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SUPERSTRING COLLISIONS AT PLANCKIAN ENERGIES

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A B S T R A C T

By evaluating and resumming the large s behaviour of string loops to all orders we are able to obtain an explicitly unitary operator for the light (closed) superstring S-Matrix above Planckian energies: it is dominated by graviton exchange at large impact parameters and by absorption at small impact parameters. In an intermediate, eikonal region a semi-classical description emerges as if, for small deflection angles at least, each string was moving in a static Schwarzschild metric.

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Superstrings[1] are presently attracting much attention as possibly consistent fundamental theories giving rise, at low energies, to gauge interactions of the usual type as well as to gravity. It is obviously at high energies - or short distances - i.e., around Planckian scales that strings should reveal their virtues, offering novel solutions to the long lasting problems of classical singularities and quantum infinities in general relativity.

Not much work has been devoted so far to these regimes, in spite of the fact that the finiteness of superstring loops is usually regarded as a consequence of their very soft short-distance behaviour evidentiated, for instance, in the exponential drop of fixed angle scattering or large p_T inclusive distributions.

There is another regime, the high s, small t limit of two-body scattering, where, instead, tree-level string collisions do not differ much from their field theoretic counterpart. The partial wave amplitudes grow too fast with energy, due to graviton exchange, crying for loop effects to restore unitarity bounds.

This is the regime we shall study here. One would like to know, for instance, if the price to pay for unitarity is a disastrous mass shift for the graviton. More generally, at high energy one is entering a strong coupling regime and loops become essential: what is saved of the tree level picture?

Quite unexpectedly, we were able to go quite a long way into evaluating these effects, as we shall now discuss.

In order to define carefully the relevant kinematical region, we remind that the theory contains, besides the fundamental scale α', a dimensionless coupling, the string loop expansion parameter, and some other parameters expressing, in units of α', the size of 10-D compactified dimensions. Out of these we can express the D-dimensional gauge coupling g^2 and the Newton constant G_N which are related by:

$$\frac{g^2}{4\pi}\alpha' = 4\,G_N = 4\,M_P^{2-D} \quad . \tag{1}$$

While considering the loop expansion parameter as small ($G_N\alpha'^{1-\frac{1}{2}D} \ll 1$), we look at energies for which the tree amplitude is large, i.e., $G_N s\ \alpha'^{2-\frac{1}{2}D} \gg 1$. Therefore, we are interested in the region where

$$\alpha' s \gg \left(M_P \sqrt{\alpha'}\right)^{D-2} \gg 1 \ , \tag{2}$$

and

$$|t| < 1/\alpha' \ , \ |t| < M_c^2 \ , \tag{3}$$

where M_c^{-1} is the compactification radius. This last restriction insures that compactified momenta are not appreciably excited.

The smallness of $G_N \alpha'^{1-\frac{1}{2}D} = g^2\alpha'^{2-\frac{1}{2}D}$ in (2) allows us to define a hierarchy of contributions to the high energy limit. In fact (in units of $\alpha'_c = \frac{1}{2}\alpha' = 1$) the leading (graviton) trajectory is at $\alpha(t) = 2+t$, and therefore the h-loop contributions are expected to generate N-graviton Regge cuts ($N = h+1$) at $\alpha_N(t) = 1+N+(t/N)$. The energy dependence will therefore be given by a sum of terms of type

$$(\log s)^{-p} (g^2 s)^N s^{1+t/N} \left(1 + O(g^2)\right) \ , \quad N \geq 1 \ , \tag{4}$$

where the $O(g^2)$ terms are generated by contributions which are subleading by powers of s (like, e.g., gravitino exchanges) that will not be considered in the following.

One could evaluate the large s behaviour of amplitudes from the string multi-loop expression. An alternative way is to use well-known Regge-Gribov techniques[2] in order to obtain multi-Regge exchanges by sewing superstring tree amplitudes. At the one-loop level, which has been recently analysed[3], the two methods yield eventually the same result[4]. We will then show how the Regge-Gribov techniques can be extended to any number of loops, leading to a resummation of the whole series.

Let us concentrate on type II superstrings (to avoid tachyons), where the four-graviton tree amplitude is asymptotically given by

$$A_{tree}(ab \to cd) = a_{tree}(s,t)\ (\varepsilon_a \cdot \varepsilon_d)(\varepsilon_b \cdot \varepsilon_c) \ ,$$
$$a_{tree} \simeq 2g^2 \frac{\Gamma(-t/2)}{\Gamma(1+t/2)} \left(\frac{s}{2}\right)^{2+t} e^{-i\frac{\pi}{2}t} \ , \tag{5}$$

ε_i being the graviton polarization tensors. It is a simple matter to show[5] that the impact parameter transform of (5) [given below, Eq. (23)] violates partial wave unitarity at high energies.

In order to compute the two-graviton Regge cut (Fig. 1), let us first obtain the two-"graviReggeon" (GR) amplitude $A_{\alpha_1\alpha_2}(M^2)$ by factorizing the six-graviton tree amplitude at the Regge poles $J_1 = \alpha(t_1)$, $J_2 = \alpha(t_2)$ (Fig. 2). This amplitude can be written in terms of the matrix element

$$\langle \varepsilon_a, k_a | W_{\varepsilon_1}(k_1, 1)\, W_{\varepsilon_2}(k_2, z_1)\, W_{\varepsilon_3}(k_3, z, \zeta)\, W_{\varepsilon_4}(k_4, z_1 z_2 \zeta) | \varepsilon_d, k_d \rangle, \tag{6}$$

where $W_\varepsilon(k,z)$ is the vertex for graviton emission and z_i are Koba-Nielsen variables to be integrated over all the complex plane.

Note now that the Regge behaviour $\sim s_1^{\alpha_1} s_2^{\alpha_2}$ is controlled, as usual, by the $z_1 \to 1$, $z_2 \to 1$ region of integration. Therefore, $A_{\alpha_1\alpha_2}$ is obtained by isolating the leading $(1-z_i)$ singularities given by the operator product expansion of pairs of vertices W_ε. By the methods of Ref. 6) we find

$$W_{\varepsilon_1}(k_1, 1)\, W_{\varepsilon_2}(k_2, z) = (\varepsilon_1 \cdot \varepsilon_2)\, (1 - k_1 \cdot k_2)^2\, |1-z|^{-2-\alpha(-(k_1+k_2)^2)}\, W_o(k_1+k_2, 1) + \cdots, \tag{7}$$

where

$$W_o(q, \zeta) = \,:e^{iqX(\zeta, \bar\zeta)}: \tag{8}$$

is the vertex for an off-shell scalar emission of the closed string and the factor $(1-k_1k_2)^2 = (1/4)\alpha^2[-(k_1+k_2)^2]$, due to SUSY, cancels the would-be tachyon pole.

We thus obtain $A^{ad}_{\alpha_1\alpha_2} = (\varepsilon_a \cdot \varepsilon_d) a_{\alpha_1\alpha_2}$, where

$$a_{\alpha_1\alpha_2}(M^2, q_1, q_2) = (\varepsilon_a \cdot \varepsilon_d)^{-1} \int \frac{d^2\zeta}{\pi |\zeta|^2} |\zeta|^{q_2^2} \langle \varepsilon_a k_a | W_o(q_1, 1) \cdot$$

$$\cdot\, W_o(q_2, \zeta) | \varepsilon_d k_d \rangle = \int \frac{d^2\zeta}{\pi} |\zeta|^{-\alpha(M^2)} |1-\zeta|^{2q_1q_2}. \tag{9}$$

In order to define the angular momentum representation - J conjugate to M^2 - it is useful to consider, instead of a Mellin transform, the beta-transformed[7)] amplitude $\tilde{a}_J(q_1,q_2)$, which has the exact form[4)]

$$\tilde{a}_J(q_1, q_2) = \Gamma\left(J + \alpha(t_1) + \alpha(t_2) - \alpha(t)\right) \Gamma(J+1) \left[\Gamma\left(J + 1 + \frac{\alpha(t_1) + \alpha(t_2) - \alpha(t)}{2}\right)\right]^{-2}, \tag{10}$$

where $t_1 = -q_1^2$, $t_2 = -(q-q_1)^2$, $t = -q^2$.

The expression for $\tilde{a}_J$ shows a Regge pole at $J = -2+t-t_1-t_2$ and a fixed pole at $J = -1$ with a residue V_2, where

$$V_2(q_1,q_2) = \frac{\Gamma(1+t_1+t_2-t)}{\left[\Gamma\left(1+\frac{1}{2}(t_1+t_2-t)\right)\right]^2} = \sum_n \left[\frac{1}{n!}\,\frac{\Gamma(-q_1q_2+n)}{\Gamma(-q_1q_2)}\right]^2$$
$$= \int_0^\infty \frac{dM^2}{\pi}\,\mathrm{Im}\, a_{\alpha_1\alpha_2}(M^2;q_1,q_2) \quad , \tag{11}$$

is what we call the 2-GR (GR) vertex since it provides the leading (graviton) trajectory contribution.

We have shown in (11) the usual sum rule for the $J = -1$ fixed pole in order to recognize the "diffractive" excitation spectrum of the string. Furthermore, the sum rule allows us to extract the residue directly from Eq. (9), i.e.

$$V_2 = \int_{-i\infty}^{+i\infty} \frac{dM^2}{2\pi i}\, a_{\alpha_1\alpha_2}(M^2) = \int_0^{2\pi} \frac{d\sigma}{2\pi}\,\left|1-e^{i\sigma}\right|^{2q_1\cdot q_2} \quad , \tag{12}$$

thus realizing that V_2 is simply obtained by setting $|\zeta| = 1$ in the integral representation of $a_{\alpha_1\alpha_2}$.

It is important to note that $V_2 = 1 + O[(q_1\cdot q_2)^2]$ for small q_1q_2, so that the inelastic channels decouple in the soft graviton limit ($|\underset{\sim}{q}_1| \to 0$, or $|\underset{\sim}{q}_2| \to 0$).

The loop representation in the t-channel angular momentum J (conjugate to s) is directly obtained from Eq. (10) by known methods[2)] and is given by

$$A_J^{(h=1)}(t) = \frac{g^4}{(2\pi)^{D-2}}\,(\varepsilon_a\varepsilon_d)(\varepsilon_b\varepsilon_c)\int \frac{d^{D-2}q_1}{J-3-t_1-t_2}\,\frac{\Gamma(-t_1/2)}{\Gamma(1+t_1/2)}\,\frac{\Gamma(-t_2/2)}{\Gamma(1+t_2/2)}\,\cdot$$
$$\cdot\, 2^{-\alpha(t_1)-\alpha(t_2)}\cos\frac{\pi}{2}(t_1+t_2)\;\Gamma^2(J-2-t)\;\Gamma^{-4}\left(J-2-\frac{1}{2}(t+t_1+t_2)\right) . \tag{13}$$

We recognize the J-plane singularities generated by the 2-GR cut with tip at $J = \alpha_2(t) = 3+(t/2)$, and by simple and double poles at $J = \alpha(t) = 2+t$. The latter are subleading contributions, vanishing at $q_1q_2 = t-t_1-t_2 = 0$ due to the Γ^{-4} factor[*)]. The cut represents the leading contribution, and is given in terms of the square of the 2-GR vertex, i.e.,

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*) We know that the pole residues at the one-loop level vanish exactly at $t = 0$, due to SST no-renormalization theorems[8)]. This cancellation of course needs other subleading contributions at $J_i = \alpha(t_i)-n_i$, that we have not considered here.

$$A^{(h=1)}(s,q) \to (\varepsilon_a\varepsilon_d)(\varepsilon_b\varepsilon_c)\,\frac{i}{2s}\int \frac{d^{D-2}q_1\, d^{D-2}q_2}{2\quad (2\pi)^{D-2}}\,\delta^{D-2}(\underset{\sim}{q}-\underset{\sim}{q_1}-\underset{\sim}{q_2})\,\cdot$$

$$\cdot\, a_{tree}(s,q_1)\; a_{tree}(s,q_2)\;\left[V_2(q_1,q_2)\right]^2 \,, \tag{14}$$

V_2 being given in Eq. (11).

Therefore, apart from the vertex correction V_2^2, the loop amplitude, including phase, becomes a simple convolution of the tree amplitudes in Eq. (5).

Before generalizing the preceding method to higher loops, let us remark that a direct analysis of the one-loop expression in superstrings, carried out in part in Ref. 3), leads[4)] to exactly the same result as in Eq. (14). The corner in parameter space responsible for the large s behaviour of the amplitude is related to a limit configuration of the torus ($\Pi_i z_i \to 0$, i.e., $\mathrm{Im}\tau \to \infty$) far away from the singularity of the partition function ($\Pi_i z_i \to 1$).

Since in the standard representation the loop momentum is integrated first, compactification effects enter simply through the standard factor[8)] $F_2^{10-D} \underset{\mathrm{Im}\tau \to \infty}{\sim} (M_c\sqrt{\mathrm{Im}\tau})^{10-D}$. The M_c^{10-D} factor is needed to convert g_{10}^2 into our D-dimensional coupling g^2, while the $(\mathrm{Im}\tau)^{5-(D/2)}$ factor makes up for the D-dependence of the loop phase space.

The agreement of the two calculations is an explicit check of perturbative s-channel unitarity, coupled to Regge behaviour of tree amplitudes, which are the ingredients used in Eq. (14).

The advantage of the Regge-Gribov approach is its simple generalization to the multi-loop case. Indeed the N-Reggeon cut can be obtained in terms of N-GR vertices $V_N(q_1,\ldots,q_N)$ which are residues at multiple fixed poles of the Reggeon amplitudes $A_{\alpha_1\ldots\alpha_N}$ $(N = h+1)$. The latter are obtained in turn from (2N+2)-graviton tree amplitudes by the OPE of N pairs of graviton emission operators. Finally, the N-GR vertex is obtained from the integral representation of the corresponding amplitude by setting $|\zeta_i| = 1$, i.e.,

$$V_N(q_1\cdots q_N) = \int \prod_{j=1}^{N-1} \frac{dM_j^2}{2\pi i}\; a_{\alpha_1\cdots\alpha_N}(M_1^2\cdots M_{N-1}^2\,;\,q_1\cdots q_N) =$$

$$= \int_0^{2\pi} \prod_{j=1}^{N} \frac{d\sigma_j}{2\pi}\,\langle 0|\prod_{j=1}^{N} :e^{iq_j\hat{X}(\sigma)}:|0\rangle = \int_0^{2\pi}\prod_{j=1}^{N}\frac{d\sigma_j}{2\pi}\prod_{1\le j<\ell\le N}\left|e^{i\sigma_j}-e^{i\sigma_\ell}\right|^{2q_j\cdot q_\ell}\,, \tag{15}$$

where $\hat{X}(\sigma)$ is the non-zero mode contribution to the usual closed string position operator at $\tau = 0$:

$$\hat{X}^i(\sigma) = i \sum_{n \neq 0} \frac{1}{n} \left(\alpha_n^i e^{in\sigma} + \tilde{\alpha}_n^i e^{-in\sigma} \right). \tag{16}$$

[The zero mode, at $|\zeta_i| = 1$, just provides the momentum and helicity conserving factor $\varepsilon_a \cdot \varepsilon_d \ \delta(k_a + k_d + \Sigma q_i)$.]

The leading asymptotic expression for the h-loop contribution to the four-graviton amplitude is then

$$A^{(h)}(s,q) \underset{s\to\infty}{\longrightarrow} (\varepsilon_a \varepsilon_d)(\varepsilon_b \varepsilon_c)\left(\frac{i}{2s}\right)^h \int \frac{d^{D-2}q_1 \cdots d^{D-2}q_{h+1}}{(h+1)!\ (2\pi)^{(D-2)h}} \ \delta^{(D-2)}(q - \Sigma q_i) \cdot$$
$$\cdot \prod_{j=1}^{h+1} a_{tree}(s, q_j) \int \prod_{j=1}^{h+1} \frac{d\sigma_j \, d\sigma_j'}{(2\pi)^2} \langle 0 | \prod_j : e^{i q_j (\hat{X}(\sigma_j) - \hat{X}(\sigma_j'))} : | 0 \rangle . \tag{17}$$

Equations (15) and (17) for the N-GR vertex and the h-loop amplitude, being written in operatorial form, show very interesting properties.

To start with, the N-GR vertex V_N is operatorially factorized, and is symmetrical in the N-Reggeon momenta, as expected in a theory of closed strings. Furthermore, it shows the infra-red decoupling, in the sense that

$$V_{N+1}(q_1 \cdots q_{N+1}) \underset{q_{N+1}\to 0}{\longrightarrow} V_N(q_1 \cdots q_N), \quad V_0 = 1,$$
$$V_N = 1 + O\left(\sum_{j<\ell} (q_j \cdot q_\ell)^2 \right) . \tag{18}$$

Finally, the operator factorization implies that $A^{(h)}(s,q)$ in Eq. (17) is a multiple convolution, thus diagonalized by an impact parameter transformation. By defining

$$\frac{1}{s} A(s,q) = (\varepsilon_a \varepsilon_d)(\varepsilon_b \varepsilon_c)\, 4 \int d^{D-2}b \ e^{i q \cdot b} a(s,b) \tag{19}$$

we obtain

$$a^{(h)}(s,b) = \frac{(2i)^h}{(h+1)!} \langle 0 | \left[\delta(s,b;\hat{X},\hat{X}') \right]^{h+1} | 0 \rangle , \tag{20}$$

with

$$\delta = \frac{1}{4}\int \frac{d^{D-2}q}{(2\pi)^{D-2}}\;\frac{1}{s}\,a_{tree}(s,q)\int\frac{d\sigma\, d\sigma'}{(2\pi)^2} : e^{i\underset{\sim}{q}\cdot\left(\underset{\sim}{b}+\hat{\underset{\sim}{X}}(\sigma)-\hat{\underset{\sim}{X}}'(\sigma')\right)} : \tag{21}$$

being an operator functional of the string field $\hat{X}(\sigma)$. The $1/(h+1)!$ in Eq. (20) has the physical meaning of a Bose counting factor of $(h+1)$ identical Reggeons, probably related to the $\frac{1}{2}$ factor noticed in the literature[9] at one-loop level.

Summing Eq. (20) over loops yields the final result for the amplitude

$$a(s,b) = \sum_{h=0}^{\infty} a^{(h)}(s,b) = \langle 0|\frac{1}{2i}\left(\exp\left[2i\delta(s,b;\hat{X},\hat{X}')\right]-1\right)|0\rangle , \tag{22}$$

which has an operator eikonal form. This fact, together with the simple dependence of the "phase" δ on $\hat{X}$, shows that the collision process can be interpreted as a rescattering series at displaced impact parameters $(\underset{\sim}{b}+\hat{\underset{\sim}{X}}-\hat{\underset{\sim}{X}}')$, as pictured in Fig. 3. Furthermore, due to factorization, the S-operator in (22) - or the amplitude in (17) - satisfies s-channel unitarity[4], if use if made of the AGK rules[2] in order to relate the phases of "inelastic" cuts (through a_j^{tree} in Fig. 3) to the ones of "diffractive" cuts (through string states in $\hat{X}$).

In order to discuss the physical content of our results, let us start by neglecting string excitations in the vertex, which amounts to setting $\hat{X} = 0$ in Eq. (21). In this case

$$\delta(b,s) = a_{tree}(b,s) = \frac{g^2 s}{16\,\pi^{\frac{1}{2}(D-2)}}\, b^{4-D}\int_0^{\frac{1}{4}b^2(Y+i\frac{\pi}{2})^{-1}} dt\; t^{\frac{D}{2}-3}\, e^{-t} \simeq$$

$$\underset{b\gg 2\sqrt{Y}}{\simeq} \left(\frac{b_c}{b}\right)^{D-4} + i\,\frac{\pi g^2 s}{32\,(\pi Y)^{\frac{1}{2}D-1}}\cdot \exp\left(-\frac{b^2}{4Y}\right), \tag{23}$$

where $Y = \log s$, $b_c^{D-4} = g^2 s/(8\pi\Omega_{D-4})$ has a pole at $D = 4$, and $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$ is the solid angle in d dimensions. Equation (23) is restricted to $D > 4$, in order to avoid the infrared (IR) divergence: we shall discuss later on the limiting $D = 4$ case.

It appears from the expression (23) that $\mathrm{Im}\delta$ is large for $b \lesssim b_I = 2Y$, dying out exponentially for $b > b_I$. Therefore, $\exp(2i\delta) \to 0$ $(a \to i/2)$ for $b < b_I$, leading to an absorptive black disk, expanding logarithmically with energy. The cross-section in this region is mostly inelastic, with

$$\sigma_{in}(s) \simeq \frac{2}{D-2}\,\Omega_{D-2}\,(2\log s)^{D-2} . \tag{24}$$

On the other hand, for $b > b_I$, δ is mostly real, with $\mathrm{Re}\,\delta$ decreasing as a power, controlled by $b_c \sim (g^2 s)^{1/(D-4)}$. Therefore, only for $b > b_c$ is the perturbative expansion in powers of δ justified. In the remaining (eikonal) region $b_I < b < b_c$, the unitary amplitude $a(b,s)$ is characterized by a large real phase and has therefore an oscillatory behaviour.

Going back to q-space by a Bessel transform of Eq. (22), we obtain the asymptotic result

$$\frac{1}{s} a(s,q) \simeq 2i\left(\frac{2\pi b_I}{q}\right)^{\frac{1}{2}D-1} J_{\frac{1}{2}D-1}(b_I q) \exp\left[2i\left(\frac{b_c}{b_I}\right)^{D-4}\right] + \\ + \frac{g^2 s}{q^2} 2^{\frac{1}{2}D-2}\Gamma\left(\tfrac{1}{2}D-1\right)\int_{b_I q}^{\infty} dx\, x^{-\frac{1}{2}D+2} J_{\frac{1}{2}D-1}(x) \exp\left[2i\left(\frac{b_c q}{x}\right)^{D-4}\right]. \tag{25}$$

The first term shows the diffractive pattern of the black disk mentioned above, while the second (eikonal) term summarizes contributions for $b > b_I$, that we shall now analyse.

For $q < b_c^{-1} \sim (g^2 s)^{-1/(D-4)}$, i.e., in a region of very small $q^2 = -t$, shrinking with s as a power, the loop expansion of the eikonal controls the $t = 0$ singularities. At one-loop level we obtain the behaviour

$$\frac{1}{s} a(s,q) \xrightarrow[t\to 0]{} \frac{g^2 s}{|t|} + i\,\mathrm{const}\left[\frac{(g^2 s)^2}{(D-4)(6-D)}|t|^{\frac{1}{2}D-3} + \frac{(g^2 s)^{\frac{D-2}{D-4}}}{(D-4)(D-6)}\right]. \tag{26}$$

We can see that the graviton pole is still there, with unrenormalized residue, while ImA has an IR $|t|^{(D/2)-3}$ singularity for $D < 6$ ($\log|t|$ at $D = 6$), and therefore yields an infinite elastic cross-section (no optical theorem). This is the usual infinity connected with the Coulomb singularity*). For $D > 6$, σ_{el} is finite and increases as $s^{(D-2)/(D-4)}$, thus violating the Froissart bound. This is not surprising, in view of the exchange of a massless particle.

For larger values of t, i.e., $b_c^{-1} \ll q \lesssim 1$, the eikonal term in Eq. (25) has a large phase, whose variation compensates the diffractive phase of the Bessel function around a saddle point in b located at

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*) Other IR t-singularities, starting at $D = 4+(2/h)$ arise at h-loop level, accumulating at $D = 4$. This explains why the loop expansion is actually never valid at $D = 4$ (see below).

$$q = \frac{g^2 s/2}{b^{D-3}\Omega_{D-2}} = \frac{8\pi G_N (k_a k_b)}{b^{D-3}\Omega_{D-2}} , \tag{27}$$

corresponding to the behaviour

$$\frac{1}{s} a(s,q)\Big]_{eik} \simeq \frac{g^2 s}{|t|} e^{i\phi_D} \left(\frac{g^2 s |t|^{\frac{1}{2}D-2}}{2\Omega_{D-2}}\right)^{-\frac{D-4}{2(D-3)}} \cdot \left(\frac{1}{2}\right)^{-\frac{D-4}{2}} \frac{\Gamma\left(\frac{D-2}{2}\right)}{\sqrt{D-3}} , \tag{28}$$

with

$$\phi_D = \left(\frac{g^2 s |t|^{\frac{1}{2}D-2}}{2\Omega_{D-2}}\right)^{\frac{1}{D-3}} \cdot \frac{D-3}{D-4} . \tag{29}$$

Let us first note that Eqs. (27) to (29) are suitable for discussing the D = 4 limit [which is never perturbative, due to the divergent (Coulomb) phase (29)]. In fact, by factorizing the infinite part of the phase, i.e., by subtracting the D = 4 pole in (29), we get the results

$$\begin{aligned} q &= 4G_N (k_a k_b)\, b^{-1} , \\ s^{-1} a(s,q)\Big]_{eik} &= \frac{8\pi G_N s}{|t|} \exp\left(i G_N s \log|t|\right) , \quad (D=4) . \end{aligned} \tag{30}$$

This means that the eikonal contribution reduces to the tree term, modified by the well-known[10] Coulomb-phase factor.

In general, Eq. (27) defines a relation between impact parameter and momentum transfer which is typical of a classical scattering process. This is confirmed by the fact that the amplitude (28) yields an elastic cross-section

$$d\sigma_{el} = d^{D-2} b(q) \tag{31}$$

which is of Rutherford type, i.e., given by the surface element in D-2 transverse dimensions implied by the relation (27).

It is interesting to note that the same saddle point (27) also dominates the eikonal amplitude at small, but fixed, deflection angle $\theta = q/k = 2q/\sqrt{s}$, provided the stationary value b in (27), i.e.,

$$b^{D-3} = \frac{8\pi G_N \sqrt{s}}{\Omega_{D-2}\,\theta} , \tag{32}$$

is much larger than b_I[11]. This can be roughly understood by noting that the eikonal sums a rescattering series through angles $\theta_1,\ldots,\theta_n$, where each partial angle $\theta_i \approx \theta/\langle n\rangle \approx \theta(g^2 s q^{D-4})^{-1/(D-3)}$ is much smaller than θ.

Furthermore, the deflection angle (32) is identical to the one of a massless particle which scatters, at impact parameter b, off a mass $M = \sqrt{s}$ *), to first non-trivial order in the Schwarzschild metric.

More precisely, the angle (32) equals the one obtained classically from the effective potential

$$V_{eff}(b,z) = \frac{\sqrt{s}}{2}\left(\frac{r_s}{r}\right)^{D-3}\frac{1}{2}\left(1+\frac{z^2}{r^2}\right) \quad , \quad (r^2 = b^2+z^2) \; , \qquad (33)$$

where

$$r_s^{D-3} = \frac{8\pi G_N \sqrt{s}}{(D-2)\,\Omega_{D-1}} \qquad (34)$$

is the Schwarzschild radius in D space-time dimensions. Both terms in the potential (33) contribute in a non-trivial way to an overall factor $\sim (D-2)/(D-3)$ which is important in order to get (32) for any D.

In other words, the high energy scattering of two (massless) gravitons in the eikonal region seems to be equivalent, up to this order, to classical scattering in a non-trivial metric, obtained from the Einstein equations.

Let us now recall that Eq. (25), whose physical content we have just discussed, has been obtained by neglecting higher string contributions to the N-GR vertex V_N. The latter will give corrections, due to "diffractive" excitation of massive string states, whose magnitude at small q_i's is given by Eq. (18). Hence, there will be corrections of relative order $(\sqrt{\alpha'}/b)^4$ and higher to both $\mathrm{Im}\delta$ and $\mathrm{Re}\,\delta$. Such terms could modify some quantitative features of our results (e.g., the black disk could turn into a grey one).

Furthermore, there is a whole lot of terms, subleading by powers of s that we have not considered (like gravitino and charged state exchange), among which are residual graviton pole contributions to the loops, away from t = 0.

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*) More generally, in an arbitrary Lorentz boosted frame, each string "sees" a Schwarzschild metric with a mass equal to twice the energy of the other string.

To sum up, we have obtained a simple picture for the scattering of massless states (gravitons) at energies $s \gg M_p^2$. High-order contributions are important and, as expected, restore unitarity. The resulting amplitude, obtained by summing loop effects, has an operator eikonal form that leads to a simple picture in impact parameter space. Inelastic scattering occurs mostly at small b, corresponding to an expanding black (or grey) disk. Elastic scattering occurs at larger b values and has a classical geometrical interpretation.

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FIGURE CAPTIONS

Fig. 1: Large energy behaviour of one-loop four graviton amplitude in closed superstrings, as given by the two-graviton Regge cut in terms of the gravireggeon vertex V_2.

Fig. 2: Factorization of the six-graviton tree amplitude at the leading Regge trajectories $J_1 = \alpha_1$, $J_2 = \alpha_2$, to yield the 2-GR amplitude $A_{\alpha_1\alpha_2}$.

Fig. 3: Picture of loop contributions to asymptotic string collisions, as given by N-GR vertices V_N in terms of the impact parameter displacement $\hat{\underset{\sim}{X}}(\sigma)-\hat{\underset{\sim}{X}}'(\sigma')$.

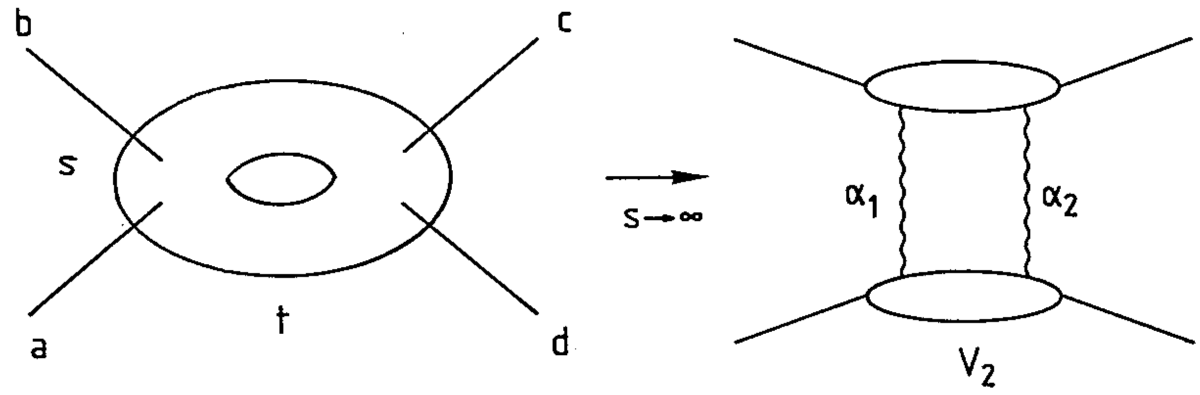

Fig. 1

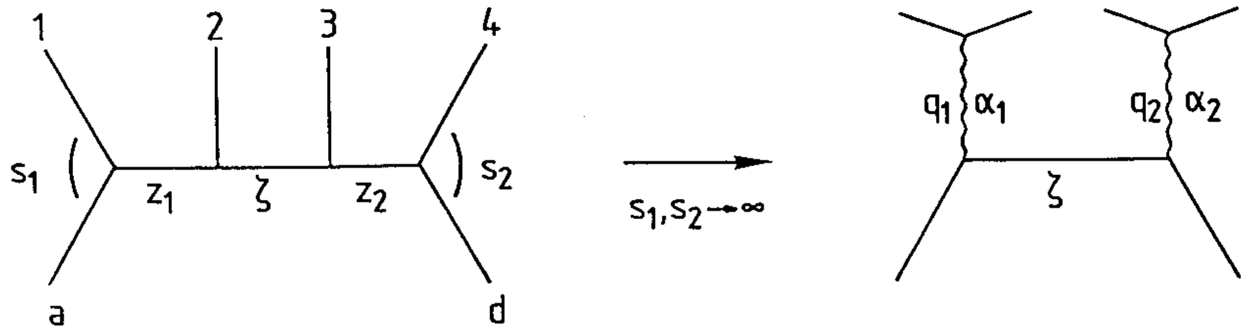

Fig. 2

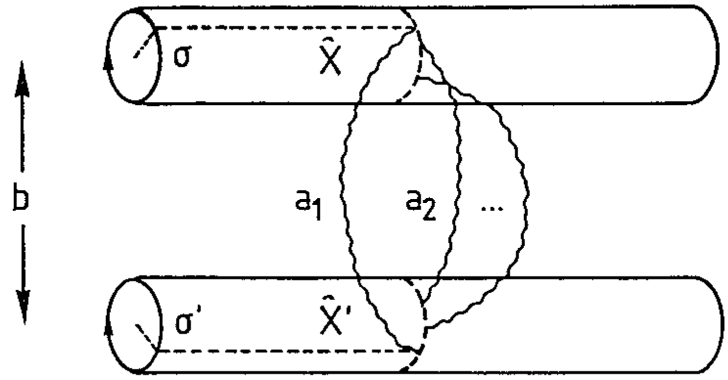

Fig. 3