimal management of microgrids with task interruptions

Javier Silvente, Lazaros G. Papageorgiou*

Centre for Process Systems Engineering, Department of Chemical Engineering, University College London, Torrington Place,
London WC1E 7JE, United Kingdom.

*corresponding author: l.papageorgiou@ucl.ac.uk

## Abstract

This work is focused on the optimal management of electricity and heat generation and demand in microgrids. The objective of the proposed mathematical model is to adjust energy and heat availability profiles resulting from the use of renewable energy sources and flexible energy and heat demands. The optimisation of the resulting short-term problem is addressed through a Mixed-Integer Linear Programming (MILP) mathematical model to minimise the operational cost of the microgrid. Delays in the energy demands are allowed to tackle flexible demand profiles, under penalties in the objective function. An additional characteristic was the consideration of non-constant profiles in the considered tasks. Also, this model takes into account eventual interruptions in the tasks, applying penalties in the economic objective function. The main decisions to be made includes the schedule of tasks, as well as energy and heat generation levels, purchases from and exportation to the power grid, and storage levels.

**Keywords:** Energy planning, Scheduling, Mathematical programming, Microgrid, MILP, Optimisation.

## 1. Introduction

The use of energy at domestic and industrial levels is essential. However, as global population and standard of life increases, the energy demand also increases. It is expected that global demand will be increasing at an average of 1.5% every year to 2035 (Orr, 2013), which can involve a failure to match this growing energy demand if the energy supply is not sufficient and sustainable.

Energy Systems Engineering is receiving increasing attention (Pfenninger et al., 2014). This awareness is due to the diminution in the operational costs and the reduction in the environmental impact as a result of the exploitation of renewable energy sources. Therefore, new advances have been produced in the development of efficient, sustainable and environmentally friendly energy networks, which includes energy sources as well as the transmission and delivery to industries and particular costumers. The main three goals of managing energy systems are focused on decreasing costs associated with the use of the energy, reducing the environmental impact due to the generation and transmission of energy as well as satisfying the energy demand subjected to unforeseen disturbances.

One area of study of Energy Systems Engineering is the management of microgrids. These kinds of grids are decentralised networks which may integrate electricity and heat generation systems close to the consumption points (Hodge et al., 2011). Microgrids may include several renewable and non-renewable sources that can be operated in parallel with the main utility grid or isolated. Some advantages of microgrids are the reduction of energy losses in power transmission as well as a significant gases emission decrease due to the use of renewable sources. Another benefit of microgrids is the improvement in the responsiveness to electricity and heat demand fluctuations (Wang and Singh, 2009; Tong et al., 2015). The dimension of a microgrid may differ according to its application and its characteristics. For example, a microgrid may refer to a smart grid or a smart building, such as the home/residential energy management.

The appropriate design and operational decision-making of microgrids are required to guarantee their optimal management. On the one hand, the design of a microgrid contemplates the capacity and location of the elements to be installed in the network (Pruitt et al., 2013), as well as their characteristics, such as the capacity (Li et al., 2017). For example, Asano et al. (2007) developed a mathematical formulation to determine the number and characteristics (e.g., capacity) of the equipment units in a microgrid with combined heat and power (CHP) systems in order to minimise the annual cost. The impact of the exploitation of CHP has been studied by Keirstead et al. (2012). On the other hand, the scheduling of a microgrid at operational level is focused on the energy generation and demand at local level, given the main elements that constitute the microgrid under study (Manfren et al., 2011; Mehleri et al., 2012; Sou et al., 2011; Prodan et al., 2014).

Several approaches have been used to manage microgrids. The solution technique to solve an optimisation problem depends on the characteristics of each problem. The application of mathematical programming approaches implies the development of a mathematical framework and the use of an optimisation algorithm.

Although this work has been focused on solution approaches based on a Mixed-Integer Linear Programming, other mathematical programming techniques are applied, including Linear Programming (Hawkes and Leach, 2009), Non-Linear Programming (Derakhshandeh et al., 2013) and Mixed-Integer Non-Linear Programming (Ghazvini et al., 2012).

Other solution methods for dealing with optimisation problems are used for the management of microgrids, such as Genetic Algorithms and Logic-based techniques. Genetic Algorithms are an alternative approach that can be used when large mathematical formulations requiring large computational effort to reach the optimal solution are used (Mohamed and Koivo, 2012; Chen, 2013). Logic-based optimisation techniques are used to facilitate the modelling, reducing the combinatorial search efforts as well as improving the handling of non-linearities. One of the most typical logic-based optimisation techniques in the management of microgrids is the Constraint Programming (Ji et al., 2015). Other approaches are applied in this area, such as Lagrangian relaxation (Han et al., 2013) and Benders decomposition (Yang et al., 2016), among other techniques. A review considering different optimisation techniques applied to microgrid planning has been presented by Gamarra and Guerrero (2015) and Nosratabadi et al. (2016).

Regarding the scheduling of microgrids under limited resources, different formulations have been developed to minimise the operational cost of a network, considering the management of electricity and heat. Some works are focused on the energy generation management side. For example, Carrión and Arroyo (2006) developed a discrete-time Mixed-Integer Linear Programming (MILP) formulation to minimise the operational cost for an established energy demand. Also, Bagherian and Tafreshi (2009) proposed a methodology for the optimal operation of a microgrid, considering energy generators and storage systems in order to satisfy different consumption tasks. Kopanos et al. (2013) presented a mixed integer formulation model to minimise the operational cost for the energy generation associated with an energy network based on a residential microgrid, considering CHP systems. Also, the importance of the exploitation of renewable sources to produce energy has received attention. Several decision-making models have been developed to determine the optimal operating conditions of the energy generation sources, such as which generator to use in each period (Coroamă et al., 2013; Chicco et al., 2009; Xiao et al., 2011; Ahadi et al., 2016). Furthermore, regarding the management of the energy generation in smart houses, Sun and Huang (2012) reviewed optimisation procedures for the energy management, including neural network, fuzzy logics as well as evolutionary approaches. Other works are focused on the management of energy generation for transport purposes (Honarmand et al., 2014; Bracco et al., 2015).

The management of the energy demand represents an emerging challenge. The domestic consumptions correspond to an average of 30-40% of the world´s energy consumption (Lior, 2010). According to Escrivá-Escrivá (2011), there are several actions to improve the energy consumption efficiency of residential houses. In this sense, the implementation of information technology (IT) can be used for a better management of energy and heat generation and distribution in order to match supply and demand between generation sources and consumers. This includes the integration of advanced digital metres (or smart meters), distribution automation and communication systems. These energy meters are used to achieve reliable information on energy consumption tasks. Thus, the exploitation of data from information technology can reduce operational costs, anticipating future requirements at local level (Krishnan, 2008; Kabalci, 2015). Therefore, obtaining real-time data becomes indispensable to take advantage of managing the network, to enhance the efficiency of the network, to organise the proactive maintenance, and to achieve customer savings (Krishnamurti et al., 2012). In the industrial area, Kato et al. (2011) developed an MILP model to optimise the energy generation and storage levels in a demand based energy supply approach, to decrease the energy consumption. In the area of smart houses, Nistor et al. (2011) presented an approach for the scheduling of tasks, where these tasks may be delayed according to the real-time energy price. Rastegar et al. (2016) presented an MILP formulation in order to manage the human behaviour to better satisfy the energy demand to respond to real-time prices, by incorporating priority in the tasks. Zhang et al. (2016) proposed a multi-criteria optimisation model to analyse the trade-off between economics and environmental sustainability while scheduling the energy tasks within smart houses in a microgrid.

Another important subject of study is the diminution of the peak demands. Thus, a peak-load shaving on-line scheduling model was proposed by Costanzo et al. (2011). Also, Caprino et al., (2014) proposed a real-time scheduling formulation to control and model the energy availability, reducing energy peaks of demand. More recently, Zhang et al. (2013) proposed an MILP model to manage the scheduling of tasks within a network, as well as to reduce the peak demand. These peaks of power are adverse for energy suppliers, for the main grid and for the consumers, because the peaks decrease the power grid efficiency (grid must be designed considering peaks of energy) as well as because the price of energy is based on the presence peaks.

Notice that the integrated management of energy and heat generation and demand introduces a degree of freedom in the management of microgrids, which may allow reaching further benefits. These benefits might comprise the integration of renewable sources, which are usually intermittent, at the distribution level (Ipakchi and Albuyeh, 2009; Chiaroni et al., 2014), and so the diminution of non-renewable sources, such as fossil fuel based sources (Baños et al., 2011; Iqbal et al., 2014). This extra flexibility may be significantly enhanced by the exploitation of energy and heat storage systems. These systems can be used to detach generation and demand peaks, as well as to deal with the variable availability of renewable sources (Wang et al., 2016; Yin et al., 2017).

The management of generation and demand has been investigated in a sequential way. This is given by adjusting the process schedule to the availability of energy. For instance, Nolde and Morari (2010) developed an MILP formulation to minimise the energy cost by optimising the schedule of energy tasks. The objective of the proposed model was to adjust the schedule of a steel plant. A penalisation for deviations from the previously contracted energy consumption to the provider was introduced. Other works are based on adjusting the schedule of a process to the variable real-time price of the energy. For example, Mitra et al. (2012) presented an MILP to adjust the generation planning based on the electricity price for a continuous process. Moreover, Hadera et al. (2015) proposed a mathematical formulation to deal with the demand response, in order to minimise the energy costs, taking into account the schedule of a steel plant. Mohsenian-Rad and León-García (2010) presented a residential energy task scheduling formulation considering energy pricing models for smart houses.

Silvente et al. (2015) presented an MILP formulation for the coordinated management of generation and demand in a microgrid, taking into account renewable energy sources and the interconnection to the power grid. Penalty costs were applied in case of delays from the nominal target in the initial starting time of different energy tasks. Zhang et al. (2013) contemplated non-constant profiles in the energy demand requirements.

Thus, electricity and heat generation management has been studied in the last years. However, the optimal management of a microgrid considering simultaneously both generation and demand (including eventual shiftable and interruptible demands) has not been reported and still constitutes an open issue to the research community. This work focuses on the home energy management within a microgrid, by the development of an electricity and heat planning and scheduling model to better manage the generation, storage and use of energy and heat within a network. The operations of the energy and heat generators and storage devices, purchases and sales to an external grid as well as the domestic appliances are going to be scheduled. The assessment of the presented mathematical formulation is presented throughout an illustrative example addressing the optimal management of the energy and heat generation, storage and consumption of energy and heat consumption tasks within a microgrid. This work constitutes an extension of the above mentioned works. The main novelty of this paper is the coordinated management of energy and heat generation and demand within a scheduling, considering the generation through renewable and non-renewable sources and the presence of flexible energy requirements, limited by a time window. An additional feature of this model is that interruptions in the energy demand are considered, under penalties in the economic objective function.

This paper is organised as follows. Section 1 has described the main recent advances in the area of management of smartgrids/microgrids. Section 2 describes briefly the assumptions of the problem as well as the constraints and the objective associated with the mathematical model. In Section 3, the mathematical model is provided. The illustrative example is described in Section 4. The obtained results are discussed in Section 5. Finally, concluding remarks are presented in Section 6.

## 2. Problem statement

Different aspects related to the management of a microgrid are considered, including generation, purchases, sales and storage levels and consumption of electricity and heat. The objective is to minimise the operational cost of the microgrid. The energy demand management is taken into account through the possibility of modifying the timing of the tasks. The mathematical model contemplates both electrical and heat balance constraints to describe their flows (i.e., generation, purchases, sales, storage and consumption) and equipment constraints (i.e., capacity constraints) involved in the microgrid. The problem is presented regarding the following items. Given:

(i) A scheduling horizon which is divided into a set of time intervals $t \in T$.
(ii) A set of homes $k \epsilon K$.
(iii) A set of electricity and heat generation sources, as well as their characteristics (i.e., capacities, efficiencies, etc.).
(iv) A set of electricity and heat storage systems, and their characteristics (i.e., capacities, efficiencies, charge/discharge limit rates, etc.).
(v) A set of equipment units $j \epsilon J$ to perform the considered tasks.
(vi) A set of energy tasks $i \epsilon I$ which define the overall energy requirements. Each task is performed in a given equipment unit $j \epsilon J$. A desirable starting time (i.e., minimum starting time) of each task $Ts_{k,i}^{min}$and its processing time $PT_{k,i}$ are established. Each task is constrained within a time window delimited by the desirable/minimum staring time and a maximum ($Ts_{k,i}^{min}$ and $Ts_{k,i}^{max}$). If the starting time $Ts_{k,i}$ of a task is located outside the time window then an extra cost is applied.
(vii) The task consumption resource profile $C_{i,\theta}$ (which can be constant or variable) is also given. Notice that the index $\theta \in \Theta_{ik}$ denotes the operation period associated with a task $i$. For a better understanding, Figure 1 represents consumption resource profile $C_{i,\theta}$ for a task $i$.
(viii) The overall heat demand, $H_t$.
(ix) The price of the electricity in the power grid, $b_t$.
(x) The wind forecast speed, $v_t$.

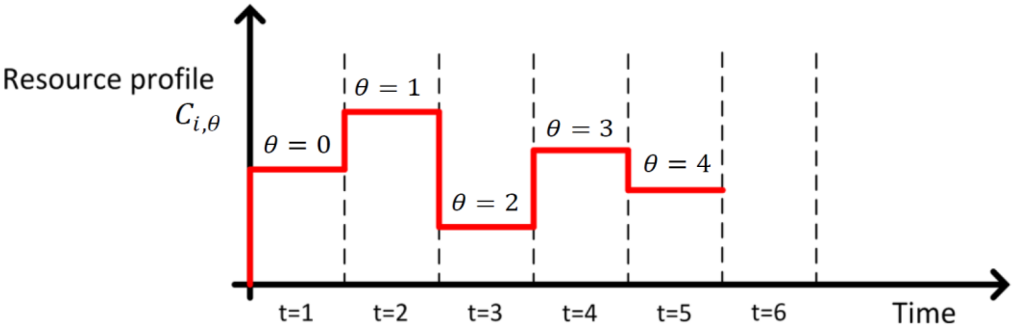

Figure 1. Time-varying resource profile for a task.

Note that although the resource profile of a task can be non-constant over its processing time, its value is assumed to be constant within a time interval. If more accuracy is required to represent the capacity profile, one alternative is to reduce the duration of the time interval. However, this may involve more computational time to reach the solution. The effect of the duration of the time interval in a discrete time formulation was studied by Silvente et al. (2015). Other alternatives are to use hybrid and continuous time formulations, at the expense of an increase in the complexity of the mathematical model, increasing the computation effort.

The objective is to determine the energy generation plan, the electricity purchases to the external power grid, the schedule of tasks, the electrical and thermal/heat storage plan and the electricity to export to the power grid so as to minimise the operational cost of the microgrid. Therefore, the main decisions to be made include:

(i) The status (on/off) of each task $i$ at home $k$ in each time interval $t$, $W_{k,i,\theta,t}$ and $Z_{k,i,\theta,t}$, starting inside and outside the time window, respectively. The binary variable $W_{k,i,\theta,t}$ takes value 1 if task $i$ at home $k$ is active at time interval $t$ for the operation period $\theta$ for tasks started inside the corresponding time window. Also, binary variable $Z_{k,i,\theta,t}$ is used for tasks started outside the established time window. These variables determine the schedule of consumption tasks within the microgrid.
(ii) The electric and thermal power generation levels through the CHP $P_t^{CHP}$, boiler $P_t^B$ and wind turbines $P_t^W$ at time interval $t$.
(iii) The electric power to be imported from and exported to the power grid, $Im_t$ and $Ex_t$ respectively, at time interval $t$.
(iv) The electrical and heat storage levels, $S_t^E$ and $S_t^T$ respectively, at time interval $t$.
(v) The electric power charge and discharge rates ($S_t^{EC}$ and $S_t^{ED}$) and the thermal power charge and discharge rates ($S_t^{TC}$ and $S_t^{TD}$) at time interval $t$.
(vi) The extra energy load from the power grid $\gamma_t$ over a given threshold $\beta$ at time $t$.

One feature of the proposed formulation is that different tasks in the same equipment unit are allowed, establishing the sequence of operations and avoiding any eventual overlap. Another characteristic is the fact that any consumption task profile can be non-constant, considering operation profiles $\theta$.

The presented MILP is based on a discrete-time representation. The scheduling horizon is divided into a finite number time intervals, considering the same duration for all intervals. Discrete-time formulations constrain all tasks to start at the beginning of a time interval. However, the processing time of tasks is assumed to be continuous, allowing tasks to finish at any time. Notice that the computational effort to solve discrete-time models depends on the size of the problem.

Therefore, the scheduling of the microgrid is based on the features of each task (i.e., consumption task profile, earliest starting and latest ending times), renewable energy sources forecast (i.e., wind speed forecast) and real-time energy prices (purchases and sales) at each time interval (time-varying electricity price). Also, peak demand charges are also considered, to reduce the peak electric power demand from the grid. This means that an extra fee is charged to the over-threshold amount when the overall demand is over the above mentioned threshold. Generation of electricity and heat is scheduled from generators, as well as purchases to the power grid. Also, electricity sales to the power grid are also allowed.

## 3. Mathematical model

The microgrid management problem is formulated as an MILP. The time representation is modelled according to equal-size time intervals. The aim is to minimise the operational cost of the microgrid, according to constraints associated with the management of the microgrid, which are presented next.

### 3.1. *Wind generator output*

The electric power generated through the exploitation of the wind turbines $P_t^W$ is calculated through equation (1). This equation takes into account the wind blade area of the wind turbine, the wind speed and the wind turbine efficiency. The electric power generation through wind turbines is delimited by the cut-in speed $\left(V^{cut-in}\right)$ and cut-out speed $(V^{cut-out})$ associated with each wind turbine. The cut-in speed corresponds to the minimum wind speed in which the turbine will generate its designated rated power. On the other hand, the cut-out speed corresponds to the maximum wind speed in which the wind turbine can operate under normal conditions. The wind turbine must be shut down for safety reasons when the wind turbine exceeds this limit. If the wind turbine remains active in this situation, it may be damaged (Villanueva and Feijóo. 2010). Therefore, the wind turbine does not generate electric power if the wind speed is under the cut-in speed or above the cut-out speed. Furthermore, notice that if the wind speed is above the nominal wind $V^{nom}$, the power output is at the maximum output level, which corresponds to the power generated at the nominal wind speed (Zhang et al., 2013).

$$P_t^W = \begin{cases} 0.5 \cdot \rho \cdot A \cdot \eta^W \cdot min(v_t, V^{nom})^3 & \forall t, V^{cut-in} \leq v_t \leq V^{cut-out} \\ 0 & \forall t, v_t < V^{cut-in}, v_t > V^{cut-out} \end{cases} \tag{1}$$

### 3.2. *Generation and storage capacity constraints*

The output from each equipment (i.e., generator, storage system) should not exceed its designed capacity. This means that the electric and thermal power generation and the storage levels are bounded by a maximum level, which cannot be exceeded. This is given by equations (2), (3), (4) and (5) for the CHP generator $P_t^{CHP}$, boiler $P_t^B$, electrical storage $S_t^E$ and heat/thermal storage $S_t^T$, respectively.

$$P_t^{CHP} \leq C^{CHP} \qquad \forall t \tag{2}$$

$$P_t^B \leq C^B \qquad \forall t \tag{3}$$

$$S_t^E \leq C^E \qquad \forall t \tag{4}$$

$$S_t^T \leq C^T \qquad \forall t \tag{5}$$

### 3.3. *Energy and heat storage constraints*

The electricity stored in the electrical storage system at time $t$, $S_t^E$, is calculated in equation (6). The storage level is equal to the storage level at the previous time, $S_{t-1}^E$, plus the difference between the electricity charged and discharged in the equipment. The efficiency of the charge and discharge of the electricity storage system, $\eta^E$, has been taken into account, which allows considering electricity losses. According to equation (7), the electrical storage level at the end of the scheduling horizon $S_{t=T}^E$ must return to the initial value $S_{t=0}^E$. This avoids net electricity accumulation. Also, electric power discharge or charge rates ($S_t^{ED}$ and $S_t^{EC}$) cannot exceed the corresponding storage discharge and charge limits ($S^{EDmax}$ and $S^{ECmax}$). These rates are defined by the electrical storage characteristics and are used to avoid excessive discharge and charge rates, which can damage the storage system or even may reduce its capacity.

$$S_t^E = S_{t-1}^E + \delta \cdot \eta^E \cdot S_t^{EC} - \frac{\delta}{\eta^E} \cdot S_t^{ED} \qquad \forall t \qquad (6)$$

$$S_{t=0}^E = S_{t=T}^E \qquad (7)$$

$$S_t^{ED} \leq S^{EDmax} \qquad \forall t \qquad (8)$$

$$S_t^{EC} \leq S^{ECmax} \qquad \forall t \qquad (9)$$

Similarly, the heat stored in the thermal storage system at time $t$ is given by equation (10). The heat storage level $S_t^T$ is equal to the storage level at the previous period, $S_{t-1}^T$, plus the difference between the heat charged and discharged in the heat storage system. The heat losses during the heat storage are also modelled considering the efficiency of the charge and discharge, $\eta^T$. Analogously to the electricity storage, heat stored at the end of the scheduling horizon $S_{t=T}^T$ must return to the initial state $S_{t=0}^T$. This is given by equation (11). Consequently, there is no heat accumulation over the scheduling horizon. Furthermore, thermal power discharge and charge rates ($S_{s,t}^{TD}$ and $S_{s,t}^{TC}$) cannot exceed the corresponding thermal storage discharge and charge limits ($S^{TDmax}$ and $S^{TCmax}$), given by equations (12) and (13).

$$S_t^T = S_{t-1}^T + \delta \cdot \eta^T \cdot S_t^{TC} - \frac{\delta}{\eta^T} \cdot S_t^{TD} \qquad \forall t \qquad (10)$$

$$S_{t=0}^T = S_{t=T}^T \qquad (11)$$

$$S_t^{TD} \leq S^{TDmax} \qquad \forall t \qquad (12)$$

$$S_t^{TC} \leq S^{TCmax} \qquad \forall t \qquad (13)$$

### 3.4. *Starting time and finishing time of tasks*

A task $i$ should start within a given time window or outside the time window, according to equation (14a). If the task starts within the given time window, the initial time is constrained by a minimum and maximum starting times. Then, in the starting point of a given task, the value of binary variable $W_{k,i,\theta,t}$ is 1 for the first $\theta$, which corresponds to the first time operation period. If task starts before or after the time window, the binary variable $Z_{k,i,\theta,t}$ takes value 1 in the starting time for the first $\theta$. Notice that all tasks are forced to start. If the demand management is not allowed, which means that all tasks must start at the pre-established starting point (i.e., minimum starting time), equation (14b) is applied instead of equation (14a). Equation (15) is required to ensure that all tasks will be finished before the conclusion of the overall scheduling horizon. This equation avoids a task to start in a period of time where cannot be completed.

$$\sum_{\substack{t \geq Ts_{k,i}^{min} \\ t \leq Ts_{k,i}^{max}}} W_{k,i,\theta,t} + \sum_{\substack{t < Ts_{k,i}^{min} \\ t > Ts_{k,i}^{max}}} Z_{k,i,\theta,t} = 1 \qquad \forall k, i \epsilon I_k, \theta = 0 \qquad (14a)$$

$$\sum_{t = Ts_{k,i}^{min}} W_{k,i,\theta,t} = 1 \qquad \forall k, i \epsilon I_k, \theta = 0 \qquad (14b)$$

$$\sum_{t > SH - \overline{PT}_{k,i}} Z_{k,i,\theta,t} = 0 \qquad \forall k, i \epsilon I_k, \theta = 0 \qquad (15)$$

### 3.5. *Penalisation for delays in the starting time of tasks*

The binary variables $W_{k,i,\theta,t}$ or $Z_{k,i,\theta,t}$, if $\theta = 0$, are active when a task $i$ at home $k$ starts at time interval $t$. These restrictions have been formulated as a set of big-M constraints, given by equations (16) and (17). Also, a penalty cost is established considering the delay in the starting time from the minimum starting time and given by equation (18). This penalty has been assigned to each task according to its characteristics. This means penalty costs of high priority tasks are always more expensive than lower priority ones.

$$Ts_{k,i} \geq T_t - M \cdot \left(1 - W_{k,i,\theta,t} - Z_{k,i,\theta,t}\right) \qquad \forall k, i \epsilon I_k, \theta = 0, t \tag{16}$$

$$Ts_{k,i} \leq T_t + M \cdot \left(1 - W_{k,i,\theta,t} - Z_{k,i,\theta,t}\right) \qquad \forall k, i \epsilon I_k, \theta = 0, t \tag{17}$$

$$CPen_{k,i} = \left(Ts_{k,i} - Ts_{k,i}^{min}\right) \cdot \mu_{k,i} \qquad \forall k, i \epsilon I_k \tag{18}$$

### 3.6. *Processing time and sequence of tasks*

Equation (19) forces all tasks to be completed. According to this equation, the summation of the active periods of time where a task $i$ takes place must be equal to the (roundup) processing time of the task, $\overline{PT}_{k,i}$. Furthermore, equations (20) and (21) avoid any overlap in the task, establishing that only one operation period can be performed at time $t$. Moreover, equations (22) and (23) determine the sequence of the time operation period within and outside the given time window, respectively. This set of two equations forces to follow the pre-established order in the consumption task profile. Furthermore, equation (24) is used to avoid any overlaps of tasks $i$ in the same equipment unit $j$. In other words, this equation forces that a task $i'$ cannot start if a previous task $i < i'$ is not completed, avoiding an overlap or a change in the determined task sequence.

$$\sum_{t} \sum_{\theta=0}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t} + \sum_{t} \sum_{\theta=0}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t} = \overline{PT}_{k,i} \qquad \forall k, i \epsilon I_k \tag{19}$$

$$\sum_{\theta=0}^{\overline{PT}_{k,i}-1} \left(W_{k,i,\theta,t} + Z_{k,i,\theta,t}\right) \leq 1 \qquad \forall k, i \epsilon I_k, t \tag{20}$$

$$\sum_{t} \left(W_{k,i,\theta,t} + Z_{k,i,\theta,t}\right) \leq 1 \qquad \forall k, i \epsilon I_k, \theta \in \Theta_{ik} \tag{21}$$

$$\sum_{\substack{t' \\ t' \leq t}} \left(W_{k,i,\theta,t'} - W_{k,i,\theta+1,t'}\right) \geq 0 \qquad \forall k, i \epsilon I_k, \theta \in \Theta_{ik}, t \tag{22}$$

$$\sum_{\substack{t' \\ t' \leq t}} \left(Z_{k,i,\theta,t'} - Z_{k,i,\theta+1,t'}\right) \geq 0 \qquad \forall k, i \epsilon I_k, \theta \in \Theta_{ik}, t \tag{23}$$

$$\sum_{\substack{t'\\ t'\le t}} \left(W_{k,i,\theta,t'} + Z_{k,i,\theta,t'}\right) \ge W_{k,i',\theta',t'} + Z_{k,i',\theta',t'} \qquad \forall k, i\epsilon I_j, i'\epsilon I_j, i' > i, \theta = \overline{PT}_{k,i} - 1, \theta = 0, t \qquad (24)$$

## *3.7. Interruption of tasks*

The following equations are implemented to manage any eventual interruption in an energy task $i$ at home $k$. Thus, equation (25) is used to determine the potential status $\widehat{W}_{k,i,t}$ (on/off) of task $i$ within the time window. This takes into account the first and the last periods of time where the considered task is active, given by the minimum and maximum $\theta$ for each task. This equation is going to be used to determine if any task $i$ is interrupted or remains interrupted.

$$\widehat{W}_{k,i,t} = \sum_{\substack{t'\\ t'\le t}} W_{k,i,\theta,t'} - \sum_{\substack{t'\\ t'<t}} W_{k,i,\theta',t'} \qquad \forall k, i\epsilon I_k, t, \theta = 0, \theta' = \overline{PT}_{k,i} - 1 \qquad (25)$$

Equation (26) determines that if the potential status of a task is not active ($\widehat{W}_{k,i,t} = 0$), there is neither interruption nor remaining interruptions ($\lambda i^W_{k,i,t} = \lambda s^W_{k,i,t} = 0$). Equation (27) is used to determine that an interruption is produced ($\lambda i^W_{k,i,t+1} = 1$) when a task is active at time $t$ but inactive at time $t+1$ ($W_{k,i,\theta,t} = 1, W_{k,i,\theta,t+1} = 0$) if the potential status of a task is active ($\widehat{W}_{k,i,t} = 1$). This avoids considering the end of a task as an interruption. Equation (28) is applied to determine if a task remains interrupted at $t+1$ ($\lambda s^W_{k,i,t+1} = 1$), for tasks which are not active at $t$ and at $t+1$ ($W_{k,i,\theta,t} = W_{k,i,\theta,t+1} = 0$), but its potential status is active ($\widehat{W}_{k,i,t} = 1$). This constrains follow a general disjunctive programming (GDP) formulation. Further information can be found in Appendix A.

$$\widehat{W}_{k,i,t} \ge \lambda i^W_{k,i,t+1} + \lambda s^W_{k,i,t+1} \qquad \forall k, i\epsilon I_k, t \qquad (26)$$

$$\sum_{\theta=0}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t+1} - \sum_{\theta=0}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t} - \widehat{W}_{k,i,t} + \lambda i^W_{k,i,t+1} + 1 \ge 0 \qquad \forall k, i\epsilon I_k, t \qquad (27)$$

$$\sum_{\theta=0}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t} + \sum_{\theta=0}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t+1} - \widehat{W}_{k,i,t} + \lambda s^W_{k,i,t+1} \ge 0 \qquad \forall k, i\epsilon I_k, t \qquad (28)$$

Figure 2 represents the concept of binary variables $W_{k,i,\theta,t}$, $\widehat{W}_{k,i,t}$, $\lambda i^W_{k,i,t}$ and $\lambda s^W_{k,i,t}$. As example, a task whose processing time is 4 time intervals is represented. This involves that the task starts when $\theta = 0$ and finishes when $\theta = 4 - 1 = 3$. Thus, $\widehat{W}_{k,i,t} = 1$ during the first and the last time interval of the mentioned task. The figure highlights only the variables that are equal to 1.

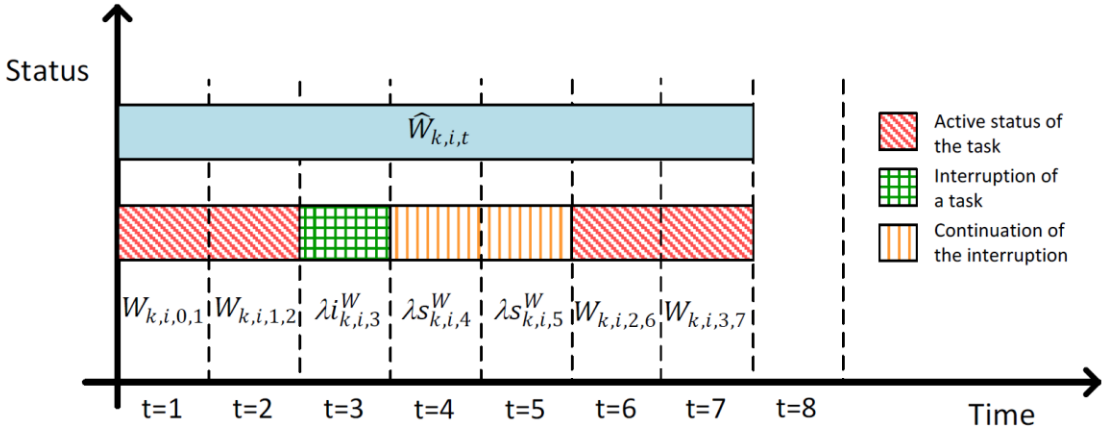

Figure 2. Explanation of binary variables involved in the interruption of tasks.

Analogously, constraints formulated in equations (25) to (28) can be applied for tasks started outside the time window, by using equations (29) to (32).

$$\hat{Z}_{k,i,t} = \sum_{\substack{t' \\ t' \le t}} Z_{k,i,\theta,t'} - \sum_{\substack{t' \\ t' < t}} Z_{k,i,\theta',t'} \qquad \forall k, i \epsilon I_k, t, \theta = 0, \theta' = \overline{PT}_{k,i} - 1 \tag{29}$$

$$\hat{Z}_{k,i,t} \ge \lambda i^Z_{k,i,t+1} + \lambda s^Z_{k,i,t+1} \qquad \forall k, i \epsilon I_k, t \tag{30}$$

$$\sum_{\theta=0}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t+1} - \sum_{\theta=0}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t} - \hat{Z}_{k,i,t} + \lambda i^Z_{k,i,t+1} + 1 \ge 0 \qquad \forall k, i \epsilon I_k, t \tag{31}$$

$$\sum_{\theta=0}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t} + \sum_{\theta=0}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t+1} - \hat{Z}_{k,i,t} + \lambda s^Z_{k,i,t+1} \ge 0 \qquad \forall k, i \epsilon I_k, t \tag{32}$$

### *3.8. Energy and heat balances*

The total electricity consumption for tasks started within the established time window is calculated as the summation of the power consumption profile capacities $C_{i,\theta}$ from all considered tasks $i$ of all homes $k$ in each time interval $t$ according to equation (33). This equation establishes that the amount of electricity consumptions corresponds to the difference between the electricity supplied by the wind turbine and the CHP generators ($P_t^W$ and $P_t^{CHP}$), the electricity received from the power grid $Im_t$ and the discharge of the electrical storage $S_t^{ED}$, minus the energy sent to charge the electrical storage system $S_t^{EC}$ and to the grid $Ex_t$. Notice that the balance constraint considers that power consumption profiles of tasks may vary over the operation time periods (see Figure 1). On the other hand, if task $i$ has started outside the time window, it is not allowed to use the generators of the microgrid, and energy purchases to the grid are required to satisfy the demand, according to equation (34).

$$\delta \cdot \sum_{k} \sum_{i \epsilon I_k} \left[ \sum_{\theta=0}^{\overline{PT}_{k,i}-1} C_{i,\theta} \cdot W_{k,i,\theta,t} - C_{i,\theta'} \cdot W_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i}\right) \right] = \delta \cdot (P_t^{W} + P_t^{CHP} + S_t^{ED} + Im_t - S_t^{EC} - Ex_t) \qquad \forall t, \theta' = \overline{PT}_{k,i} - 1 \qquad (33)$$

$$\delta \cdot \sum_{k} \sum_{i \epsilon I_k} \left[ \sum_{\theta=0}^{\overline{PT}_{k,i}-1} C_{i,\theta} \cdot Z_{k,i,\theta,t} - C_{i,\theta'} \cdot Z_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i}\right) \right] = \delta \cdot Im_t^{Z} \qquad \forall t, \theta' = \overline{PT}_{k,i} - 1 \qquad (34)$$

The terms $C_{i,\theta'} \cdot W_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i}\right)$ and $C_{i,\theta'} \cdot Z_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i}\right)$ in equations (33) and (34) are used to determine the exact energy consumption of a task $i$ in each time interval $t$. This is useful when the electrical consumption of task $i$ does not occur during an overall time interval. This allows enhancing the overall energy balance, due to a better adjustment of tasks. The meaning of this term is explained in Figure 3.

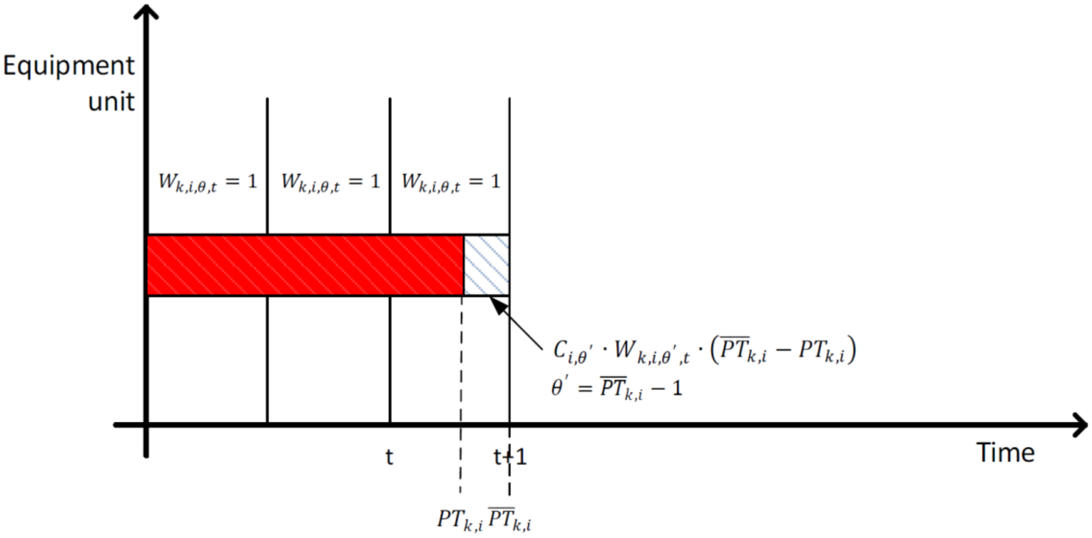

Figure 3. Time adjustment.

The heat consumed during a time interval $H_t$ corresponds to the difference between the heat provided by the CHP generator and boiler ($P_t^{CHP}$ and $P_t^{B}$), heat received from the discharge of the thermal storage system $S_t^{TD}$, minus heat transmitted to the thermal storage $S_t^{TC}$. Also, any eventual dissatisfaction in the heat demand $UH_t$ is considered.

$$\delta \cdot H_t = \delta \cdot (\alpha \cdot P_t^{CHP} + P_t^{B} + S_t^{TD} - S_t^{TC} + UH_t) \qquad \forall t \qquad (35)$$

### *3.9. Peak demand charge*

Equation (36) is introduced in the model with the aim of reducing the electric power peak demand from the power grid. Therefore, if energy purchases from grid exceed an agreed threshold $\beta$ at time $t$, the extra amount $\gamma_t$ over threshold $\beta$ is penalised with an extra rate. On the contrary, if the energy purchases are below the above mentioned threshold $\beta$, the normal electricity price is applied, since $\gamma_t$ takes value zero.

$$\gamma_t \geq Im_t - \beta \qquad \forall t \qquad (36)$$

### *3.10. Objective function*

The objective function is to minimise the operational cost $\phi$ of the microgrid. This is calculated by equation (37), including all operation and maintenance costs of generators and storage systems, purchases, penalties and revenues from electricity exported to the grid. Notice that since the equipment capacities are fixed, their capital costs do not depend on the schedule and are not considered. Particularly, the following elements are taken into account in the objective function:

| | |
|---|---|
| $\phi =$ | Operational cost |
| $\delta \cdot \sum_t \frac{ng}{\alpha} \cdot P_t^{CHP}$ | Operation and maintenance costs of the CHP generator |
| $+\delta \cdot \sum_t m^W \cdot P_t^W$ | Operation and maintenance costs of the wind turbine |
| $+\delta \cdot \sum_t \frac{ng}{\eta^B} \cdot P_t^B$ | Operation and maintenance costs of the boiler |
| $+\delta \cdot \sum_t m^E \cdot S_t^{ED}$ | Electrical storage costs |
| $+\delta \cdot \sum_t m^T \cdot S_t^{TD}$ | Thermal storage costs |
| $+\delta \cdot \sum_t b_t \cdot Im_t$ | Cost of energy imported from the grid |
| $+\delta \cdot \sum_t m^Z \cdot b_t \cdot Im_t^Z$ | Cost of energy imported from the grid for tasks staring outside the time windows |
| $+\delta \cdot \sum_t pk \cdot \gamma_t$ | Revenues from electricity exported to the grid |
| $+\delta \cdot \sum_t \mu H \cdot UH_t$ | Penalty cost for non-satisfying the heat demand |
| $-\delta \cdot \sum_t q \cdot Ex_t$ | Revenues from electricity exported to the grid |
| $+\sum_k \sum_{i \in I_k} \sum_t \mu i_i^W \cdot \lambda i_{k,i,t}^W$ | Penalty cost if task starting inside the time windows is interrupted |
| $+\sum_k \sum_{i \in I_k} \sum_t \mu s_i^W \cdot \lambda s_{k,i,t}^W$ | Penalty cost if task starting inside the time windows remains interrupted |
| $+\sum_k \sum_{i \in I_k} \sum_t \mu i_i^Z \cdot \lambda i_{k,i,t}^Z$ | Penalty cost if task starting outside the time windows is interrupted |

$$+\sum_{k}\sum_{i\epsilon I_k}\sum_{t}\mu s_i^Z \cdot \lambda s_{k,i,t}^Z \qquad \text{Penalty cost if task starting outside the time windows remains interrupted}$$

$$+\sum_{k}\sum_{i\epsilon I_k} CPen_{k,i} \qquad \text{Penalties in case of deviation from the initial starting time} \tag{37}$$

Notice that this work is not focused on the design of the microgrid, and only short-term decisions are considered. Thus, the model does not take into account fixed costs associated with the investment and installation of electricity and heat generator sources. This work addresses the management of the existing generators in the microgrid, and the design of a new/extended microgrid is out of the current scope of the work. Moreover, the model does not consider more detailed generation constraints, such as ramping constraints, minimum up/down time constraints, non-convex generation costs and time-dependent startup costs. This kind of constraints introduces non-linearities in the problem formulation, involving more computational effort to obtain the optimal solution. Only linear constraints have been taken into account since the objective is to study benefits of managing electricity and heat supply and demand within a microgrid.

## 4. Illustrative example

The proposed MILP model is applied to a microgrid formed of 1, 5, 10, 15 and 20 homes. A schematic representation of the microgrid is given in Figure 4. This illustrative example has considered common electricity and heat generators, as well as representative household equipment units that define the overall demand. Thus, all these physical elements are realistic and may constitute a real domestic microgrid.

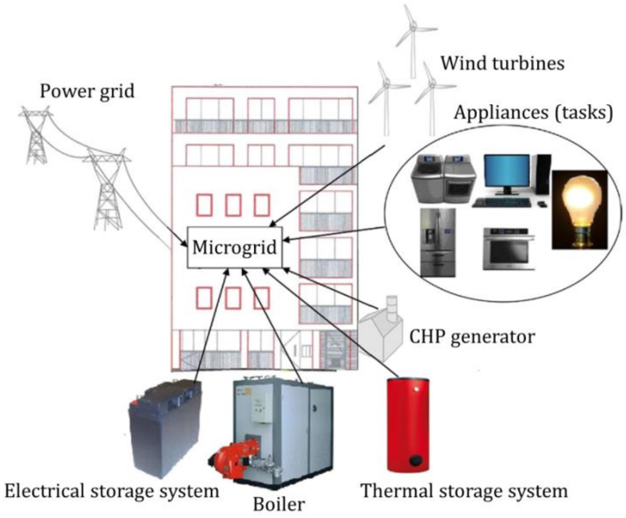

Figure 4. Schematic representation of the microgrid (Zhang et al., 2013).

The technical parameters and costs are next detailed:

- One CHP generator with a maximum capacity of 1.2 $kW_e$ per house and electrical efficiency of 35%. Heat to power ratio is assumed to be constant and equal to 1.3.

- One wind turbine per home, with a maximum capacity of 10 $kW_e$ and a maintenance cost of 0.005 $£/kWh_e$. The power coefficient is 47%, and the blade diameter is 4.0 $m$. The cut-in and cut-out wind speeds are 5.0 $m/s$ and 25.0 $m/s$, respectively, whereas the nominal wind speed is 12.0 $m/s$.
- One boiler with a capacity of 2.8 $kW_{th}$ per house. Natural gas cost is 0.027 $£/kWh$.
- One electrical storage unit with a maximum capacity of 0.500 $kWh_e$. Charge and discharge efficiencies are assumed to be 95%, discharge limit and charge limits are 0.333 $kW_e$, and the maintenance cost is 0.005 $£/kWh_e$.
- One thermal storage unit with a maximum capacity of 0.700 $kWh_{th}$. Charge and discharge efficiencies are assumed to be 98%, discharge limit and charge limits are 0.667 $kW_{th}$, and the maintenance cost is 0.001 $£/kWh_{th}$.
- The connection to the power grid. This connection allows importing and exporting electricity. The real-time energy price at different times is collected from Balancing Mechanism Reporting System. Furthermore, an extra cost of 0.05 $£/kWh_e$ is charged if electricity demand per house from the grid is over 1 $kW_e$. Electricity may also be sold to the power grid with 0.01 $£/kWh_e$.

Equal-size time intervals are considered. The length of the time intervals is 30 minutes. The considered scheduling horizon is one day, resulting 48 time intervals.

Regarding the electricity demand, 12 equipment units $j$ per home $k$ are considered to perform 16 different tasks $i$. The description of the equipment units that constitute the illustrative example can be found in Table 1.

Table 1. Description of the equipment units involved in the illustrative example.

| Equipment unit $j$ | Description |
|---|---|
| $j1$ | Dishwasher |
| $j2$ | Washing machine |
| $j3$ | Spin dryer |
| $j4$ | Cooker hob |
| $j5$ | Cooker oven |
| $j6$ | Microwave |
| $j7$ | Interior lighting |
| $j8$ | Laptop |
| $j9$ | Desktop |
| $j10$ | Vacuum cleaner |
| $j11$ | Fridge |
| $j12$ | Electric car |

All tasks, except $i1$ and $i2$, have constant power consumption profiles. Data related to energy requirements, earliest and latest starting time and processing time of tasks can be found in Table 2. The power consumption for the non-constant profile tasks can be found in Table 3.

Table 2. Power requirements, earliest and latest starting time and processing time of tasks.

| Task $i$ | Equipment unit $j$ | Power, $C_{i,\theta}$ $(kW)$ | Earliest starting time, $Ts_{k,i}^{min}$ $(h)$ | Latest starting time, $Ts_{k,i}^{max}$ $(h)$ | Processing time, $PT_{k,i}$ $(h)$ |
|---|---|---|---|---|---|
| $i1$ | $j1$ | Variable | 0.5 | 7.0 | 2.0 |
| $i2$ | $j2$ | Variable | 1.0 | 4.0 | 1.5 |
| $i3$ | $j3$ | 2.50 | 5.0 | 9.0 | 1.2 |
| $i4$ | $j4$ | 3.00 | 0.0 | 4.5 | 1.0 |
| $i5$ | $j5$ | 5.00 | 10.0 | 14.5 | 0.7 |
| $i6$ | $j6$ | 1.70 | 0.0 | 2.0 | 0.5 |
| $i7$ | $j7$ | 0.84 | 10.0 | 10.5 | 6.0 |
| $i8$ | $j8$ | 0.10 | 9.0 | 16.0 | 2.3 |
| $i9$ | $j9$ | 0.30 | 9.0 | 13.5 | 3.0 |
| $i10$ | $j10$ | 1.20 | 0.0 | 9.5 | 0.8 |
| $i11$ | $j11$ | 0.30 | 0.0 | 24.0 | 24.0 |
| $i12$ | $j12$ | 3.50 | 10.0 | 14.0 | 3.1 |
| $i13$ | $j3$ | 2.50 | 10.5 | 15.5 | 1.8 |
| $i14$ | $j6$ | 1.70 | 6.5 | 12.0 | 0.9 |
| $i15$ | $j9$ | 0.30 | 16.5 | 21.0 | 3.4 |
| $i16$ | $j12$ | 3.50 | 17.5 | 22.0 | 1.5 |

Table 3. Variable power requirements.

| Operation period $\theta$ | Task $i1$ | Task $i2$ |
|---|---|---|
| $\theta 0$ | 1.80 | 2.15 |
| $\theta 1$ | 0.22 | 0.21 |
| $\theta 2$ | 1.80 | 0.45 |
| $\theta 3$ | 0.22 | |

Also, data related to heat demand, wind forecast speed as well as electricity prices can be found in Table B1 in Appendix B. Furthermore, penalty costs for interrupting a task $i$ or for remaining interrupted, within and outside the time window can be found in Table B2 in Appendix B. If the heat demand is not satisfied, a penalty cost of 0.3 $£/kWh_{th}$ is applied. Also, the table above contains the penalty cost from any eventual deviation from the desired initial starting time of task $i$ at home $k$. If any task $i$ starts outside the established time window, an extra cost factor $m^Z$ is applied. This parameter takes value 1.50, which means an extra 50% on the price of the purchases of energy to the power grid.

## 5. Results and discussion

The proposed model has been applied to a microgrid of 1, 5, 10, 15 and 20 homes to analyse the characteristics of the proposed model. Different scenarios have been considered, comprising:

A. the absence of demand management (tasks cannot be postponed)
B. simultaneous generation, purchases and demand management of uninterruptible tasks
C. simultaneous generation, purchases and demand management of interruptible tasks

In the first scenario, only electricity and heat generation management is taken into account. Thus, the storage systems are the only resource to decouple generation and demand, since demand requirements may not match the availability of generation sources. One characteristic of the obtained solution is the fact that the combination of electricity and heat generation sources and storage systems do no satisfy the overall demand. Consequently, purchases to the power grid are required. Figures 5a and 5b show the electric power generation (and purchases) and the thermal power generation profiles, respectively. The electricity and heat storage level profiles are represented in Figure 5c. Also, Figure 5d plots the schedule of tasks. Only the solution for one home is presented in this scenario since the aim is to show how the management of the demand may improve the optimal solution. Each colour in this plot denotes a task.

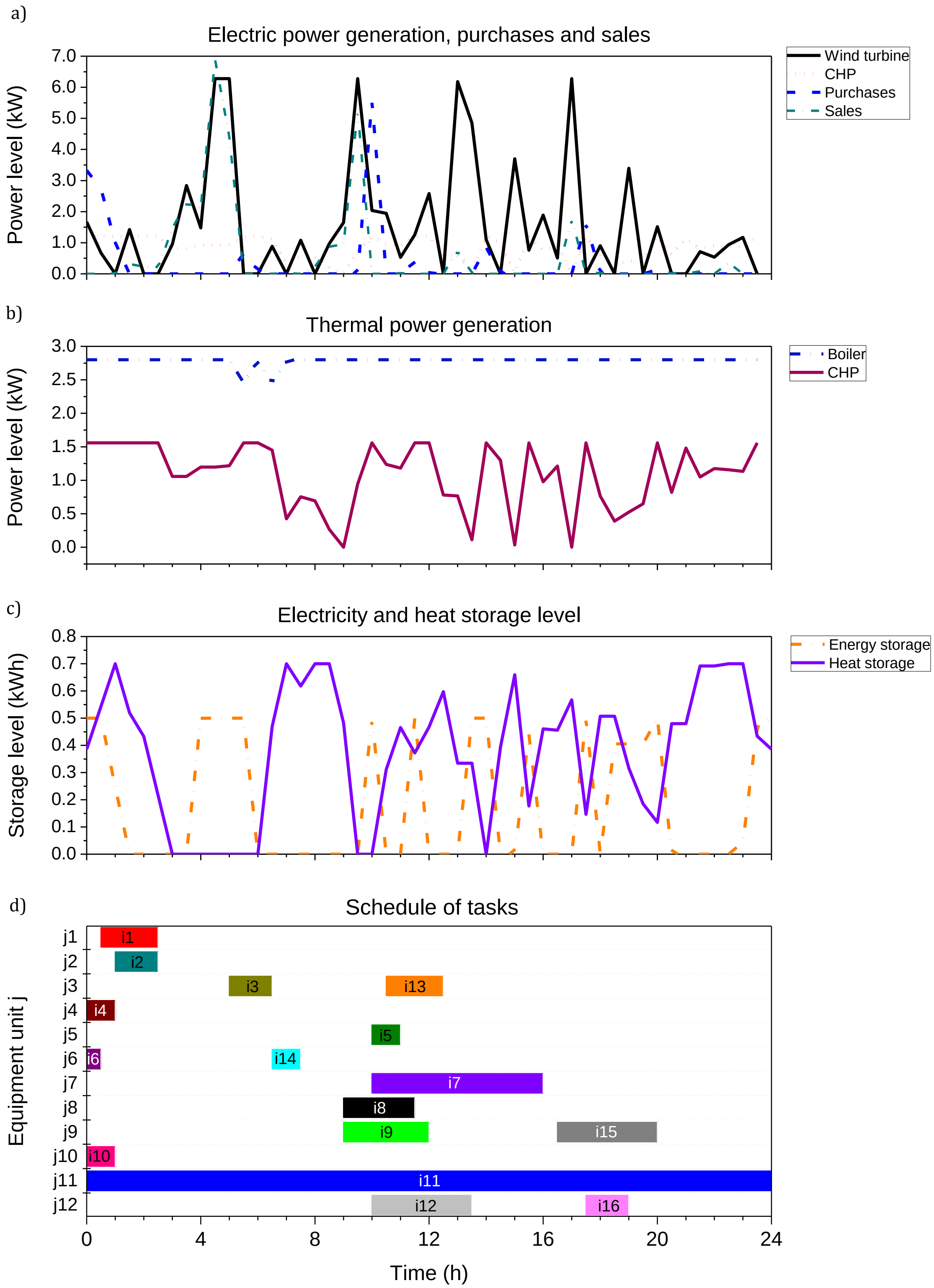

Figure 5. a) Electric power generation, purchases and sales; b) Thermal power generation profiles; c) Electricity and heat storage level profiles; d) Schedule of uninterruptible and non-manageable tasks for 1 home.

The second scenario allows delaying the starting time of tasks, at the payment of a penalty cost for each delay. However, once a task is started, this task must be finished with no interruptions during its processing time. So, the binary variables associated to any eventual interruption ($\lambda i_{k,i,t}^{W}, \lambda s_{k,i,t}^{W}, \lambda i_{k,i,t}^{Z}, \lambda s_{k,i,t}^{Z}$) are forced to be zero. The optimal decisions in scenario B lead to a reduction in the dependence on the power grid, which means less external purchases to satisfy the overall electricity and heat demand (Figure 6a). Figure 6b plots the thermal power generation level for scenario B. The energy and heat storage level profiles for this scenario are represented in Figure 6c. Moreover, Figure 6d shows the schedule of uninterruptable tasks. Delays are produced in tasks $i4$, $i5$, $i12$, $i13$, $i14$ and $i16$. These delayed tasks have reduced penalty cost in case of variation from the initial target. Notice that if this penalty is expensive or infinite, this task will not be postponed. Furthermore, it is worth mentioning that all tasks have been performed inside the considered time window since this is more economical than operating outside the time window. Also, notice that the starting time of all tasks is located inside the established time window. This is because perform the consumption inside the time window is more economical that performing it outside the time window, since this requires to purchase the energy to be consumed.

The third scenario also allows postponing the initial time of tasks, applying a penalisation in case of variation from the target. The different between the second and the third scenarios is the fact that tasks can be interruptible, applying a penalty cost in case of any interruption in a task. Although a task can be interrupted, this task must finish, and must re-start again from the moment where this task was interrupted. The optimal solution in scenario C improves the solution in scenario B (as well as in scenario A). The optimal decisions in scenario C lead to a reduction in the dependence on the power grid, which means less external purchases to satisfy the overall electricity and heat demand (Figure 7a). Figure 7b shows the thermal power generation profile for scenario C. The energy and heat storage levels for the third scenario are plotted in Figure 7c. Finally, Figure 7d represents the schedule of tasks, where some of these tasks are delayed and interrupted. Delays in the starting time of tasks were produced in tasks $i4$, $i5$, $i12$, $i14$ and $i16$. In comparison with the previous scenario, in scenario C there is no delay in task $i14$, and the delays in tasks $i12$ and $i16$ are reduced in 3 and 1 hours, respectively. The delays in the starting times are reduced by 40 %. This diminution is due to a better accommodation of the runtimes of tasks since they can be disrupted. The interruptions of the considered tasks are produced in $i7$, $i10$, $i12$, $i14$, $i15$ and $i16$. Table 4 contains the interruptions of the tasks. These interrupted tasks have reduced penalty cost in case of disruption. Notice that if this penalty is expensive or infinite, this task will not be interrupted. Furthermore, the interruptions in tasks $i12$ and $i16$ have involved the reduction in the delay associated with their starting times.

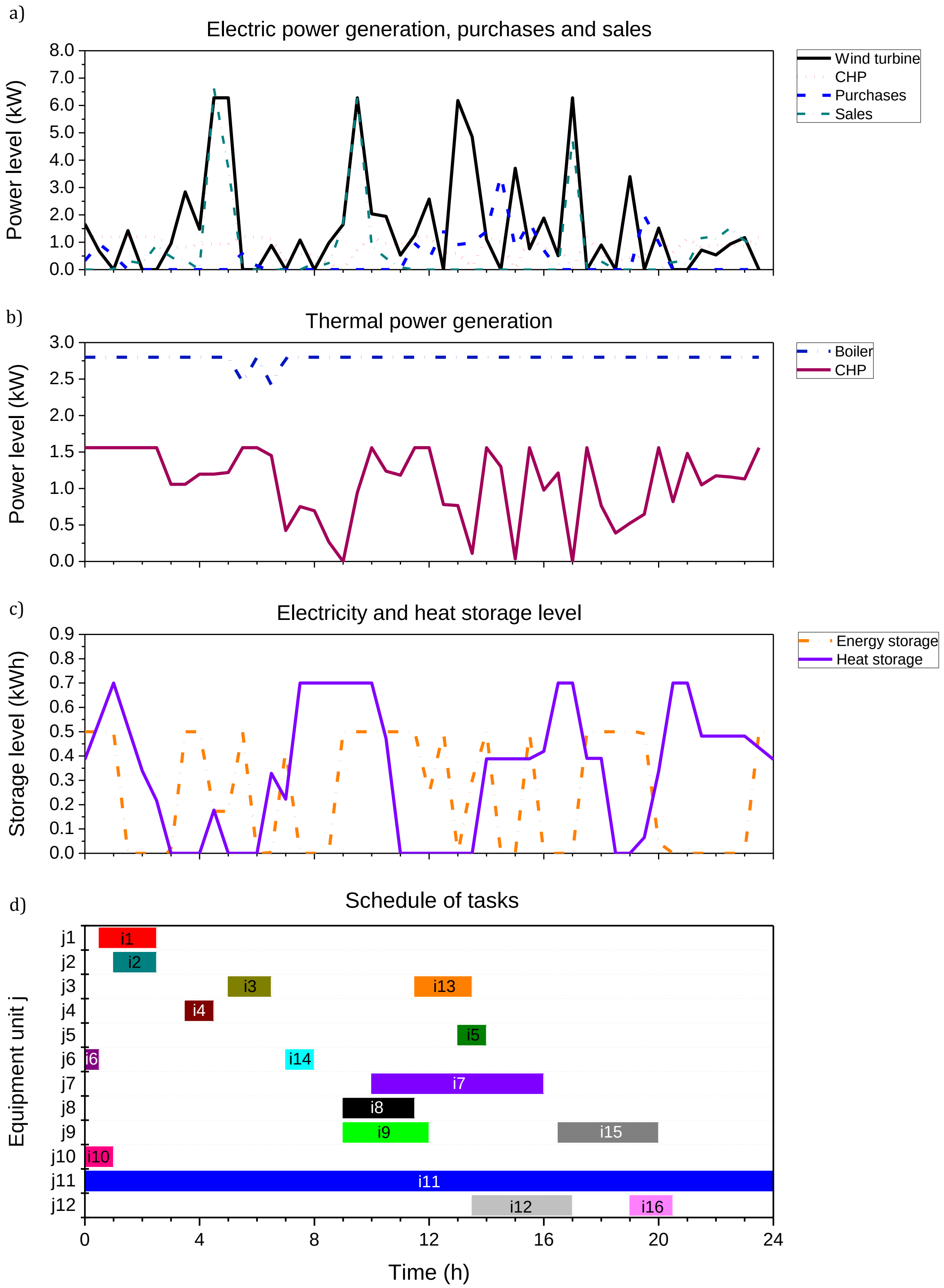

Figure 6. a) Electric power generation, purchases and sales; b) Thermal power generation profiles; c) Electricity and heat storage level profiles; d) Schedule of uninterruptible and manageable tasks for 1 home.

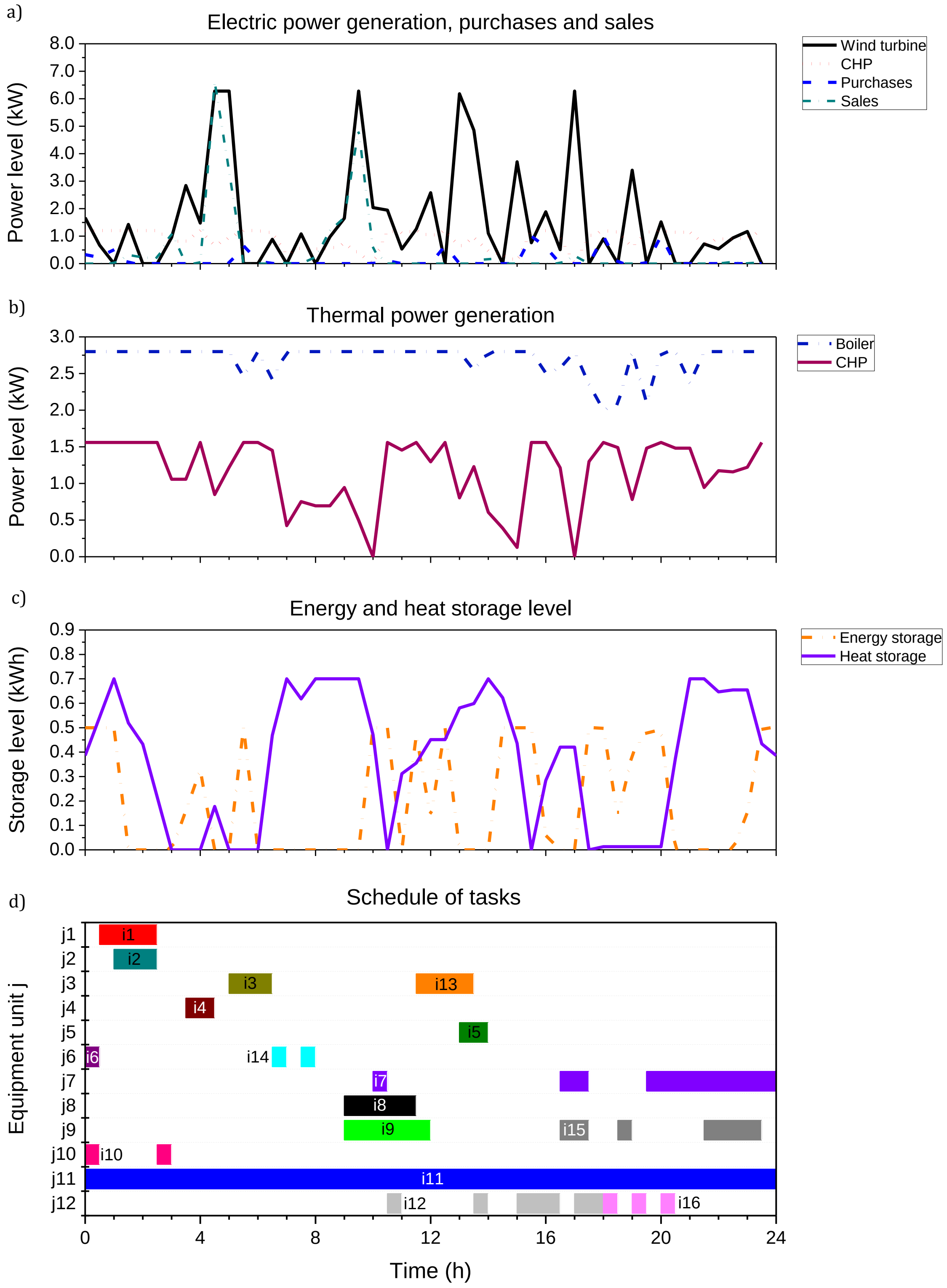

Figure 7. a) Electric power generation, purchases and sales; b) Thermal power generation profiles; c) Electricity and heat storage level profiles; d) Schedule of interruptible and manageable tasks for 1 home.

Table 4. Interruptions associated with different tasks for 1 home.

| Task | Number of interruptions | Time interrupted (h) |
|---|---|---|
| $i7$ | 2 | 8.0 |
| $i10$ | 1 | 2.0 |
| $i12$ | 3 | 4.0 |
| $i14$ | 1 | 0.5 |
| $i15$ | 2 | 3.5 |
| $i16$ | 2 | 1.0 |

Table 5 compares the electricity generation, purchases and sales as well as the heat generation for 1 home. The administration of interruptible and manageable tasks involves less necessity of energy (the amount of electricity generation and purchases), reducing the costs of the network. This is due to a better adjustment of tasks according to the availability and prices of energy, which improves the optimal solution. In scenario B (uninterruptible tasks that can be delayed), there is an increase in the electricity purchases to the grid. However, the increase in the electricity sales involves an enhancement in the management of the considered microgrid. This means that some of the delays were due to the lack of availability of electricity generated inside the microgrid, and other delays were produced in order to sell energy to the power grid in more profitable periods of time.

Table 5. Electricity and heat generation, purchases and sales for 1 home.

| Energy and heat levels | Scenario A | Scenario B | Scenario C |
|---|---|---|---|
| Electricity via wind turbine (kWh) | 37.6 | 37.6 | 37.6 |
| Electricity via CHP (kWh) | 20.0 | 20.0 | 21.5 |
| Electricity purchases (kWh) | 8.2 | 9.2 | 3.1 |
| Electricity sales (kWh) | 14.2 | 16.9 | 10.5 |
| Heat via boiler (kWh) | 66.8 | 66.8 | 64.9 |
| Heat via CHP (kWh) | 26.0 | 26.0 | 27.9 |

Moreover, a different number of homes within the microgrid have been considered. Table 6 shows the solutions of the three scenarios (A, B and C) for 1, 5, 10, 15 and 20 homes. As expected, the consideration of interruptible and manageable tasks improves the optimal solution, regardless of the number of considered homes. Particularly, the solution is improved by 5% comparing uninterruptible and interruptible manageable tasks. This table also contains the total delays in the starting time of all tasks and the total interrupting time of tasks for the considered number of homes within the microgrid. Note that delays for scenario A and interruptions for scenarios A and B have not been considered, because on the one hand, scenario A does not allow delays nor interruptions; and on the other hand, scenario B does not allow interruptions. Thus, their values are zero.

Table 6. Daily cost, delays and interruptions for a different number of homes under different scenarios.

| Number of homes | Daily cost (£) | | | Total delays (h) | | Total interruptions (h) |
|---|---|---|---|---|---|---|
| | Scenario A | Scenario B | Scenario C | Scenario B | Scenario C | Scenario C |
| 1 home | 4.93 | 4.78 | 4.45 | 13.0 | 8.5 | 19.0 |
| 5 homes | 31.72 | 23.10 | 21.95 | 67.5 | 41.0 | 104.5 |
| 10 homes | 63.43 | 46.16 | 43.83 | 137.0 | 78.0 | 214.0 |
| 15 homes | 95.15 | 69.31 | 65.71 | 203.5 | 122.0 | 313.0 |
| 20 homes | 126.87 | 92.17 | 87.66 | 281.0 | 159.0 | 418.5 |

Moreover, different approaches have been contemplated to analyse the optimal solution when all homes are solved simultaneously instead of solved one by one. Thus, the management of the microgrid can be considered by solving the following approaches:

A. Approach A: Simultaneous management of all homes.
B. Approach B: Sequential management of all homes. This approach is solved iteratively, starting by the first home, fixing the decisions concerning the above home, and iterating one by one until the last home.

Table 7 summaries the obtained results for these approaches, considering 20 homes. The optimal solution is improved by 1.9% when the simultaneous homes are considered instead of solving the model home by home. This difference is because approach A finds the optimal solution for all homes. However, approach B seeks the optimal solution for the homes taken into account. This means that the solution in each iteration tries to optimise the use of the elements of the microgrid, without considering other homes. Thus, the schedule of tasks in each iteration is accommodated to the available resources, disregarding other homes. Moreover, although the electricity demand in the overall scheduling horizon is the same for both approaches, the difference on the schedule of tasks leads to obtain different energy requirements in each period of time. This difference may involve changes in the electricity purchases, which impact on the value of the optimal solution.

Table 7. Results for the optimal management considering different approaches for 20 homes.

| Costs and electricity and heat levels | Approach A | Approach B |
|---|---|---|
| Objective function (£) | 87.88 | 89.55 |
| Electricity via wind turbine (kWh) | 752.5 | 752.5 |
| Electricity via CHP (kWh) | 441.5 | 458.4 |
| Electricity purchases (kWh) | 18.8 | 45.4 |
| Electricity demand (kWh) | 181.0 | 225.7 |
| Energy sales (kWh) | 1025.1 | 1025.1 |
| Heat via boiler (kWh) | 1278.2 | 1255.6 |
| Heat via CHP (kWh) | 574.0 | 595.9 |

The mathematical models have been implemented in GAMS 24.7 and solved using CPLEX 12.6, in an Intel® Xenon® CPU E5-1650 v3 @ 3.50 GHz, with 32.00 GB of installed memory (RAM). Table 8 summarises the main model statistics associated with the solution of the mathematical model for scenario C (simultaneous generation/purchase and demand management of interruptible tasks) considering a different number of homes. Obviously, the consideration of more homes involves more equations, continuous variables and binary variables. This increase in the size of the problem implies an exponential augment in the computational time to reach the optimal solution. Thus, the relative gap has been modified in comparison to the case that considers only one home, to obtain a solution in a reasonable time. The optimisation procedure was interrupted after 3600 seconds when 15 and 20 homes were considered. The reported relative gap corresponds to the gap when the process was aborted.

Table 8. Model and computational statistics.

| Model statistics | 1 home | 5 homes | 10 homes | 15 homes | 20 homes |
|---|---|---|---|---|---|
| Equations | 83,842 | 416,318 | 831,913 | 1,247,508 | 1,663,103 |
| Continuous variables | 79,572 | 395,348 | 790,068 | 1,184,788 | 1,579,508 |
| Discrete variables | 76,800 | 384,000 | 768,000 | 1,152,000 | 1,536,000 |
| Resource time (CPU, s) | 66.0 | 727.4 | 2971.7 | 3600.0 | 3600.0 |
| Relative gap (%) | 0.00 | 1.00 | 1.00 | 1.21 | 2.38 |

## 6. Concluding remarks

This work addresses the management of a microgrid to determine optimal energy and heat generation, purchases, sales, storage and consumption. Different tasks have been scheduled considering consumption profiles, time windows, possibility to interrupt them, non-constant grid energy prices and peak demand extra penalisation to reduce the operational cost. As a consequence, important cost savings and have been accomplished.

The results reveal that the proposed MILP formulation is capable of dealing with the operational scheduling problem of a microgrid to exploit the benefits of the flexibility in the energy demand, such as delays in the starting times and eventual interruptions during the performance of any task. The simultaneous management of generation, purchases, sales, storage and demand in microgrids have allowed improving their efficiency and self-sufficiency. In the suggested illustrative example, the results exhibit that the flexibility in the management of the microgrid allows significant cost savings. This involves reducing the operational cost of the proposed illustrative example by 5 % on average, as well as reducing the energy purchases to the power grid.

The results expose that the proposed formulation may also be used as the basis for solving more complex problems, such as industrial cases. Further work embraces incorporating uncertainty in the model, affecting both generation and demand of energy and heat (i.e., weather conditions, consumption habits). Furthermore, the mathematical model can be extended by considering a start-up consumption profile in case of any interruption.

## Nomenclature

### *Indexes and sets*

| | |
|---|---|
| $i$ | task |
| $j$ | equipment unit |

| | |
|---|---|
| $k$ | home in the microgrid |
| $t$ | time interval |
| $\theta$ | task operation period |
| $i\epsilon I_j$ | subset of tasks $i$ performed in equipment unit $j$ |
| $i\epsilon I_k$ | subset of tasks $i$ performed at home $k$ |
| $j\epsilon J_k$ | subset of equipment units $j$ available at home $k$ |

### *Parameters*

| | |
|---|---|
| $A$ | wind generator blade area ($m^2$) |
| $b_t$ | electricity buying price from power grid at time $t$ (£/$kWh_e$) |
| $C_{i,\theta}$ | power consumption capacity of task $i$ at operation period $\theta$ ($kW_e$) |
| $C^{CHP}$ | combined heat and power (CHP) generator capacity ($kW_e$) |
| $C^W$ | wind generator capacity ($kW_e$) |
| $C^B$ | boiler capacity ($kW_{th}$) |
| $C^E$ | electrical storage capacity ($kWh_e$) |
| $C^T$ | thermal storage capacity ($kWh_{th}$) |
| $H_t$ | heat demand at time $t$ ($kW_{th}$) |
| $m^E$ | cost per unit input (maintenance) for electrical storage unit (£/$kWh_e$) |
| $m^T$ | cost per unit input (maintenance) for thermal storage unit (£/$kWh_{th}$) |
| $m^W$ | wind generator maintenance cost (£/$kWh_e$) |
| $m^Z$ | extra cost factor due to starting the task outside the time window |
| $ng$ | price of natural gas (£/$kWh$) |
| $pk$ | difference between peak and base electricity demand price from grid (£/$kWh_e$) |
| $P_t^W$ | electric power generated from wind generator at time $t$ ($kW_e$) |
| $PT_{k,i}$ | processing time of task $i$ at home $k$ |
| $\overline{PT}_{k,i}$ | roundup value of the processing time of task $i$ at home $k$ |
| $q$ | electric power selling price to grid (£/$kWh_e$) |
| $S^{EDmax}$ | electrical power storage discharge limit ($kW_e$) |
| $S^{TDmax}$ | thermal power storage discharge limit ($kW_{th}$) |
| $S^{ECmax}$ | electrical power storage charge limit ($kW_e$) |
| $S^{TCmax}$ | thermal power storage charge limit ($kW_{th}$) |
| $SH$ | scheduling horizon ($h$) |
| $Ts_{k,i}^{max}$ | latest starting time of task $i$ at home $k$ |
| $Ts_{k,i}^{min}$ | earliest starting time of task $i$ at home $k$ |
| $v_t$ | wind speed at time $t$ ($m/s$) |
| $V^{nom}$ | nominal wind speed ($m/s$) |
| $V^{cut-in}$ | cut in wind speed ($m/s$) |
| $V^{cut-out}$ | cut out wind speed ($m/s$) |
| $\alpha$ | CHP heat-to-power ratio |
| $\beta$ | agreed electric power peak demand threshold from the grid ($kW_e$) |
| $\delta$ | time interval duration ($h$) |
| $\eta^B$ | boiler efficiency |
| $\eta^{CHP}$ | CHP generator electrical efficiency |
| $\eta^E$ | electrical storage charge/discharge efficiency |
| $\eta^T$ | thermal storage charge/discharge efficiency |
| $\eta^W$ | wind generator efficiency |
| $\mu_{k,i}$ | penalty cost for delay in the starting time of operation $i$ at home $k$ (£/$h$) |
| $\mu h$ | penalty cost for unsatisfied heat demand (£/$kWh_{th}$) |

| | |
|---|---|
| $\mu i_i^W$ | penalty cost for interrupting task $i$ started within the time window (£) |
| $\mu s_i^W$ | penalty cost for remaining interrupted task $i$ started within the time window (£) |
| $\mu i_i^Z$ | penalty cost for interrupting task $i$ started outside the time window (£) |
| $\mu s_i^Z$ | penalty cost for remaining interrupted task $i$ started outside the time window (£) |
| $\rho$ | air density ($kg/m^3$) |

*Variables*

| | |
|---|---|
| $CPen_{k,i}$ | penalty cost for delays in the starting time of task $i$ at home $k$ (£) |
| $Ex_t$ | electric power exported to the grid at time $t$ ($kW_e$) |
| $Im_t$ | electric power imported from the grid at time $t$ ($kW_e$) |
| $P_t^B$ | thermal power generated from boiler at time $t$ ($kW_{th}$) |
| $P_t^{CHP}$ | electric power generated from CHP generator at time $t$ ($kW_e$) |
| $S^{IE}$ | initial state of electrical storage ($kW_e$) |
| $S^{IT}$ | initial state of thermal storage ($kW_{th}$) |
| $S_t^E$ | electricity in storage at time $t$ ($kWh_e$) |
| $S_t^{EC}$ | electric power storage charge rate at time $t$ ($kW_e$) |
| $S_t^{ED}$ | electric power storage discharge rate at time $t$ ($kW_e$) |
| $S_t^T$ | heat in storage at time $t$ ($kWh_{th}$) |
| $S_t^{TC}$ | thermal power storage charge rate at time $t$ ($kW_{th}$) |
| $S_t^{TD}$ | thermal power storage discharge rate at time $t$ ($kW_{th}$) |
| $Ts_{k,i}$ | starting time of task i at home k (h) |
| $UH_t$ | unsatisfied thermal power demand at time $t$ ($kW_{th}$) |
| $\gamma_t$ | extra electric power from grid over the agreed threshold $\beta$ at time $t$ ($kW_e$) |
| $\phi$ | daily cost of the microgrid (£) to be minimised |

*Binary variables*

| | |
|---|---|
| $W_{k,i,\theta,t}$ | 1 if task $i$ at home $k$ is active in operation period $\theta$ at time $t$ starting within the time window, 0 otherwise |
| $Z_{k,i,\theta,t}$ | 1 if task $i$ at home $k$ is active in operation period $\theta$ at time $t$ starting outside the time window, 0 otherwise |
| $\widehat{W}_{k,i,t}$ | 1 if the potential task $i$ at home $k$ is active at time $t$ starting within the time window, 0 otherwise |
| $\hat{Z}_{k,i,t}$ | 1 if the potential task $i$ at home $k$ is active at time $t$ starting outside the time window, 0 otherwise |
| $\lambda i_{k,i,t}^W$ | 1 if task $i$ at home $k$ within the time window is interrupted at time $t$, 0 otherwise |
| $\lambda s_{k,i,t}^W$ | 1 if task $i$ at home $k$ within the time window remains interrupted at time $t$, 0 otherwise |
| $\lambda i_{k,i,t}^Z$ | 1 if task $i$ at home $k$ outside the time window is interrupted at time $t$, 0 otherwise |
| $\lambda s_{k,i,t}^Z$ | 1 if task $i$ at home $k$ outside the time window remains interrupted at time $t$, 0 otherwise |

## Acknowledgements

The authors would like to thank the financial support received from the UK Engineering and Physical Sciences Research Council (under the project EP/M028240/1). Also, authors would like to appreciate Dr. Georgios M. Kopanos (Cranfield University) and Dr. Di Zhang (University College London) for useful discussions.

## References

Ahadi, A., Kang, S.K. & Lee, J.H. (2016). A novel approach for optimal combinations of wind, PV, and energy storage system in diesel-free isolated communities. Applied Energy, 170, 101-115.

Asano, H., Bando, S. & Watanabe, H. (2007). Methodology to design the capacity of a microgrid. IEEE International Conference on System of Systems Engineering, 1-6.

Bagherian, A. & Tafreshi, S.M.M. (2009). A developed energy management system for a microgrid in the competitive electricity market. IEEE Bucharest PowerTech, 1-6.

Baños, R., Manzano-Agugliaro, F., Montoya, F.G., Gil, C. Alcayde, A. & Gómez, J. (2011). Optimization methods applied to renewable and sustainable energy: A review. Renewable and Sustainable Energy Reviews, 15, 1753-1766.

Bracco, S., Delfino, F., Pampararo, F., Robba, M. & Rossi, M. (2015). A dynamic optimization-based architecture for polygeneration microgrids with tri-generation, renewables, storage systems and electrical vehicles. Energy Conversion and Management, 96, 511-520.

Caprino, D., Della Vedova, M.L. & Facchinetti, T. (2014). Peak shaving through real-time scheduling of household appliances. Energy and Buildings, 75, 133-148.

Carrión, M. & Arroyo, J.M. (2006). A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Transactions on Power Systems, 21, 1371-1378.

Charoni, D., Chiesa, V., Colasanti, L., Cucchiella, F., D'Adamo, I. & Frattini, F. (2014). Evaluating solar energy profitability: A focus on the role of self-consumption. Energy Conversion and Management, 88, 317-331.

Chen, H.C. (2013). Optimum capacity determination of stand-alone hybrid generation system considering cost and reliability. Applied Energy, 103, 155-164.

Chicco, G. & Mancarella, P. (2009). Matrix modelling of small-scale trigeneration systems and application to operational optimization. Energy, 34, 261-73.

Coroamă, I., Chicco, G., Gavrilaş M. & Russo, A. (2013). Distribution system optimisation with intra-day network reconfiguration and demand reduction procurement. Electric Power Systems Research, 98, 29-38.

Costanzo, G.T., Kheir, J. & Zhu, G. (2011). Peak-load shaving in smart homes via online scheduling. IEEE international symposium on industrial electronics, 1347-1352.

Derakhshandeh, S.Y., Masoum, A.S., Deilami, S., Masoum, M.A.S. & Golshan, M.E.H. (2013). Coordination of generation scheduling with PEVs charging in industrial microgrids. IEEE Transactions on Power Systems, 28, 3451-3461.

Escrivá-Escrivá, G. (2011). Basic actions to improve energy efficiency in commercial buildings in operation. Energy and Buildings, 43, 3106-3111.

Gamarra, C. & Guerrero, J.M. (2015). Computational optimization techniques applied to microgrids planning: A review. Renewable and Sustainable Energy Reviews, 48, 413-424.

Ghazvini, M.A.F., Morais, H. & Vale, Z. (2012). Coordination between mid-term maintenance outage decisions and short-term security-constrained scheduling in smart distribution systems. Applied Energy, 96, 281-291.

Han, Y., Young, P. & Zimmerle, D. (2013). Microgrid generation units optimum dispatch for fuel consumption minimization. Journal of Ambient Intelligence and Humanized Computing, 4, 685-701.

Hadera, H., Harjunkoski, I., Sand, G., Grossmann, I.E. & Engell, S. (2015). Optimization of steel production scheduling with complex time-sensitive electricity cost. Computers & Chemical Engineering, 76, 117-136.

Hawkes, A.D. & Leach, M.A. (2009). Modelling high level system design and unit commitment for a microgrid. Applied Energy, 86, 1253-1265.

Hodge, B.M., Huang, S., Siirola, J.D., Pekny, J.F. & Reklaitis, G.V. (2011). A multi-paradigm modeling framework for energy systems simulation and analysis. Computers & Chemical Engineering, 35, 1725-1737.

Honarmand, M., Zakariazadeh, A. & Jadid, S. (2014). Integrated scheduling of renewable generation and electric vehicles parking lot in a smart microgrid. Energy Conversion and Management, 86, 745-755.

Ipakchi, A. & Albuyeh, F. (2009). Grid of the future. IEEE Power and Energy Magazine, 7, 52-62.

Iqbal, M., Azam, M., Naeem, M., Khwaja, A.S., & Anpalagan, A. (2014). Optimization classification, algorithms and tools for renewable energy: A review. Renewable and Sustainable Energy Reviews, 39, 640-654.

Ji, L., Niu, D.X., Xu, M. & Huang, G.H. (2015). An optimization model for regional micro-grid system management based on hybrid inexact stochastic-fuzzy chance-constrained programming. International Journal of Electrical Power & Energy Systems, 64, 1025-1039.

Kabalci, E. (2015). A smart monitoring infrastructure design for distributed renewable energy systems. Energy Conversion and Management, 90, 336-346.

Keirstead, J., Samsatli, N., Shah, N. & Webber, C. (2012). The impact of CHP (combined heat and power) planning restrictions on the efficiency of urban energy systems. Energy, 41, 93-103.

Kopanos, G.M., Georgiadis, M.C. & Pistikopoulos, E.N. (2013). Energy production planning of a network of micro combined heat and power generators. Applied Energy, 102, 1522-1534.

Krishnamurti, T., Schwartz, D., Davis, A., Fischhoff, B., Bruin, W.B., Lave, L. & Wang, J. (2012). Preparing for smart grid technologies: A behavioral decision research approach to understanding consumer expectations about smart meters. Energy Policy, 41, 790-797.

Krishnan, R. (2008). Meters of Tomorrow. IEEE Power and Energy Magazine, 92-94.

Li, B., Roche, R. & Miraoui, A. (2017). Microgrid sizing with combined evolutionary algorithm and MILP unit commitment. Applied Energy, 188, 547-562.

Lior, N. (2010). Sustainable energy development: the present (2009) situation and possible paths to the future. Energy, 35, 3976-3994.

Manfren, M., Caputo, P. & Costa, G. (2011). Paradigm shift in urban energy systems through distributed generation: Methods and models. Applied Energy, 88, 1032-1048.

Mehleri, E.D., Sarimveis, H., Markatos, N.C. & Papageorgiou, L.G. (2012). A mathematical programming approach for optimal design of distributed energy systems at the neighbourhood level. Energy, 44, 96-104.

Mitra, S., Grossmann, I.E., Pinto, J.M. & Arora, N. (2012). Optimal production planning under time sensitive electricity prices for continuous power-intensive processes. Computers & Chemical Engineering, 38, 171-184.

Mohamed, F.A. & Koivo, H.N. (2012). Online management genetic algorithms of microgrid for residential application. Energy Conversion and Management, 64, 562-568.

Mohsenian-Rad, A. & León-García, A. (2010). Optimal residential load control with price prediction in real-time electricity pricing environments. IEEE Transactions on Smart Grid, 1, 120-133.

Nistor, S., Wu, J., Sooriyabandara, M. & Ekanayake, J. (2011). Cost optimization of smart appliances. IEEE PES international conference and exhibition on innovative smart grid technologies, 1-5.

Nolde, K. & Morari, M. (2010). Electrical load tracking scheduling of a steel plant. Computers & Chemical Engineering, 34, 1899-1903.

Nosratabadi, S.M., Hooshmand, R.A. & Gholipour, E. (2016). A comprehensive review on microgrid and virtual power plant concepts employed for distributed energy resources scheduling in power systems. Renewable and Sustainable Energy Reviews, 67, 341-363.

Orr, L. (2013). Changing the world's energy systems, Stanford University Global & Energy Project. SRI Consulting.

Pfenninger, A., Hawkes, A. & Keirstead, J. Energy systems modeling for twenty-first century energy challenges. Renewable and Sustainable Energy Reviews, 33, 74-86.

Pruitt, K.A., Braun, R.J. & Newman, A.M. (2013). Evaluating shortfalls in mixed-integer programming approaches for the optimal design and dispatch of distributed generation systems. Applied Energy, 102, 386-398.

Prodan, I. & Zio, E. (2014). On the microgrid energy management under a predictive control framework. IEEE International Conference on Control Applications, 861-866.

Rastegar, M., Fotuhi-Firuzabad, M. & Zareipour, H. (2016). Home energy management incorporating operational priority of appliances. International Journal of Electrical Power & Energy Systems, 74, 286-292.

Silvente, J., Aguirre, A.M., Zamarripa, M.A., Méndez, C.A., Graells, M. & Espuña, A. (2015). Improved time representation model for the simultaneous energy supply and demand management in microgrids. Energy, 87, 615-627.

Sou, K.C., Weimer, J., Sandberg, H. & Johansson, K.H. (2011). Scheduling smart home appliances using mixed integer linear programming. IEEE conference on decision and control and European control conference, 5144-5149.

Sun, H.C. & Huang, Y.C. (2012). Optimization of power scheduling for energy management in smart homes. Procedia Engineering, 38, 1822-1827.

Tong, C.D., Palazoglu, A., El-Farra, N.H. & Yan, X.F. (2015). Energy demand management for process systems through production scheduling and control. AIChE Journal, 61, 3756-3769.

Villanueva, D. & Feijóo, A. (2010). Wind power distributions: A review of their applications. Renewable and Sustainable Energy Reviews, 14, 1490-1495.

Wang, L. & Singh, C. (2009). Multicriteria design of hybrid power generation systems based on a modified particle swarm optimization algorithm. IEEE Transactions on Energy Conversion, 24, 163–172.

Wang, Y., Wang, B., Chu, C., Pota, H. & Gadh, R. Energy management for a commercial building microgrid with stationary and mobile battery storage. Energy and Buildings, 116,141-150.

Xiao, J., Hodge, B.M.S., Pekny, J.F. & Reklaitis, G.V. (2011). Operating reserve policies with high wind power penetration. Computers & Chemical Engineering, 35, 1876-1885.

Yang, Z.Y., Wu, R., Yang, J.F., Long, K.Y. & You, P.C. (2016). Economical operation of microgrid with various devices via distributed optimization. IEEE Transactions on Smart Grid, 7, 857-867.

Yin, C.J., Wu, H.W., Locment, F. & Sechilariu, M. (2017). Energy management of DC microgrid based on photovoltaic combined with diesel generator and supercapacitor. Energy Conversion and Management, 132, 14-27.

Zhang, D., Shah, N. & Papageorgiou, L.G. (2013). Efficient energy consumption and operation management in a smart building with microgrid. Energy Conversion and Management, 74, 209-222.

Zhang, D., Evangelisti, S., Lettieri, P. & Papageorgiou, L.G. (2016). Economic and environmental scheduling of smart homes with microgrid: DER operation and electrical tasks. Energy Conversion and Management, 110, 113-124.

## Appendix A. Modelling of interruptions of tasks

Equations (26), (27) and (28) follow a general disjunctive programming (GDP) formulation. This constraints are used to determine any eventual in a task. Particularly:

(i) If the potential status of a task is not active ($\widehat{W}_{k,i,t} = 0$), there is neither interruption ($\lambda i^{W}_{k,i,t} = 0$) nor remaining interruptions ($\lambda s^{W}_{k,i,t} = 0$).

$$\neg \widehat{W}_{k,i,t} \Rightarrow \neg \lambda i^{W}_{k,i,t} \wedge \neg \lambda s^{W}_{k,i,t}$$

(ii) If the potential status of a task is active ($\widehat{W}_{k,i,t} = 1$), the status of the task is active at time $t$ ($W_{k,i,\theta,t} = 1$) and not active at time $t+1$ ($W_{k,i,\theta,t+1} = 0$), an interruption is produced ($\lambda i^{W}_{k,i,t+1} = 1$).

$$\widehat{W}_{k,i,t} \wedge W_{k,i,\theta,t} \wedge \neg W_{k,i,\theta,t+1} \Rightarrow \lambda i^{W}_{k,i,t}$$

(iii) If the potential status of a task is active ($\widehat{W}_{k,i,t} = 1$), and the status of the task is not active neither at time $t$ nor at time $t+1$ ($W_{k,i,\theta,t} = W_{k,i,\theta,t+1} = 0$), a remaining interruption is considered ($\lambda s^{W}_{k,i,t+1} = 1$).

$$\widehat{W}_{k,i,t} \wedge \neg W_{k,i,\theta,t} \wedge \neg W_{k,i,\theta,t+1} \Rightarrow \lambda s^{W}_{k,i,t+1}$$

## Appendix B. Input data

Table B1. Heat demand per house, electricity price and wind forecast speed at each time interval.

| Time interval $t$ | Heat demand per house, $H_t$ $(kW)$ | Wind forecast speed, $v_t$ $(m/s)$ | Electricity buying price from grid, $b_t$ $(£/kWh)$ |
|---|---|---|---|
| $t1$ | 4.03956 | 7.7189 | 0.051650 |
| $t2$ | 4.03956 | 5.7020 | 0.052120 |
| $t3$ | 4.71396 | 0.0000 | 0.055490 |
| $t4$ | 4.71396 | 7.3189 | 0.055810 |
| $t5$ | 4.78394 | 0.0000 | 0.056070 |
| $t6$ | 4.78394 | 0.0000 | 0.071547 |
| $t7$ | 3.85706 | 6.4022 | 0.080241 |
| $t8$ | 3.85706 | 9.2115 | 0.054940 |
| $t9$ | 3.99744 | 7.4104 | 0.054850 |
| $t10$ | 3.99744 | 12.0000 | 0.053990 |
| $t11$ | 4.01842 | 12.0000 | 0.053580 |
| $t12$ | 4.01842 | 0.0000 | 0.053410 |
| $t13$ | 3.40232 | 0.0000 | 0.053300 |
| $t14$ | 3.40232 | 6.2422 | 0.052590 |
| $t15$ | 3.38558 | 0.0000 | 0.065953 |
| $t16$ | 3.38558 | 6.6764 | 0.081068 |
| $t17$ | 3.49448 | 0.0000 | 0.086111 |
| $t18$ | 3.49448 | 6.4401 | 0.085419 |
| $t19$ | 3.74536 | 7.6899 | 0.096813 |
| $t20$ | 3.74536 | 12.0000 | 0.105791 |
| $t21$ | 3.72398 | 8.2480 | 0.116452 |
| $t22$ | 3.72398 | 8.1244 | 0.116298 |
| $t23$ | 4.16544 | 5.2636 | 0.109260 |
| $t24$ | 4.16544 | 7.0204 | 0.062010 |
| $t25$ | 4.09506 | 8.9224 | 0.057140 |
| $t26$ | 4.09506 | 0.0000 | 0.054320 |
| $t27$ | 3.56726 | 11.9390 | 0.044720 |
| $t28$ | 3.56726 | 11.0150 | 0.044050 |
| $t29$ | 3.55678 | 6.7188 | 0.042760 |
| $t30$ | 3.55678 | 0.0000 | 0.042190 |

| | | | |
|---|---|---|---|
| $t31$ | 3.78082 | 10.0640 | 0.038460 |
| $t32$ | 3.78082 | 5.9410 | 0.039470 |
| $t33$ | 3.78708 | 8.0399 | 0.041150 |
| $t34$ | 3.78708 | 5.1846 | 0.040130 |
| $t35$ | 3.62412 | 12.0000 | 0.041310 |
| $t36$ | 3.62412 | 0.0000 | 0.040140 |
| $t37$ | 3.56462 | 6.2849 | 0.039360 |
| $t38$ | 3.56462 | 0.0000 | 0.039100 |
| $t39$ | 3.58136 | 9.7792 | 0.039370 |
| $t40$ | 3.58136 | 0.0000 | 0.037970 |
| $t41$ | 3.61988 | 7.4742 | 0.038070 |
| $t42$ | 3.61988 | 0.0000 | 0.037580 |
| $t43$ | 3.84896 | 0.0000 | 0.038440 |
| $t44$ | 3.84896 | 5.8136 | 0.038490 |
| $t45$ | 3.95828 | 5.2846 | 0.042830 |
| $t46$ | 3.95828 | 6.3642 | 0.045050 |
| $t47$ | 4.45378 | 6.8540 | 0.050750 |
| $t48$ | 4.45378 | 0.0000 | 0.054810 |

Table B2. Penalty costs.

| Task $i$ | Penalty cost for interruption a task $i$ | | | | Penalty cost from deviation to the target, $\mu_{k,i}$ (£/$h$) |
|---|---|---|---|---|---|
| | Within the time window | | Outside the time window | | |
| | Interruption, $\mu i_i^W$ (£) | Remain interrupted, $\mu s_i^W$ (£) | Interruption, $\mu i_i^Z$ (£) | Remain interrupted, $\mu s_i^Z$ (£) | |
| $i1$ | 0.0500 | 0.0050 | 0.5000 | 0.0500 | 0.0500 |
| $i2$ | 0.0100 | 0.0020 | 0.1000 | 0.0200 | 0.0100 |
| $i3$ | 0.0300 | 0.0010 | 0.3000 | 0.0100 | 0.0200 |
| $i4$ | 0.0800 | 0.0900 | 0.8000 | 0.9000 | 0.0080 |
| $i5$ | 0.0100 | 0.0010 | 0.1000 | 0.0100 | 0.0100 |
| $i6$ | 0.0100 | 0.0010 | 0.1000 | 0.0100 | 0.0200 |
| $i7$ | 0.0100 | 0.0010 | 0.1000 | 0.0100 | 0.0060 |
| $i8$ | 0.0200 | 0.0070 | 0.2000 | 0.0700 | 0.0040 |
| $i9$ | 0.0100 | 0.0010 | 0.1000 | 0.0100 | 0.0090 |
| $i10$ | 0.0100 | 0.0010 | 0.1000 | 0.0100 | 0.1000 |
| $i11$ | 0.1000 | 0.1000 | 0.1000 | 1.0000 | 0.1000 |
| $i12$ | 0.0200 | 0.0100 | 0.2000 | 0.1000 | 0.0200 |
| $i13$ | 0.0500 | 0.0010 | 0.5000 | 0.0100 | 0.0080 |
| $i14$ | 0.0100 | 0.0020 | 0.1000 | 0.0200 | 0.0100 |
| $i15$ | 0.0001 | 0.0001 | 0.0010 | 0.0010 | 0.0300 |
| $i16$ | 0.0010 | 0.0001 | 0.0100 | 0.0010 | 0.0100 |