Methods of nonlinear dynamics and the construction of cryptocurrency crisis phenomena precursors

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Abstract. This article demonstrates the possibility of constructing indicators of critical and crisis phenomena in the volatile market of cryptocurrency. For this purpose, the methods of the theory of complex systems such as recurrent analysis of dynamic systems and the calculation of permutation entropy are used. It is shown that it is possible to construct dynamic measures of complexity, both recurrent and entropy, which behave in a proper way during actual pre-crisis periods. This fact is used to build predictors of crisis phenomena on the example of the main five crises recorded in the time series of the key cryptocurrency bitcoin, the effectiveness of the proposed indicators-precursors of crises has been identified.

Keywords: cryptocurrency, bitcoin, complex system, measures of complexity, nonlinear dynamics, recurrence plot, recurrence quantification analysis, entropy, permutation entropy, crisis, indicator-precursor.

1   Introduction

Bitcoin is an important electronic and decentralized cryptographic currency system proposed by Satoshi Nakamoto as the "greatest technological breakthrough since the Internet" [1]. It is based on a peer-to-peer network architecture and secured by cryptographic protocols and there is no need for a central authority or central bank to control the money supply within the system. Bitcoin relies on a proof-of-work system to verify and authenticate the transactions that are carried out in the network. Anonymity and avoidance of double spending are realized via a block chain, a kind of transaction log that contains all transactions ever carried out in the network. For further verification purposes all transactions are public [2].

The bitcoin has emerged as a fascinating phenomenon in the financial markets. Without any central authority issuing the currency, the bitcoin has been associated with controversy ever since its popularity, accompanied by increased public interest, reached high levels.

Despite an influx of media buzz and venture capital, digital currencies face an uncertain future amid an ever-changing global landscape. Investment requires careful consideration of the potential use cases and risks associated with various cryptocurrencies.

A look back at bitcoin price swings in the last five years, which include several stomach-churning tumbles of 40% and even 50%, makes it clear the world's most popular cryptocurrency was—and is—extremely volatile. It is also apparent that most of the bitcoin crashes coincide with speculative run-ups coupled with exogenous shocks, such as a major hack or a government crackdown. Also, in most cases, bitcoin has bounced back from the crashes in months or even weeks—suggesting nervous bitcoin buyers will be okay if they are holding for the long run. On the other hand, the crashes of late 2013 and early 2014 are a cautionary tale—recall it took years for those who first bought bitcoin at $1,000 to see their investment recover.

Bitcoin attracts considerable attention of researchers of different levels, using modern methods and models of analysis of the peculiarities of the dynamics of the popular digital currency.

The authors [3] examine the relation between price returns and volatility changes in the bitcoin market using a daily database denominated in various currencies. The results for the entire period provide no evidence of an asymmetric return-volatility relation in the bitcoin market. They test if there is a difference in the return-volatility relation before and after the price crash of 2013 and show a significant inverse relation between past shocks and volatility before the crash and no significant relation after.

A noncausal autoregressive process with Cauchy errors in application to the exchange rates of the bitcoin electronic currency introduced in [4]. The dynamics of the daily bitcoin/USD exchange rate series displays episodes of local trends, which can be modelled and interpreted as speculative bubbles. The bubbles may result from the speculative component in the on-line trading.

Taking Bitcoin as a representative example, the authors [5] first uses autoregressive moving average (ARMA) functions to explain trading values, then applies log-periodic power law (LPPL) models [6] in an attempt to predict crashes. The results of ARMA modeling show that bitcoin values react to the BOE Volatility Index, suggesting that a primary force currently driving bitcoin values is speculation by investors looking outside traditional markets. In addition, the LPPL models accurately predict ex-ante the crash that occurred in December 2013, making LPPL models a potentially valuable tool for understanding bubble behavior in digital currencies.

In the work [7], a comparative correlation and fractal analysis of time series of bitcoin cryptocurrency rate and community activities in social networks associated with bitcoin was conducted. A significant correlation between the bitcoin rate and the community activities was detected. Time series fractal analysis indicated the presence of self-similar and multifractal properties. The results of researches showed that the series having a strong correlation dependence have a similar multifractal structure.

It is analyzed the time-varying behavior of long memory of returns on bitcoin and volatility 2011 until 2017, using the Hurst exponent [8]. Daily returns exhibit persistent behavior in the first half of the period under study, whereas its behavior is more informational efficient since 2014. Price volatility, measured as the logarithmic difference between intraday high and low prices exhibits long memory during all the period. This reflects a different underlying dynamic process generating the prices and volatility.

The research [9] is concerned with predicting the price of bitcoin using machine learning. The goal is to ascertain with what accuracy the direction of bitcoin price in USD can be predicted. The price data is sourced from the Bitcoin Price Index . The task is achieved with varying degrees of success through the implementation of a Bayesian optimised recurrent neural network (RNN) and Long Short Term Memory (LSTM) network. The LSTM achieves the highest classification accuracy of 52% and a RMSE of 8%. The popular ARIMA model for time series forecasting is implemented as a comparison to the deep learning models. As it is expected, the non-linear deep learning methods outperform the ARIMA forecast which performs poorly.

The bitcoin price was modeled as a geometric fBm, and price predictions were put forward through a Monte Carlo approach with 10^4 realisations [10]. The predicted mid-2017 price, based on historical values until the end of 2016, taken as the median, was slightly underestimated. This is considered as a good agreement, thus justifying the applicability of the model. Therefore, price predictions for the beginning of 2018 were made in the same way. It is found that the price predicted as the median of a log-normally distributed set of realisations is 6358 USD. On the other hand, the chance of falling below the current price of 2575.9 USD is 11.4%.

In the paper [11] it has been presented that an agent-based artificial cryptocurrency market in which heterogeneous agents buy or sell cryptocurrencies, in particular bitcoins. In this market, there are two typologies of agents, Random Traders and Chartists, which interact with each other by trading bitcoins. Each agent is initially endowed with a finite amount of crypto and/or fiat cash and issues buy and sell orders, according to the strategy and resources. The number of bitcoins increases over time with a rate proportional to the real one, even if the mining process is not explicitly modeled.

The model proposed is able to reproduce some of the real statistical properties of the price absolute returns observed in the bitcoin real market. In particular, it is able to reproduce the autocorrelation of the absolute returns, and their cumulative distribution function. The simulator has been implemented using object-oriented technology, and could be considered a valid starting point to study and analyse the cryptocurrency market and its future evolutions.

Authors [12] have reported the results of a preliminary exploratory analysis of bitcoin market value from a popular exchange market BitStamp. They have collected the data for a period of five days in January 2014 at a rate of about one minute and construct different network representation of the time series [13]. The above network representations can also model multidimensional time series, which enables the analysis of bitcoin market value and trade from several exchange markets simultaneously. Since the value can differ substantively across the markets, predicting the future fluctuations at one market from the dynamics of another could be of considerable practical value.

During the last two decades, a number of interesting methods have been proposed to detect dynamical changes. They include, among others, recurrence plots and recurrence quantification analysis [14], concept of permutation entropy (PEn) [15] as a complexity measure for time series analysis. Since we will use them in the future, it is necessary to consider the above methods in more detail.

2   Recurrence plots and recurrence quantification analysis

Recurrence plots (RPs) have been introduced to study the dynamics of complex systems that is represented in an m-dimensional phase space by its phase space trajectory $\mathbf{X}_i \in R^m$ (assuming discrete sampling, $i = 1, ..., N$) [14]. A phase space trajectory can be reconstructed from a time series u_i ($t = i\Delta t$, where Δt is the sampling time) by the time delay embedding scheme

$$\mathbf{X}_i = (u_i, u_{i+1}, ..., u_{i+(m-1)\tau}), \tag{1}$$

with m the embedding dimension and τ the embedding delay. Both parameters can be estimated from the original data using false nearest neighbors and mutual information [16].

A Recurrence Plot is a 2-dimensional representation of those times when the phase space trajectory $\mathbf{X}_i$ recurs. As soon as a dynamical state at time j comes close to a previous (or future) state at time i, the recurrence matrix $\mathbf{R}$ at (i, j) has an entry one:

$$R_{ij} = \Theta(\varepsilon - \left\| x_i - x_j \right\|), \quad i, j = 1, ..., N, \tag{2}$$

where $\| \; \|$ is a norm (representing the spatial distance between the states at times i and j), ε is a predefined recurrence threshold, and Θ is the Heaviside function (ensuring a binary $\mathbf{R}$).

The RP has a square form and usually the identity $R_{ij} \equiv 1$ is included in the graphical representation, although for calculations it might be useful to remove it [16]. The graphical representation of the RP allows to derive qualitative characterizations of the dynamical systems. For the quantitative description of the dynamics, the small-scale patterns in the RP can be used, such as diagonal and vertical lines. The histograms of the lengths of these lines are the base of the recurrence quantification analysis (RQA) developed by Webber and Zbilut and later by Marwan et al. [17-19].

The simplest measure of RQA is the density of recurrence points in the RP, the recurrence rate:

$$RR = \frac{1}{N^2} \sum_{1, j=1}^{N} R_{i,j}, \tag{3}$$

that can be interpreted as the probability that any state of the system will recur.

The fraction of recurrence points that form diagonal lines of minimal length μ is the determinism measure:

$$DET^{(\mu)} = \frac{\sum_{l=\mu}^{N} l \cdot D(l)}{\sum_{i,j}^{N} R_{i,j}} = \frac{\sum_{l=\mu}^{N} l \cdot D(l)}{\sum_{l=1}^{N} l \cdot D(l)}, \tag{4}$$

where

$$D(l) = \sum_{i,j}^{N} \left\{ (1 - R_{i-1,j-1}) \cdot (1 - R_{i+1,j+1}) \cdot \prod_{k=0}^{l-1} R_{i+k,j+k} \right\} \qquad (5)$$

is the histogram of the lengths of the diagonal lines. The understanding of 'determinism' in this sense is of heuristic nature.

3   Permutation entropy (PEn)

The PEn is conceptually simple, computationally very fast and can be effectively used to detect dynamical changes in complex time series.

The degree of disorder or uncertainty in a system can be quantified by a measure of entropy. The uncertainty associated with a physical process described by the probability distribution

$$P = \{p_i, i = 1, ..., M\} \qquad (6)$$

is related to the Shannon entropy,

$$S[P] = -\sum_{i=1}^{M} p_i \ln p_i. \qquad (7)$$

Constructing probability distributions using ordinal patterns from recorded time series was proposed by Bandt and Pompe [15]. The benefit of using this symbolic approach is improved robustness to noise and invariance to nonlinear monotonous transformations (e.g. measurement equipment drift) when compared with other complexity measures [15]. This is due to the way the ordinal patterns are constructed based on the relative amplitude of time series values and makes it particularly attractive for use on experimental data.

To obtain the ordinal pattern distribution on which to calculate entropy, one must first choose an appropriate ordinal pattern length D and ordinal pattern delay τ. There are $D!$ possible permutations for a vector of length D, so in order to obtain reliable statistics the length of the time series N should be much larger than $D!$ [20].

The time scale over which the complexity is quantified can be set by changing the ordinal pattern delay τ. This is the time separation between values used to construct the vector from which the ordinal pattern is determined. Its value corresponds to a multiple of the signal sampling period. For a given time series $\{u_t, t = 1, ..., N\}$, ordinal pattern length D, and ordinal pattern delay τ, we consider the vector

$$\mathbf{X}_S \rightarrow (u_{s-(D-1)\tau}, u_{s-(D-2)\tau}, ,,,, u_{s-\tau}, u_s). \qquad (8)$$

At each time s the ordinal pattern of this vector can be converted to a unique symbol $\pi = (r_0, r_1, ..., r_{D-1})$ defined by $u_{s-r_0\tau} \geq u_{s-r_1\pi} \geq ... \geq u_{s-r_{D-2}\tau} \geq u_{s-r_{D-1}\tau}$.

The ordinal pattern probability distribution $P = \{p(\pi), i = 1, ..., D!\}$ required for the entropy calculation is constructed by determining the relative frequency of all the

$D!$ possible permutations π_i. The normalized permutation entropy is then defined as the normalized Shannon entropy S associated with the permutation probability distribution P

$$H_S[P] = \frac{S[P]}{S_{max}} = \frac{-\sum_{i=1}^{D!} p(\pi_i) \ln p(\pi_i)}{\ln D!}.$$ (9)

This normalized permutation entropy gives values $0 \leq H_S \leq 1$ where a completely predictable time series has a value of 0 and a completely stochastic process with a uniform probability distribution is represented by a value of 1. It is important to realize that the PE is a statistical measure and is not able to distinguish whether the observed complexity (irregularity) arises from stochastic or deterministic chaotic processes. It is also important that the PEn provides means to characterize complexity on different time scales, given by the delay.

Thus H_S gives a measure of the departure of the time series under study from a complete random one: the smaller the value of H_S, the more regular the time series is. It is clear that if D is too small, such as 1 or 2, the scheme will not work, since here are only very few distinct states. In principle, using a large value of D is fine, as long as the length of a stationary time series under study can be made proportional to $D!$. In their paper [15], Bandt and Pompe recommend $D = 3, ... ,7$. We found that a value of $D = 5, 6,$ or 7 seems to be the most suitable.

4   Experimental testing of the effectiveness of indicators-precursors of crisis phenomena

We have already reached a point where the crash of the bitcoin will have serious global consequences. The degree of involvement of financial institutions in a transaction with cryptocurrencies is now unclear, and, apparently, it will be fully disclosed after the financial catastrophe. This is very similar to the situation in 2007-2008, when nobody really knew where, ultimately, subprime mortgages are concentrated. Until the crash, everyone was only wondering which financial institutions could be bankrupt. Thus, the identification of possible trends of the cryptocurrency movement, construction and modeling of indicators of stability and possible crisis states is extremely relevant.

During the entire period (16.07.2010 - 10.02.2018) of verifiable fixed daily values of the bitcoin price (BTC) (https://finance.yahoo.com/cryptocurrencies) in relative units, five crisis phenomena were recorded and marked with arrows on Fig. 1.
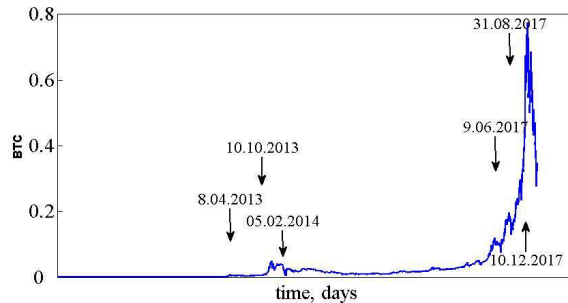
Fig. 1. Dynamics of price fluctuations of bitcoin over time. The arrows indicate the beginning of one of the five known crises

In order to study the possibility of constructing indicators of crisis phenomena in the market of cryptocurrency, the price range of bitcoin was divided into five parts in accordance with the periodization of crises [21]:

1). From 19.02.2013 to 31.05.2013.
2). From 10.10.2013 to 31.12.2013.
3). From 18.12.2013 to 02.03.2014.
4). From 22.04.2017 to 31.07.2017.
5). From 15.07.2017 to 02.10.2018.

For each of the time series phase portraits, recurrent diagrams were constructed, their quantitative analysis was carried out, and there were entropies of permutations estimated. Calculations were carried out within the framework of the algorithm of a moving window. For this purpose, the part of the time series (window), for which there were measures of complexity (RR, DET, PEn), was selected, then the window was displaced along the time series in a one-day increment and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the cryptocurrency. If this or that measures of complexity behaves in a definite way for all periods of crisis, for example, decreases or increases during the pre-crisis period, then it can serve as an indicator or precursor of such a crisis phenomenon.

We expect that the variation of RR, DET, PEn as a function of time or certain time-varying parameter can accurately indicate interesting dynamical changes in a time series.

The simulation results are quite sensitive to the window width selection. Indeed, if the window is too large, several crisis or shock (critical states) may enter it. As a result, we get an average case where it is impossible to reliably divide one crisis from another. On the contrary, when over a small window, the measures of complexity is not that exact, it fluctuates noticeably and requires smoothing.

In Fig. 2 for the first crisis the phase portrait, the recurrent diagram and the measures of complexity calculated for the window in 15 days in a one-day increments are given.
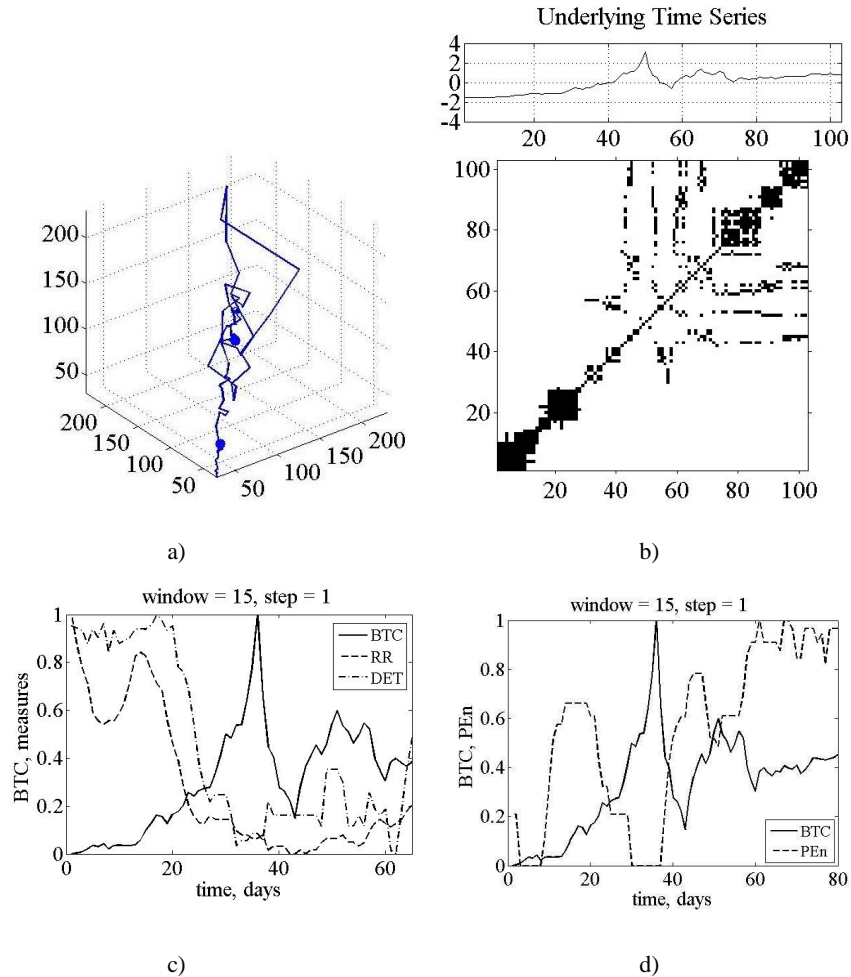
Fig. 2. Phase portrait (a), recurrence diagram (b) and, respectively, the measure of recurrence RR and determinism DET (c) and the permutation entropy PEn (d)

Unlike, for example, the stock markets, the cryptocurrencies market is more volatile, and critical phenomena are separated by a smaller time lag. This justifies the choice of the size of the window of a few days. We have proved calculations for windows in 15, 25 and 35 days. The best way is to share critical events in time when choosing a window in 15 days.

At the phase portrait there are no attractive areas, although fluctuations during the first crisis are visible both on the phase portrait and on the recurrence diagram. But the measures of complexity look interesting: before the crisis, both recurrent and entropy measures are noticeably diminished, thus signaling the oncoming crisis.
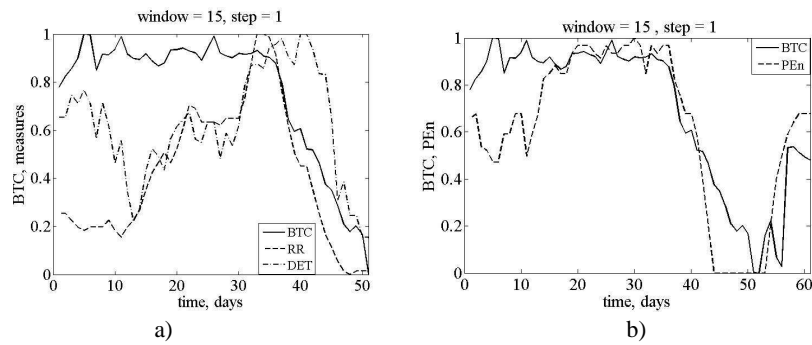
For the second crisis, the indicators-precursors produce dynamics, which is depicted in Fig. 3.
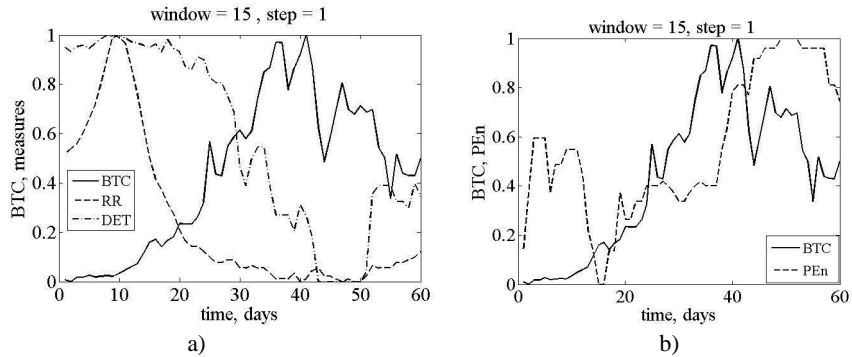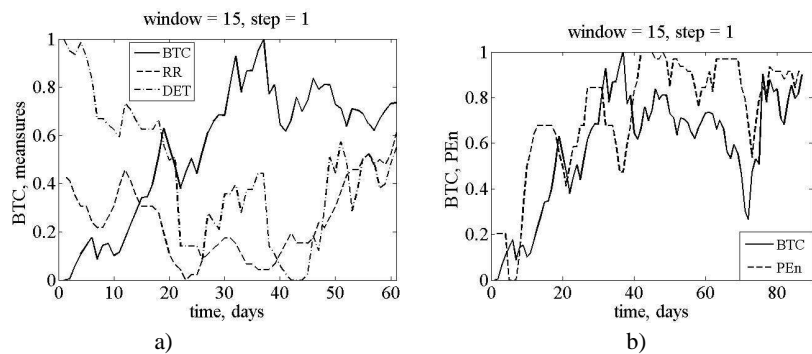
Fig. 3. Dynamics of RR and DET (a) and permutation entropy PEn (b) for the second crisis

For the third crisis, the behavior of indicators-precursors has the form, presented in Fig. 4.


Fig. 4. Dynamics of RR, DET (a) and permutation entropy PEn (b) for the third crisis

A fourth crisis could also have been predicted using the indicators introduced (Fig. 5).

Fig. 5. Dynamics of RR, DET (a) and PEn (b) for the fourth crisis

Finally, the last crisis is preceded by shock states, which are identified by the introduced indicator measures. But most clearly, they "prevent" the rapid fall of the main phase of the crisis of the end of 2017 beginning of 2018 (Fig. 6).
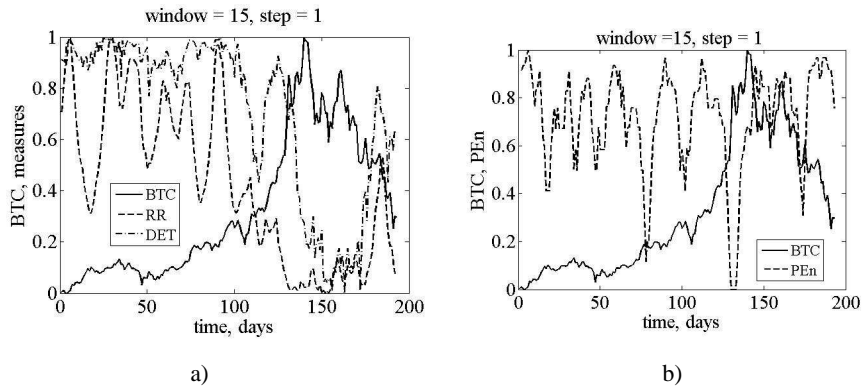
Fig. 6. Dynamics of RR, DET (a) and PEn (b) for the fifth crisis

It should be noted that other of the most capitalized cryptocurrencies, such as Ethereum, Ripple, Bitcoin Cash have coefficients of pair correlation with bitcoin at the level of 0.6-0.8 and similarly react to crisis phenomena.

5   Concluding remarks

Consequently, in this paper, we have shown that monitoring and prediction of possible critical changes on cryptocurrency is of paramount importance. As it has been shown by us, the theory of complex systems has a powerful toolkit of methods and models for creating effective indicators- precursors of crisis phenomena. In this paper, we have explored the possibility of using the recurrent and entropy measures of complexity to detect dynamical changes in a complex time series. We have shown that the measures that have been used can indeed be effectively used to detect crisis phenomena for the time series of bitcoin. Certainly there is no reason to expect that the RR, DET or PE is universally and indiscriminately applicable. It is most likely that no such measure exists; instead, various measures would have to be used in a complementary fashion, to take best advantage of their respective merits within their ranges of applicability. We have concluded though by emphasizing that the most attractive features of the RR, DET and PE, namely its conceptual simplicity and computational efficiency make it an excellent candidate for a fast, robust, and useful screener and detector of unusual patterns in complex time series.

As for the prospects for further research, we plan to investigate the fractal and network properties of cryptomarket, as well as its correlation with other sectors of the global financial market.

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