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LUDWIG BOLTZMANN AND HIS INFLUENCE ON SCIENCE *)

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A B S T R A C T

On the occasion of the 75th anniversary of his death, the life of Ludwig Boltzmann (20.2.1844-5.9.1906) and his influence on science is reviewed. This great Austrian scientist was not only the founder of statistical mechanics and a gifted experimentalist, but his pioneering ideas influenced all the physical sciences. In his honour, many Austrian research institutes carry his name, many of them belong to the life sciences. He had great influence on Albert Einstein whose first papers were, according to his own words, in the spirit of Boltzmann, and intended to prove the reality and the size of certain atoms using the molecular fluctuations postulated by Boltzmann. Max Planck was converted from a "Saulus" to a "Paulus" when he had to use Boltzmann's method to derive his famous law of radiation. In fact, Boltzmann had already used discrete molecular energy levels as early as 1872. Yet his work was heavily criticized by the neopositivists around Ernst Mach and his work seemed to obtain very little attention in the last years of his life when a great number of physicists did not believe in atoms. It is the tragedy of Boltzmann's life that he did not experience the glorious victory of his ideas, but died under the gloomy vision that the work of his whole life was doomed for oblivion.

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*) Memorial Lecture.

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1. INTRODUCTION

On 5 September this year is the 75th anniversary of Ludwig E. Boltzmann's tragic death. He was undoubtedly one of the greatest scientists ever born in Austria, if not in the whole world.

In his honour a number of Research Institutes in Austria carry his name. He was not only a brilliant theoretical physicist and a skilled experimentalist, but his thoughts included the life sciences and the philosophy of science. Most of his ideas in other fields than physics are written down in his brilliant book "Populäre Schriften" (Popular Writings)[1,2]. In the following I do not want to go into the details of his scientific work, but I want to present to you his philosophy of life. It has its roots in the ideas of the great Greek philosophers such as Democritus and Socrates.

First of all let me say a few words about his life. Boltzmann was born on 20 February 1844 in Vienna. This was just the night from Shrove Tuesday to Ash Wednesday and Ludwig Boltzmann used to say, jokingly, that his date of birth was the reason why his temperament could change so suddenly from great joy to deep grief. Boltzmann received his primary education from a private tutor in the house of his parents. Later he entered the Gymnasium in Linz. Boltzmann was very industrious and, with the exception of one term, he was always the best in his class. When Boltzmann was fifteen his father, who had worked in the Imperial Internal Revenue Service, died from tuberculosis. Boltzmann's mother spent the family's small fortune on providing her son with the best education available. In Linz he received piano lessons from Anton Bruckner and he used to play chamber music throughout his life.

In 1863 Boltzmann registered at the University of Vienna, where he studied mathematics and physics. During his studies Boltzmann appreciated particularly the close contact which Josef Stefan had with his students. In 1866 he got his doctor's degree and already one year later he qualified as a University lecturer. His appointments as a University professor are shown in the following table:

Years	University	Professor of
1869-1873	Graz	Mathematical Physics
1873-1876	Vienna	Mathematics
1876-1890	Graz	Experimental Physics
1890-1893	Munich	Theoretical Physics
1893-1900	Vienna	Theoretical Physics
1900-1902	Leipzig	Theoretical Physics
1902-1906	Vienna	Theoretical Physics

Changes in his research interests and the financial needs of his family -- he had five children -- may have been the reason why he changed his position so frequently. The lectures on the philosophy of science, which he gave from 1903 to 1906 in addition to his teaching commitment in theoretical physics, together with his trips abroad, seem to have been too heavy a burden for his weak health.

2. PHILOSOPHY OF SCIENCE

Boltzmann called himself a realist. He criticized the idealistic philosophers, in particular Hegel and Schopenhauer. He was convinced that also the human mind and science as a whole was subject to an evolution process. At the beginning of his lectures on the philosophy of science he says[3]: "Im nie ruhenden Kampf ums Dasein wird nur der Sieger bleiben, der möglichst congruent mit der Aussenwelt denkt. Nicht der menschliche Geist zwingt und meistert die Natur, nein die Dinge da draussen ausser mir modeln fortwährend an meinem Denkvermögen. Mein Kopf ist so lange gegen mich, bis er endlich nach langen Irrfahrten so denken gelernt hat, dass das geistige Abbild in mir dem Wirklichen ausser mir entspricht. Dieser Prozess des Denkanpassens dauert, seitdem überhaupt gedacht wird und wird dauern, so lange denkende Wesen existieren. Die Ergebnisse dieser Denkanpassung vererben sich und bekommen je länger sie dauern, eine desto zwingendere Gewalt. So entsteht eine Art von Überzeugung, die sich bis zum Gefühle des Axioms verdichten kann." (Only he will survive in the never-ending struggle for survival who thinks most adequately to his environment. It is not the human mind which overcomes and masters nature. No, it is the things around me which all the time shape my mind. My head is against me until it has finally learned, after long misleading trips, to shape the mental portrait inside me in such a way that it corresponds to the reality surrounding me. This process of mental adjustment has been going on since there has been thought and it will last as long as thinking beings exist. The results of this mental adjustment are handed over from generation to generation and the longer they endure the more they obtain compulsory power. This way convictions develop which at the end are coined as axioms.) This kind of modesty should be the guideline of human thinking.

In fact, it is the arrogancy of those who believe that they have a special spiritual mission on this earth, and thus all that they do must be right, which has brought so much disaster to our human race. And it is the modesty of those who always try to do their best and call for the help of God only when their wisdom is at its end, which has failed to prevent many tragedies.

3. ETHICS

Boltzmann did not believe that ethics could be derived from the physical sciences. Physics is constructed in such a way that it contains no scale of values. The values by which our actions should be determined result from conventions. These conventions have evolved from our human society and they have to be adjusted according to the needs of survival of mankind. Undoubtedly the survival of mankind is linked to the survival of most of the other living creatures and plants.

Boltzmann considered life as the central value of ethics. "Das wertvolle ist eben das, was das Leben fördert. Die Frage nach denn Wert des Lebens selbst hat keinen Sinn; dass sie sich uns aufdrängt, ist aber nach der Darwinschen Theorie leicht erklärlich. Es ist wieder ein über das Ziel Hinausschiessen einer Denkgewohnheit."*) (That thing or that action is of value which is promoting life. To ask for the value of life itself is meaningless. That nevertheless we ask this question can easily be explained by Darwin's theory. It is nothing other than a continuation of our thinking habit beyond its aim.) The question of the value of life itself is meaningless in the sense of the physical sciences, because there is no general rational answer to it. This should not prevent anybody from answering this question for himself. The answer is a matter of his personal belief.

Boltzmann was aware that neither the physical sciences nor technology could ultimately resolve the problems of our human society. But society must integrate science and technology appropriately into its educational system in order that people feel comfortable with its achievements.

4. PERCEPTION THEORY

Boltzmann calls metaphysics a "migraine of the human mind". He also agrees with Ernst Mach that economy of thought should be a guiding principle of science. But he vigorously rejected Mach's neopositivistic view that scientists should not be allowed to construct an intuitive picture of the real world.

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*) From the lecture "Über eine These Schopenhauers".

According to Mach, scientists should be restricted to correlating experimental facts. They should only use concepts directly related to measurable quantities. In fact Mach's views had a great influence on the development of the theory of relativity and on the early years of quantum mechanics, where, in order to eliminate the prejudice of classical physics, it was prohibited to think in terms of intuitive pictures and models. Ernst Mach would be shocked if he knew that nowadays we believe that the basic constituents of matter are quarks and gluons; building stones which, owing to the so-called confinement hypothesis, cannot even in principle be directly observed. The pragmatic view of American physicists such as Feynman[4)] and Gell-Mann helped to overcome certain extreme interpretations of quantum mechanics. In particular, between 1960 and 1970, many physicists followed a phenomenological approach which denied the existence of fundamental constituents of matter. Today physicists are free to construct intuitive models just as Boltzmann liked to do.

Now we experience a fruitful time in physics where it seems that all elementary interactions can be merged to make a grand unified theory. For example, one can speculate that certain details of the microcosmos, such as whether neutrinos have a mass, may have consequences for the Universe.

Such a unification is very much in the spirit of Boltzmann. He did not believe that science should be an incoherent patchwork of different disciplines based on different axioms just vaguely connected by the fact that each special field tries to grasp a detail of the human perception. He tried to find a unified model of reality -- only of course a model that constantly had to be improved. His great contribution in this direction was statistical mechanics.

5. STATISTICAL MECHANICS

Boltzmann achieved the unification of two branches of physics which seemed to have nothing in common: classical mechanics and axiomatic thermodynamics. Even the calculational methods used in these two fields were entirely different and yet they ultimately have the same root -- the motion of the constituents of matter. In classical mechanics we are only interested in the collective motion of a bulk of material and we approximate planets and stars by point-like particles moving according to simple differential equations. In thermodynamics, on the other hand, we are just interested in the properties of a bulk of material at rest. These properties can, as Boltzmann showed, under certain assumptions be derived from the motion of the individual constituents of matter: the atoms and molecules.

Certain problems can be treated by a very simple model: for example, for a train going on a rail, the motion can be predicted very accurately. It is absolutely essential for our everyday life that we are able to rely blindly on the performance of devices which our engineers have constructed. But even for rather simple mechanical systems constructed by nature, such as our planetary system, one needs to know rather few parameters in order to keep track of its motion. Furthermore, since the parameters are so few, we have the means to measure each of these parameters very accurately and keep track of its errors as time evolves. The situation is quite different if one wants to predict how many billiard balls will hit each other successively, knowing the initial velocity of the first billiard ball and the rest positions of the billiard balls to be hit later. In this case the error in our knowledge of the velocity increases exponentially with the number of hits and it is not possible to predict what will happen after a small number of hits. Even the very best billiard players in the world cannot achieve many more than four successive hits with one stroke. The mechanics of a gas resembles very much the mechanics of a billiard game, only that for one mole of gas one has to keep track of 6×10^{23} molecules instead of a few billiard balls. Thus, even if we had a rather complete information on the initial state of the gas at a certain time, this information would be lost in a millionth of a second.

In 1872 Boltzmann introduced his famous transport equation[5)] for the molecular distribution function $f(\vec{x},\vec{v},t)$ where $\vec{x}$ is the position and $\vec{v}$ the velocity. In the same paper he introduces the functional $H[f]$ given by

$$H[f] = \int_V d^3x \int_{-\infty}^{\infty} d^3v \, f \log f \qquad (1)$$

to derive the expression suggested by Maxwell for the equilibrium distribution. In the first paper this quantity is actually denoted by the letter E. Up to a sign and a numerical factor it is equal to the entropy of the system under consideration.

One of the serious arguments Boltzmann had to cope with was that the laws of classical mechanics had no preferred direction of time, while in thermodynamics the direction of increasing entropy is preferred. This question had been raised by Boltzmann's friend Loschmid. Boltzmann resolved the problem by his statistical interpretation of the second law. Another argument against statistical mechanics is the so-called recurrence paradox. Later I shall come back to these arguments. Actually the name statistical mechanics was coined by the American physicist J.W. Gibbs. Gibbs worked independently of Boltzmann in this field, but he did not fight as vigorously as Boltzmann for the existence of atoms.

In small space regions of a gas, about 10^{-3} cm in size, deviations from the second law, the so-called statistical fluctuations should, however, arise. In Brownian motion these fluctuations can be observed. It was no less than Albert Einstein and Marian von Smoluchowski who later independently made a theory of Brownian movement. Boltzmann was the first to give a fundamental law of physics a statistical interpretation. For large material systems, one can only make statistical predictions for the physical state of the system. It is hopeless to follow the individual motion of every molecule in detail. In the so-called thermodynamic limit, one can predict the general properties of a very large, ideally infinitely large, material system. The laws one obtains in this limit are just the laws of thermodynamics.

6. BOLTZMANN'S PRINCIPLE

It was this statistical interpretation of a basic law of nature which shocked many physicists -- among them Max Planck -- who, before the year 1900, believed that the second law of thermodynamics was a basic axiom given to us by God, which one had to accept as the starting point of any thermodynamic consideration. Maybe this is the point where I should explain Boltzmann's intuitive understanding of the second law of thermodynamics. It made the concept of entropy, which had been introduced by Rudolf Clausius in 1857 as a rather abstract mathematical concept, into a fact quite familiar from everyday life. In very sloppy terms, the second law of thermodynamics states: If a large system is left alone, after a short while a certain disorder will develop. It is just this which gives our poor housewives so much work.

Boltzmann found in 1877[6] that entropy is a measure of the disorder of the state of a physical system. The second law of thermodynamics simply states that if a physical system is left alone, its entropy, which is a measure of its disorder, will always increase in time or remain constant rather than decrease.

In more mathematical terms he stated that the entropy S is proportional to the logarithm of the probability W to find a physical system in a class of states:

$$S = k \log W \tag{2}$$

This formula has been called Boltzmann's principle by Albert Einstein. It is engraved on Boltzmann's tombstone in the central cemetery of Vienna, situated next to the graves of the presidents of the Austrian republic (see Fig. 1).

7. BOLTZMANN'S SCHOOL

Boltzmann's fame has attracted many brilliant students. Just as Boltzmann himself who, when he was already professor of mathematical physics, had spent some time at the Institutes of Bunsen and of Königsberger in Heidelberg, and of Helmholtz and of Kirchhoff in Berlin. Many young scientists came to Boltzmann's Institute in Graz, then in Munich, and later in Vienna. His most famous students in Graz were Svante Arhennius and Walter Nernst. In Munich the Japanese atomic physicist H. Nagaoka was at his Institute and in Vienna Paul Ehrenfest, my father Ludwig Flamm, and Lise Meitner were among his students. Stefan Mayer was his assistant in Vienna and Fritz Hasenöhrl his successor. His thoughts were not only spread out by his lectures, but even more by his writings. He published more than 4000 printed pages. Especially, Arnold Sommerfeld, who had many students, among them Werner Heisenberg and Wolfgang Pauli, had a deep appreciation for Boltzmann's work. He wrote: "Niemand auch nicht Maxwell und Gibbs, hat so tief über die Einseitigkeit der Naturvorgänge und ihre wahrscheinlichkeitstheoretische Begründung nachgedacht wie Boltzmann." (Nobody, not even Maxwell and Gibbs, has thought so deeply on the preferred direction of the processes in nature and its probabilistic explanation as Boltzmann.) Sommerfeld also was a witness to the memorable debate at the meeting of the Naturforschergesellschaft in Leipzig in 1895, where Helm and Ostwald attacked the kinetic theory while Boltzmann and Felix Klein defended it[7)].

Max Planck had been converted from a "Saulus" to a "Paulus" when he had to use Boltzmann's methods to derive his celebrated law for the black-body radiation. In fact, Boltzmann had shown that one needed a random-phase assumption for this problem. Einstein was a good friend of Boltzmann's student P. Ehrenfest, who presumably conveyed Boltzmann's ideas to him. According to Einstein's own words, his work on the Brownian motion in 1905 was in the spirit of Boltzmann[8)]. It intended to prove the reality and the size of certain atoms using the molecular fluctuations postulated by Boltzmann.

The philosophy of science advocated by Einstein in his later years coincides with Boltzmann's ideas. Also Marian von Smoluchowski, who studied in Vienna after Boltzmann's death, calls himself a student of Boltzmann.

Well-known "intellectual grandsons" of Boltzmann are Erwin Schrödinger and Victor Weisskopf. Schrödinger attended the lectures of Boltzmann's successor Hasenöhrl. Weisskopf learned much from Boltzmann's student Ehrenfest[9)]. Both Schrödinger and Weisskopf, like Boltzmann, had widespread interests and also wrote on our physical conception of life[10,11)].

After his derivation of the entropy of the black-body radiation, Boltzmann insisted already in 1886 on the importance of solar energy for photosynthesis. In 1904 he had already anticipated our current picture of chemical evolution and self organization of biological molecules[1]. In non-linear systems which are far from thermodynamic equilibrium, the statistical fluctuations predicted by Boltzmann can lead to metastable states in a limited space region, which corresponds to a considerable molecular order. Globally, however, the second law of thermodynamics is fulfilled.

Figure 2 shows Boltzmann and his collaborators in Graz. In order to give you an impression of Boltzmann's personality, Fig. 3 is a photograph of him and Figs. 4 and 5 are two cartoons drawn by his student Przibram.

8. THE STRUGGLE

Boltzmann's ideas met with violent objections coming from theoretical physicists and mathematicians. In fact, these discussions never stopped[12]. The classical objections are often formulated in the form of paradoxes. The two most famous ones are the reversibility paradox of Boltzmann's friend Loschmid and the recurrence paradox put forward by Planck's student Zermelo. Loschmid pointed out that the laws of classical mechanics had no preferred direction of time, while in thermodynamics the direction of increasing entropy is preferred. Boltzmann resolved this paradox by his statistical interpretation of the second law.

If one calculates Boltzmann's quantity H for two-dimensional hard disks which start from lattice sites with an isotropic velocity distribution, one obtains the graph shown in Fig. 6. This same figure shows what happens if the velocities of all disks are reversed at a certain time. Indeed, H increases for a short period of time. But this deviation from the second law of thermodynamics only lasts for the very short time of about 100 collisions, which corresponds to about 10^{-6} s in a dilute gas.

Zermelo argued, using a lemma of Poincaré, that every mechanical system under certain conditions would after a while come back to its initial state, thus violating the second law of thermodynamics. Boltzmann answered that the time it takes the system to return to its initial state increases exponentially with the number of particles involved. In fact, already for a small volume of gas the recurrence time is larger by far than the age of our Universe.

Boltzmann was ahead of his time and many of his fellow scientists were not ready to accept his ideas. He had already introduced discrete energy levels as early as 1872 and his statistical interpretation of the second law of thermodynamics paved the way for the statistical interpretation of quantum mechanics.

Especially towards the end of the century many physicists did not believe in atoms. On the continent, the spiritual leader of this movement against atoms was Ernst Mach. He declared that atomistics is pure metaphysics. His critical mind did a good job in cleaning up the old house of classical mechanics. But he pushed his scepticism too far. He neither accepted the existence of atoms nor Einstein's theory of relativity. Even after radioactive decay and Brownian motion had provided strong support for atomistics he did not accept it. In May 1917 Einstein wrote to his friend Michele Besso[13]: "Über das Machsche Rösslein schimpf ich nicht. Du weisst doch, wie ich darüber denke. Aber es kann nichts Lebendiges gebären, nur schädliches Gewürm ausrotten." (I do not rail against Mach's little horse. You know how I think about it. But it cannot give birth to something living, it can only extirpate harmful worms.)

It was not enough that Boltzmann had to fight for his own ideas, he had to defend the concept of atoms as a whole. It looked like the fight of a single man against a clan of people.

As on the continent, Boltzmann's struggle for his ideas seemed not to be rewarded by much success, he did not miss the opportunity to defend his case in Great Britain. The British in their fairness had invited Boltzmann to defend atomistics at a meeting of the British Association for the Advancement of Science in 1894.

In 1904 and 1905 Boltzmann even travelled to the United States of America hoping that in the new world people would have an open mind for his ideas.

It is the tragedy of Boltzmann's life that he did not experience the glorious victory of his ideas. He left this world while the decisive battle was still going on. During a vacation in Duino near Trieste, my mother Elsa Boltzmann, of whom her father used to say that she was the sunshine of his life, found her father hanged. She was only fifteen years old.

9. EPILOGUE

Still today many textbooks follow Boltzmann's derivation of the H theorem. In 1924 Bose found that for quantum statistics one just had to replace Boltzmann's particle counting by a counting of the number of cells of size h^3 in phase space[14]. The quantum mechanical expression for the entropy given by J. von Neumann is[15]

$$S = -k \, \mathrm{Tr} (\rho \log \rho) \tag{3}$$

where ρ is the density matrix. It is the analogue of Boltzmann's expression (1).

Boltzmann was always enthusiastic about new technical developments. For instance, he gave public lectures on aviation and on X-rays. I believe that even if he were confronted with our present-day problems he would remain optimistic, believing like Socrates that man is intrinsically good. If he had lived today, he would, like R.P. Feynman, have expressed the opinion that he could not imagine that any bombs, even nuclear bombs, could crush the spirit of humanity for long[4]. After all life has constantly evolved at the edge of its own destruction in the three and a half billion years of its existence. The deadly plutonium we produce today might be compared to the, at that time, deadly oxygen which the ancestors of the alga chlorella produced in the course of photosynthesis. Photosynthesis was a necessity because the reserves of primary organic compounds supplied in the early days by chemical reactions in the atmosphere of the earth and in the water had come to an end. Practically all of the oxygen in our atmosphere is due to the photosynthesis of algae chlorellae and other plants. And it is just this oxygen which was a prerequisite for the development of vertebrates and mankind.

Acknowledgement

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Figure captions

Fig. 1 : Boltzmann's tombstone.

Fig. 2 : Boltzmann and his collaborators in Graz.

Fig. 3 : Boltzmann at the age of 60 years.

Fig. 4 : Boltzmann at the blackboard (cartoon).

Fig. 5 : Boltzmann in California (cartoon).

Fig. 6 : Evolution of H with time for a system of 100 disks when the velocities are inverted after 0, 50, and 100 collisions, respectively[12].

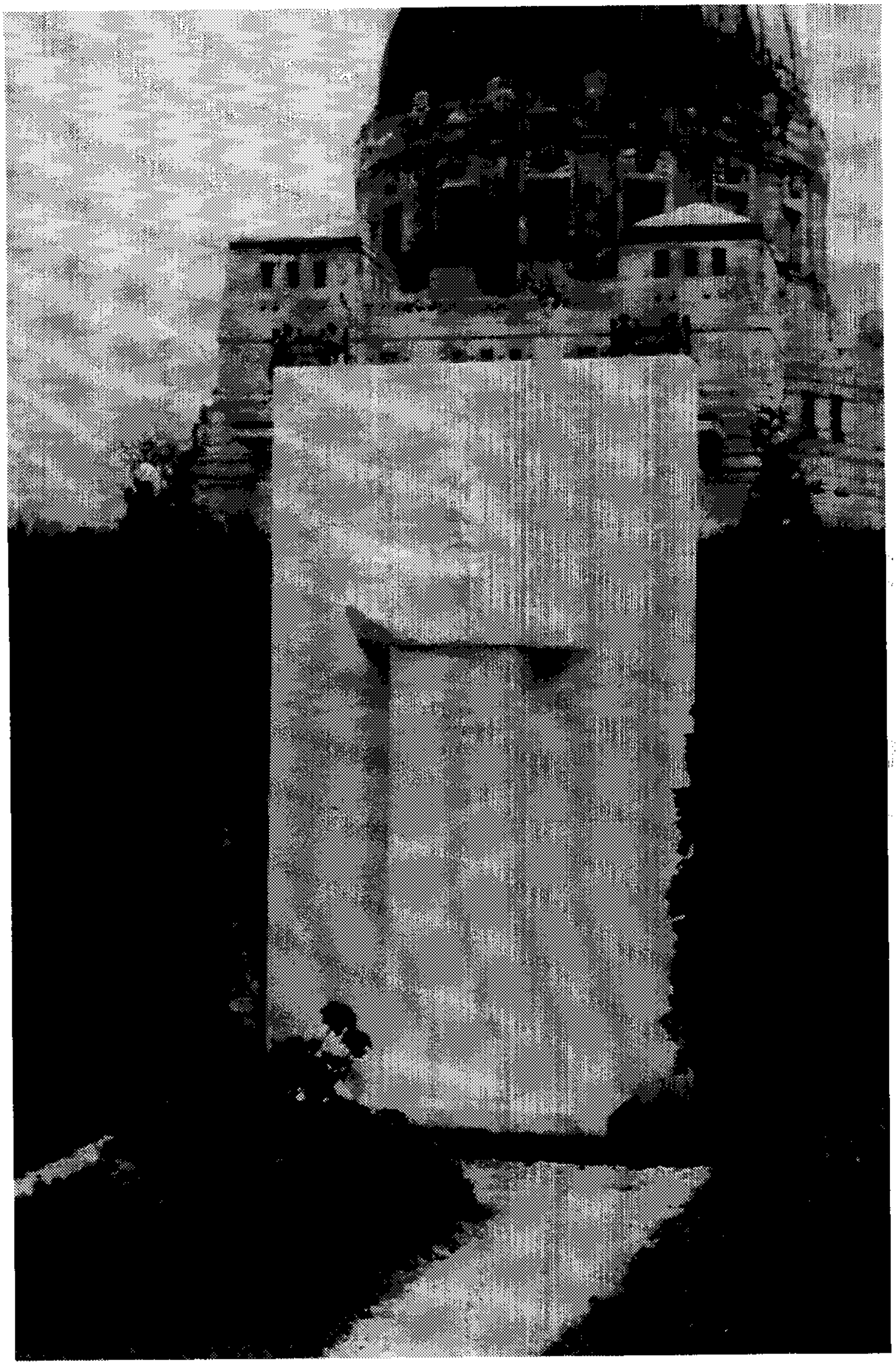

Fig. 1

Fig. 2

Fig. 3

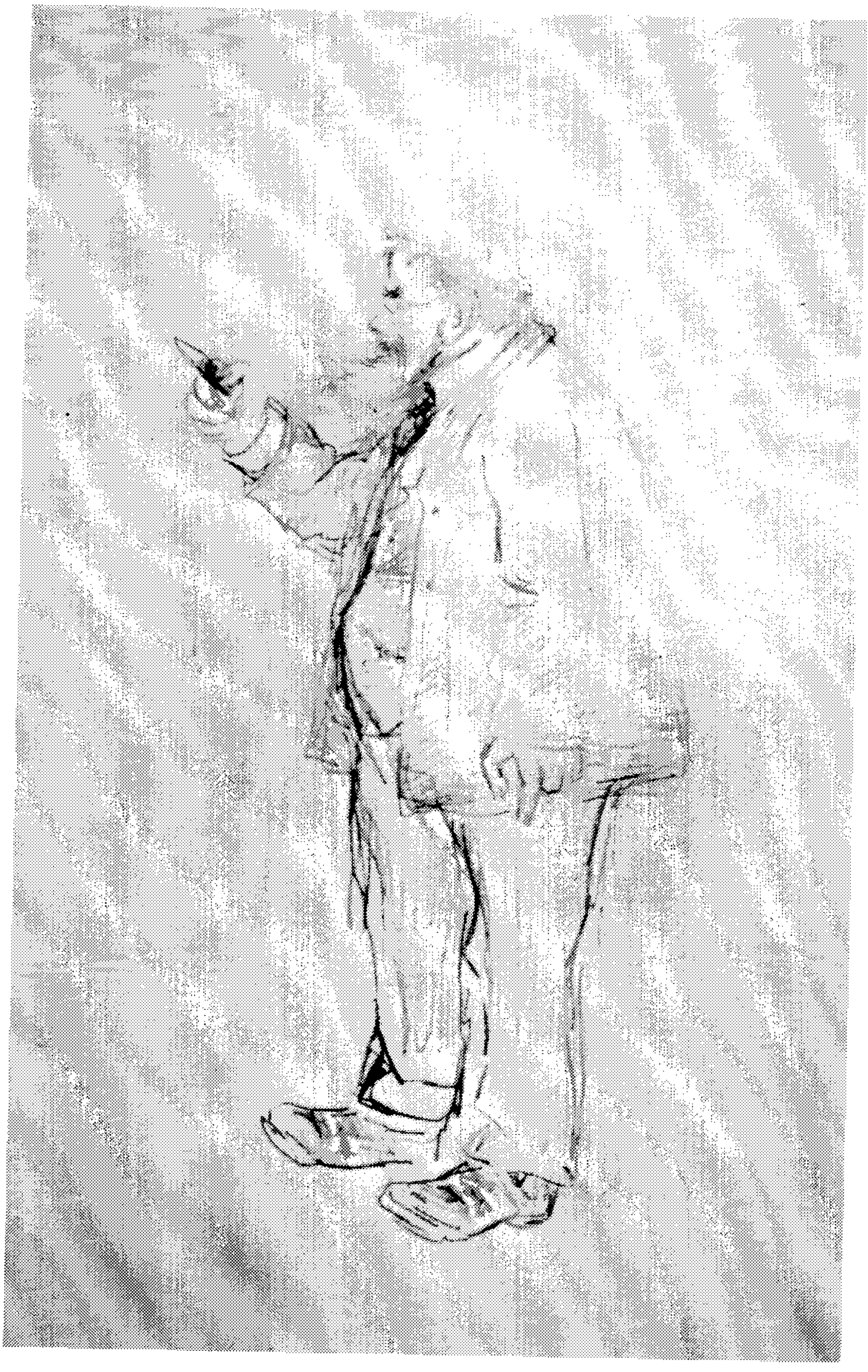

Fig. 4

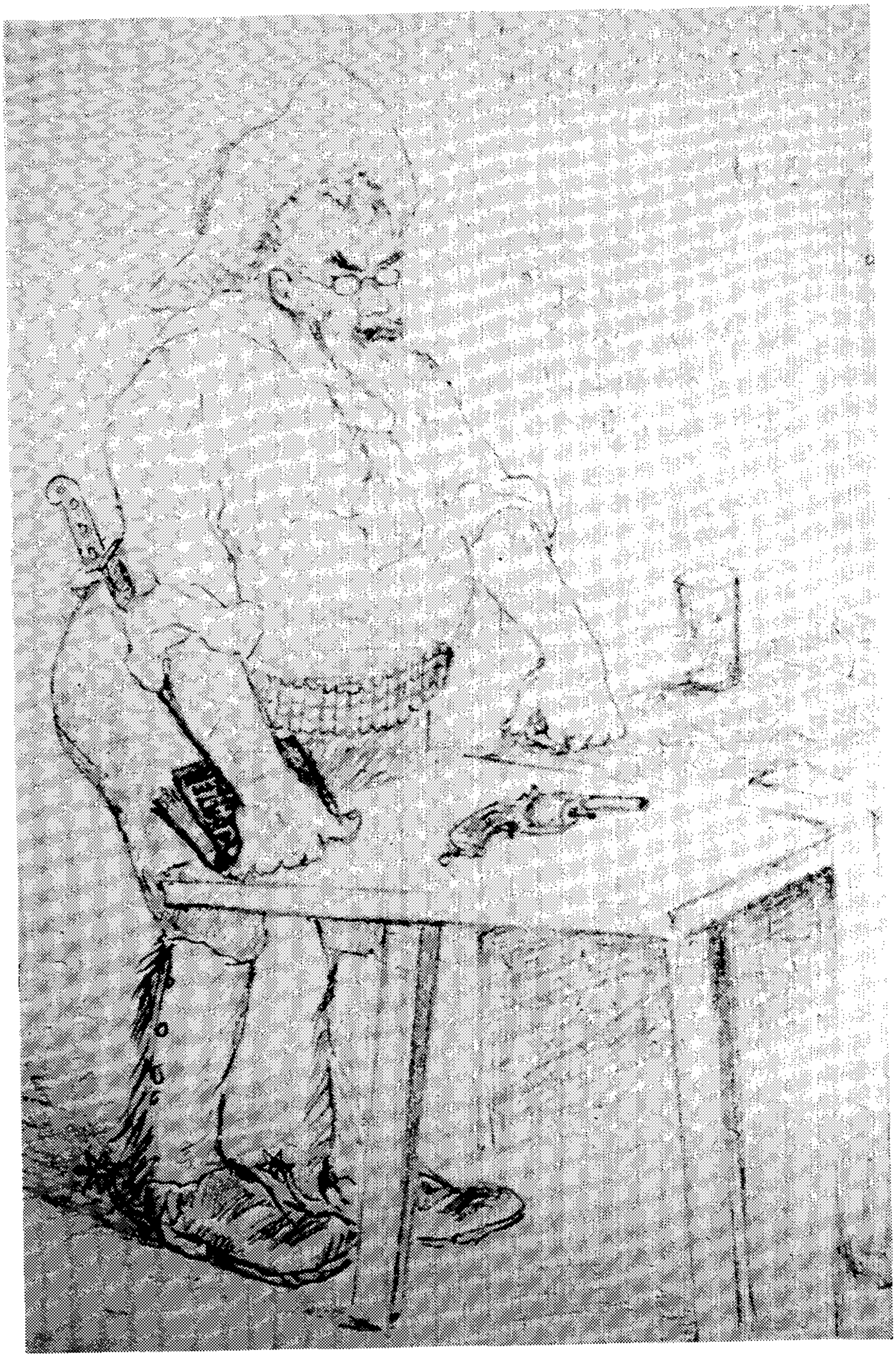

Fig. 5

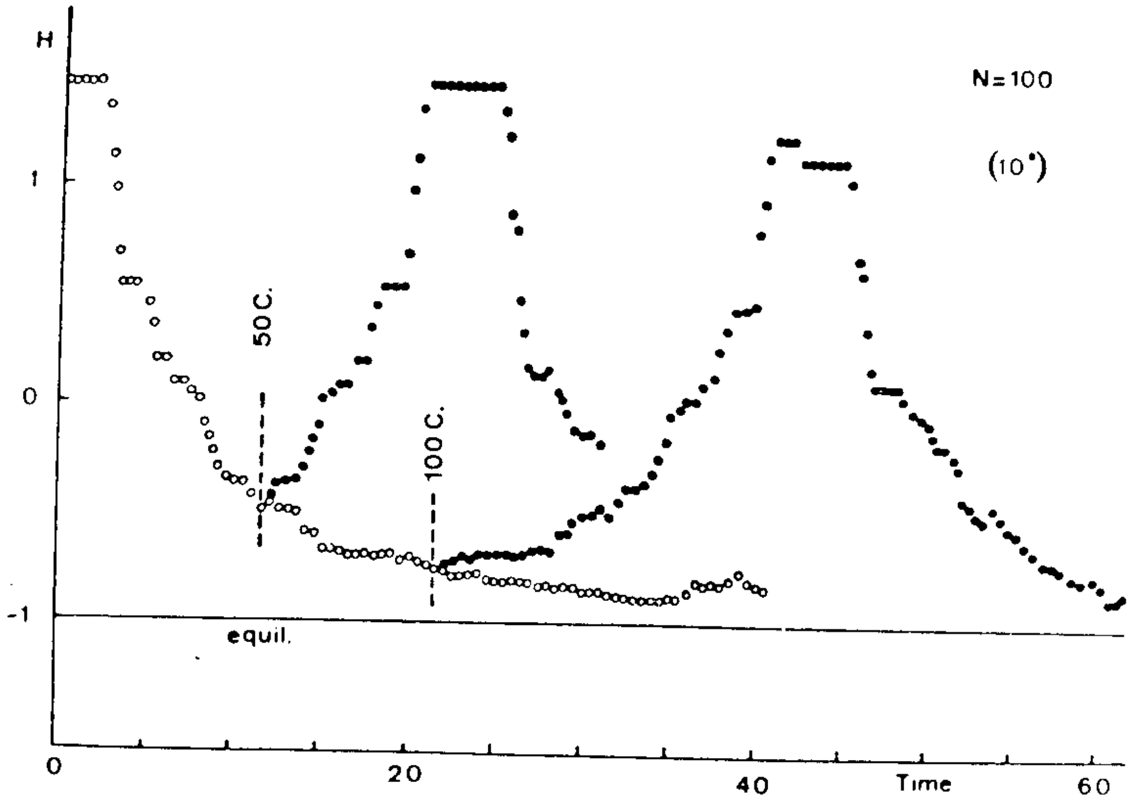

Fig. 6