Application of a Fuzzy Controller in a Warm Water Plant*

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A new method of designing a controller, based on a vague kind of information and using fuzzy set theory, shows promising results in a case study.

Summary—In many cases a human operator is far more successful in controlling a complex industrial process than a controller designed by modern control techniques. The method of expressing the strategy of a human operator using fuzzy set theory has already been proposed elsewhere. In this study this method is applied to the control of a warm water plant. Fuzzy algorithms based on linguistic rules describing the operator's control strategy are applied to control this plant. Several types of such algorithms are implemented and compared.

1. INTRODUCTION

Fuzzy set theory is a theory about vagueness, uncertainty and enables us to use nonprecise, ill-defined concepts and yet to work with these in a mathematically strict sense[1]. Automatic Control theory has developed in the last decades from an empirically oriented technique into a strongly mathematically based technique, requiring precision, well defined concepts and exact data. Nevertheless vagueness and subjectivity still play a role as is pointed out further on.

In the forties the introduction of frequency characteristics and diagrams (Nyquist, Bode, Nichols) to investigate the stability of a system created an elegant and mathematically exact tool.

The design criteria however remain vague and subjective. No precise answer can be given as to what gain and phase margin, maximum relative error etc. have to be chosen to achieve a 'good' system performance. The large spread of these criteria to be found with several authors, dependant on their personal views and experience in their field of application, is thus not surprising. Hence the introduction of different criteria like that of Ziegler, Nichols and others.

The root locus method of Evans in the fifties suffers from this same ambiguity as no exact values for the relative and absolute damping factors exist for most practical situations. The introduction of the integral error criteria was undoubtedly a step forwards in the exact determination of an optimal system, but in fact the vagueness here has been shifted to the choice of a particular criterion. The use of more complex performance criteria enables the incorporation of several desired factors in the optimisation. The decision as to which factors have to be accounted for and to what extent, is still subjective. Thus, notwithstanding the creation of numerous mathematical control techniques, the final decision about the 'goodness' of a system's behaviour remains a personal, subjective task. Under the surface of modern control techniques subjectively, vagueness—unconsciously—still does play a role. Furthermore, in non-engineering systems, the co-called 'soft systems', subjective matters are almost always predominant. A theory of vagueness could be very useful here[2].

Apart from this kind of general rationale of the incorporation of vagueness in system's design, there is a much more practical reason for the particular kind of fuzzy control system used in this research. Complex industrial plants such as chemical reaction processes often are difficult to control automatically. In some cases plant models can be derived from the underlying physical or chemical properties of the process. Usually this requires very elaborate calculations. Even under various approximations, the final model is difficult, of high order, nonlinear, time varying etc. In many cases the real process differs from this model in such a way that no more than the structure of the model can be determined. Parameter estimation methods to obtain a purely mathematically described behavioristic model may also be complex, time consuming and therefore costly. When non-linearity, time variance and stochastic distur-

bances are important, modelling methods become still more complicated. Control theory however relies on modelling as a vital step in the design process.

On the other hand it is interesting to note that in many cases the control of a process by a human operator is more successful than any such automatic control. Hence it seems worthwhile to investigate the control policy of the operator. As the strategy he uses is vague and qualitatively described, the use of fuzzy set theory in such an investigation is self evident. This was also the rationale behind the 'fuzzy logic controller' recently reported by Mamdani and Assilian[3]. In their control application of fuzzy set theory, they achieved a successful control of a small boiler–steam engine combination, even better than a conventional DDC controller. The present work follows the same idea of using fuzzy rules as a control algorithm.

A warm water plant which was difficult to control because of nonlinearity and variability, has been controlled by a fuzzy algorithm based on the experience of a human operator. From a set of linguistic rules which describes the operator's control strategy a control algorithm is constructed where fuzzy sets define the words used. Several types of such an algorithm are implemented and compared in behaviour as well as in structure. An alternative algorithm— mathematically equivalent to the other—is proposed to speed up the computation[4].

2. THE FUZZY LINGUISTIC CONTROL

The development of the theory of fuzzy sets and algorithms[5] makes it possible to build a control algorithm based on a very general kind of inexact information, namely information expressed in natural language. This linguistic information may be obtained from an experienced human process operator. This is done by asking the operator to describe the control strategy he uses, the way he reacts in a certain situation. Thus the operator may be able to express his control strategy as a set of linguistic decision rules of the form

if "increase in temperature is big" then "decrease pressure a lot", else, if "increase in temperature is low" then "decrease pressure a little", else, etc.

Clearly such expressions can be described as fuzzy sets on the universes of discourse "increase in temperature" and "decrease of pressure", respectively. Thus by defining the appropriate fuzzy sets and translating the rules as fuzzy implications of the form: if A then B, as

functions of those fuzzy sets (A and B), the human control strategy can be converted into a control algorithm and implemented on a computer as outlined below. Note that the appendix presents the precise mathematical derivation of the fuzzy control algorithm. Here a less formal outline of the method will be given.

The basis of the whole approach is the fuzzy implication (rule)

$$\text{if } A \text{ then } B$$

where A and B are fuzzy sets, like "high temperature", "small pressure", on the universes of discourse input and output respectively. Considering this rule as a kind of equivalent of a system mapping, the next question is: what will the output be to a certain input A'? In other words, given the rule: if A then B, and the input A', what will be the output B'? An expression therefore is derived using the compositional rule of inference[5] in the appendix.

The next stage is the observation that the control algorithm clearly is composed of several rules; in different situations the human operator will apply different actions. The algorithm will have a form like

$$\text{if } A_1 \text{ then } B_1, \text{ else, if } A_2 \text{ then } B_2, \text{ else} \ldots$$

This set of rules will be evaluated by identifying the 'else' connective as the union operator between fuzzy sets. The rules can be evaluated seperately and the results are combined using the max operator. Thus given a certain input A' resulting in an output of the first rule: B'_1, of the second rule: B'_2, etc., the resulting overall fuzzy output B' will be

$$B' = \max (B'_1, B'_2, \ldots)$$

The extension of this single-input–single-output type to a more complex form of system having e.g. two inputs and one output with rules like

$$\text{if } A \text{ then (if } B \text{ then } C)$$

is a straightforward one. The same approach still applies as indicated in the Appendix.

In the particular kind of application of this system concept to a process controller the input to the controller—temperature error—and the output of the controller—process input: flow— were both non fuzzy but deterministic quantities. The approach to cope with a non fuzzy input is explained in the appendix in two different ways leading to the same result. The result of evaluating the fuzzy algorithm for a particular deter-

ministic input is still a fuzzy output set ranging over the whole possible set of outputs. In order to obtain one deterministic output value from this fuzzy output set a decision procedure has to be adopted to make a choice as to which particular (non fuzzy) value is a good representative of the fuzzy set. The simple decision procedure applied here is to take that output value at which the membership function is maximal as described in the Appendix.

2.1 The process

This fuzzy system concept has been applied to design a controller for the temperature of a warm water plant, built on a laboratory scale. Figure 1 shows a schematic diagram of the plant. The warm water tank is divided into several compartments. The cold water stream enters the tank with a variable flow $F2$, passes the compartments in sequence and leaves the tank in the last compartment. This water is heated by a heat exchange unit in which hot water, at about 90°C, flows with a variable flow $F1$. The aim is to control the temperature of the water in one of the compartments for different temperatures and steady state values of the flow $F2$ by adjusting the dynamic values of $F1$ and $F2$. In this application the temperature of the water leaving the heating compartment has been controlled to minimize time delay problems. Usually a constant amount of liquid, i.e. water, of a certain temperature is required from the process, so the flow $F2$ has to be kept constant during steady state. Only during a change to another desired temperature can the flow $F2$ be changed. The main control variable however, is the flow $F1$ of the hot water.

Earlier investigations of the process had shown that this process had difficult control properties, arising from nonlinearities, asymetric behaviour for heating and cooling, noise and dead time. Also the ambient temperature influenced the process behaviour. To get a comparative idea of the performance of the fuzzy controllers an ordinary PI-controller has been implemented as well. This PI-controller has been optimally adjusted for an experimentally fitted model consisting of two equal time constants and
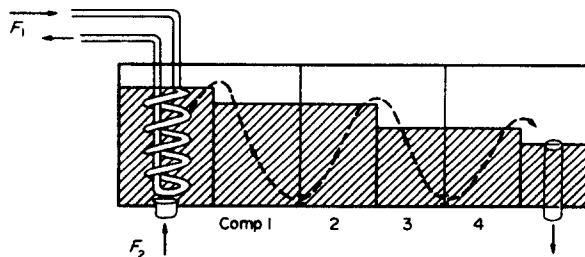

F₁

Comp I   2   3   4

F₂

FIG. 1. Schematic diagram of the plant.

time delay, with a time delay = 10 sec, the time constants = 40 sec. The optimal values of the integral gain K_I and the proportional gain K_P for three different integral error criteria, the ITAE, IAE and the ISE and a step function input of ten degrees centigrade, of this digital PI-controller are shown in Table 1.

TABLE 1. OPTIMAL K_I AND K_P VALUES FOR A DIGITAL PI-CONTROLLER (SAMPLE TIME 1 SEC)

	ITAE	ISE	IAE
K_I	0.018	0.019	0.020
K_P	1.35	3.02	1.94

One of the main difficulties of this controller was its need of adjustments to operate over a wide range of desired temperatures. It is clear that a more sophisticated controller, e.g. with a stochastic, adaptive model, than just a PI type could have a better performance. Hence the comparison betweeen the PI control and "fuzzy control" should be regarded as only a rough indication of relative performance.

2.2 The algorithm

The described fuzzy controller resulted in the following algorithm:

Every rule i associates a fuzzy flow (fl) subset to a fuzzy temperature (t) subset, represented by their membership functions:

$$\mu_i(t) \rightarrow \nu_i(fl) \quad i = 1, 2, 3, \ldots, I$$

The actual action applied, fl_o, can be computed from the measured temperature t_o as follows.

The membership values at the temperature t_o are determined for each rule

$$\mu_1(t_o), \mu_2(t_o), \ldots, \mu_I(t_o)$$

The implied fuzzy subsets for the flow fl have a membership function λ that can be calculated for each rule as

$$\lambda_i(fl) = \min [\mu_i(t_o); \nu_i(fl)] \quad i = 1, 2, \ldots, I$$

The overall fuzzy subset for flow is obtained by taking the union

$$\lambda(fl) = \max_i \min [\mu_i(t_o); \nu_i(fl)]$$

$$i = 1, 2, \ldots, I$$

The result is a fuzzy subset which ranges over all

values of the flow. As the action is taken at the maximum value of the membership function of this fuzzy subset, it can be determined directly by taking that value of the flow fl_0, for which the following holds

$$\lambda\,(fl_0) = \max_{fl} \max_i \min\,[\mu_i(t_0);\ \nu_i\,(fl\,)]$$

$$i = 1, 2, \ldots, I$$

3. THE FUZZY CONTROLLERS

3.1 The fuzzy sets

The fuzzy sets used in this application all had a continuous form. An uniform structure of the membership function for all fuzzy sets was chosen, namely the continuous function

$$\mu\,(x) = (1 + (a(x - c))^b)^{-1}$$

as shown in Fig. 2. This choice has the advantage that the desired shape of the fuzzy set can be adapted by just three parameters: c alters the point of minimum fuzziness ($\mu = 1$), a the spread and b the contrast. Because the decision procedure would become too time-consuming in the continuous case, the fuzzy output sets were
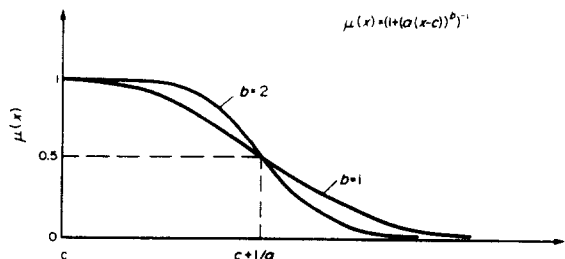

FIG. 2. Form of the membership functions used.

TABLE 2. DEFINITIONS OF THE FUZZY SETS USED FOR DIFFERENT STRATEGIES (SEE TEXT)

NAME	SUPPORT SET	MEMBERSHIP FUNCTION
not small	x	$1 - (1+0.5x)^{-1}$
small	x	$(1+0.5x)^{-1}$
very small	x	$(1+x^4)^{-1}$
slightly small	x	$(1+0.5x)^{-1}$ for $x \geqslant 1$ else 0.5
small	x	$(1+(3(x-1))^2)^{-1}$
medium small	x	$(1+(3(x-0.5))^2)^{-1}$
extremely small	x	$(1+(3x)^2)^{-1}$
small	dx	$(1+(3dx)^2)^{-1}$
medium	dx	$(1+(3(dx-0.5))^2)^{-1}$
big	dx	$(1+(dx-2)^2)^{-1}$
very big	F1	$(1+2(F1-12)^2)^{-1}$
very small	F1	$(1+2F1^2)^{-1}$
near st. state	F1'	$(1+(3(F1'-1))^2)^{-1}$
very near st. state	F1'	$(1+(3(F1'-0.5))^2)^{-1}$
small	dF1	$(1+(2(dF1-0.2))^2)^{-1}$
medium	dF1	$(1+(2(dF1-1))^2)^{-1}$
big	dF1	$(1+(dF1-3)^2)^{-1}$
very big	F2	$(1+2(F2-18)^2)^{-1}$
very small	F2	$(1+2(F2-1)^2)^{-1}$

x: temperature error, dx: change in error, $F1$: warm water flow, $dF1$: change in $F1$, $F2$: cold water flow.

calculated at finite quantized intervals of the support set, the flow. The definitions of the fuzzy sets used are shown in Table 2. *F*1 is quantized in 12 levels, d*F*1 in 15 and *F*2 in 18.

3.2 Heuristic structure

Whereas in [3] just one fuzzy control algorithm has been successfully applied to a real dynamic process, in this research three types of fuzzy algorithms have been tested. Instead of asserting one fixed structure of the human operator's control heuristics, namely that a process operator generally uses error and rate of change of error to calculate a change in the value of the process input, several different heuristics have been applied. The motive for this was the fact that one part of the control—keeping the temperature accurately at a desired value—turned out to be difficult for a human controller. It was extremely difficult to avoid oscillations around the setpoint. Hence three strategies for this 'steady state' control have been tested:

(1) the operator uses error and rate of change of error to affect a change of flow (the process input).

(2) the operator only uses the error as information and compensates by changing the flow.

(3) the operator uses error and adjusts the flow above or below neutral position.

In this third strategy the controller was supposed to know what absolute value of the flow (*F*1) was the steady state position, hence a static flow-temperature characteristic was assumed to be known. A summary of these three different strategies is given in Table 3.
Because the aim of the control was not only to keep the temperature accurately at a desired value, but also to perform step changes in temperature as fast as possible, the set point change strategy should obviously have a kind of bang-bang character, both for flows *F*1 and *F*2 where the latter is only used during the transient as stated earlier.

3.2 The rules

The first strategy resulted in the following set of rules

if *x* 'not small' then *F*1 'very big'
          then *F*2 'very small'
if *x* 'small'   then *F*1 'very small'
          then *F*2 at steady state
if*x* 'very small' then *F*2 at steady state
  then if increase of *x* 'small' then decrease of *F*1 'small'
  then if increase of *x* 'medium' then decrease of *F*1 'medium'
  then if increase of *x* 'big' then decrease of *F*1 'big'

These five rules control a temperature below setpoint while it is increasing. Apart from the second rule a symmetric set of rules was applied in the other cases.

The second strategy was realized by the following rules

if *x* 'not small'   then *F*1 'very big'
          then *F*2 'very small'
if *x* 'slightly small' then *F*1 'very small'
          then *F*2 at steady state
if *x* 'small'   then increase of *F*1 'big'
          then *F*2 at steady state
if *x* 'medium small' then increase of *F*1 'medium'
          then *F*2 at steady state
if *x* 'extremely small' then increase of *F*1 'small'
          then *F*2 at steady state

The additional refinement of the 'small' region required an appropriate modification of the previous fuzzy set 'small' as indicated in Table 2.
The third strategy which has been applied consisted of the following set of rules

if *x* 'not small' then *F*1 'very big'
          then *F*2 'very small'
if *x* 'small'   then *F*1 'near steady state'
          then *F*2 at steady state
if *x* 'very small' then *F*1 'very near steady state'
          then *F*2 at steady state

TABLE 3. CONTROL HEURISTICS

	Observation	Action
Strategy 1	error and change in error	change in flow F1
strategy 2	error	change in flow F1
strategy 3	error	flow F1 (static value known)

Because the static flow–temperature characteristic was very sensitive to the environment, the algorithm was set up to enable alterations of this characteristic during running time.

3.4 Results

The overall results of these three types of controllers have been summarized in Table 4 and compared with a PI type controller mentioned above. In view of the bang-bang rules it is not surprising that the systems with the fuzzy controllers all show much faster step responses than the classical PI type control system, i.e. for a step of 10°C about 0.3 min against 0.7 min for the PI controller. However the first two controllers behaved like the human operator in that their accuracy was poor, 1.5°C oscillations around the setpoint against 0.4° for the PI controller. The warm water process with the third type fuzzy controller showed the best performance. It combined the same high speed step response as the other fuzzy controllers, 0.3 min, with nearly the same accuracy as that of the PI controller, 0.5° variations.

3.5 Discussion

Although the last fuzzy controller showed the best results, additional information about the 'neutral' steady state flow position had to be used. The introduction of this steady state information has the disadvantage that the controller has to be readjusted for each different desired temperature value. The sensitivity of these settings to changing surroundings is another problem. The fact that the actual readjustment of these settings during running time was performed by the human operator indicates that a vague guess of this steady state flow value might be sufficient. However in some processes a guess of such steady state characteristics may be difficult. A small integral action may ease the above mentioned problem.

An intuitive way of explaining the differences in behaviour of these three fuzzy controllers could be to relate their structure to those of conventional controllers. Looking only at the 'steady state' rules, it can be observed that the inputs and output of the first type fuzzy controller are similar to those of a PI type incremental control algorithm. The input–output quantities of the second type are those of a purely I type incremental algorithm and finally the third type has an input and output identical to those of a P type controller using a positional algorithm, as indicated in Table 3. It should be emphasized that this supposed analogy lacks any rigid basis. The sort of combined bang-bang and 'PI' nature makes an explanation of the results from only this second point of view even more doubtful. Clearly more detailed study on such an analogy should therefore be done, as it is currently, before its conclusions are used to assess the accuracy and stability.

One observation which can definitely be made is that this kind of fuzzy control is very well suited for an easy implementation of a time optimal control. The calculation of a switching line for the bang-bang control of a noisy time delay system is difficult and the simplicity of this fuzzy bang-bang control is therefore an important advantage.

3.6 Further remark

It is possible to speed up this fuzzy algorithm by using an alternative approach: decide at the beginning to which fuzzy temperature subset the temperature measurement belongs. This is interpreted as that fuzzy subset where the measured point has the highest membership grade. This decision gives thus the rule number (i_0) at which

$$\mu_{i_0}(t_0) = \max_i \mu_i(t_0)$$

Having determined this rule number, the appropriate calculations are carried out for this rule only. The action is then taken at that flow fl_0 at which

$$\lambda(fl_0) = \max_{fl} \min [\mu_{i_0}(t_0); \nu_{i_0}(fl)].$$

This method not only saves a considerable amount of computing time but also has a kind of intuitive appeal. Its mathematical equivalence to the previous method can also be shown [4].

TABLE 4. PERFORMANCE OF DIFFERENT CONTROLLERS ON A STEP RESPONSE OF 10°C

Controller	Rise Time (minute)	Overshoot (centigrade)	Temp. Variations (centigrade)
classical PI type	0.7 min	1.5°	0.4°
first fuzzy type	0.3 min	less than var.	1.5°
second fuzzy type	0.3 min	"	1.5°
third fuzzy type	0.3 min	"	0.5°

4. CONCLUSIONS

A comparison has been made between the response of the system for three different fuzzy controllers and for DDC controllers of a non fuzzy nature. The DDC controllers had a PI action; the setting of this action was optimised according to the ISE-, IAE- and ITAE-criteria on a linearised model.

All the fuzzy controllers showed a faster step response of the system than was possible with the DDC-controllers. However, it was more difficult to get accurate control of the temperature as indicated in Table 4. The simplest fuzzy controller, the third type, showed the best performance and combined a high speed response with the same accuracy as that of the optimal DDC-controller. The other two fuzzy controllers showed a tendency to oscillate around the steady state value.

It has been shown that the three different types of fuzzy controllers show some similarities with proportional and integral actions. Although the results of this preliminary research on fuzzy control are promising, the accuracy and stability problem needs to be investigated more deeply. This kind of fuzzy control is essentially non-linear. It is the way the particular control algorithm is derived which is the novelty and major contribution of this method based on fuzzy set theory. The easy way of implementing the experience of a human operator in the controller makes the application of fuzzy linguistic rules attractive for those processes that are already controlled by operators. This is particularly true in cases where automatic control following the usual methods requires time consuming and complex modelling and control methoes.

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APPENDIX: FUZZY SYSTEMS

A fuzzy subset A of a universe of discourse (support set) X is characterised by a membership function $\mu_A(x)$. This function assigns to each element $x \in X$ a number $\mu_A(x)$ in the closed interval $[0, 1]$, which represents the grade of membership of x in A [5]. Three basic operators used in fuzzy set theory are defined as follows

(a) The union of the fuzzy subsets A and B of the universe of discourse X is a fuzzy subset, denoted $A \cup B$, with a membership function defined by

$$\mu_{A \cup B}(x) = \max [\mu_A(x); \mu_B(x)] \quad x \in X$$

The union corresponds to the connective 'OR'.

(b) The intersection of the fuzzy subsets A and B is a fuzzy subset, denoted $A \cap B$, with a membership function defined by

$$\mu_{A \cap B}(x) = \min [\mu_A(x); \mu_B(x)] \quad x \in X$$

The intersection corresponds to the connective 'AND'.

(c) The complement of a fuzzy subset A is a fuzzy subset, denoted $\neg A$, with a membership function defined by

$$\mu_{\neg A}(x) = 1 - \mu_A(x) \quad x \in X$$

Complementation corresponds to negation 'NOT'.

The definition of a fuzzy set enables us to deal with the information contained in the experience of a human operator.

Linguistic expressions, such as the flow is 'big', 'medium', 'small', 'not big', etc. clearly are fuzzy subsets of the universe of discourse 'flow'.

Furthermore to represent the concept of a system mapping from an input to an output set in a fuzzy way, the concept of a fuzzy conditional statement (implication) is introduced. The system is described as a set of fuzzy conditional statements of the form

if 'input is big' then 'output is medium'

The membership function corresponding to a fuzzy conditional statement S: if A then B, given the fuzzy subset A of the universe of discourse X and the fuzzy subset B of Y, is defined by [5]

$$\mu_S(y, x) = \min [\mu_A(x); \mu_B(y)] \quad x \in X, y \in Y \quad (1)$$

The complete system is described by a set of such fuzzy implications* e.g.

if 'input is big' then 'output is medium'
or (else)
        if 'input is medium' then 'output is small'

Using the above mentioned definition of the 'or' connective the final fuzzy implication S composed of two implications: if A_1 then B_1 or (else) if A_2 then B_2, has the membership function

$$\mu_S(y, x) = \max [\min [\mu_{A_1}(x); \mu_{B_1}(y)]; \min [\mu_{A_2}(x); \mu_{B_2}(y)]]$$
$$(2)$$

This can be extended to the case of more than two fuzzy implications.

Having defined the relation between fuzzy subsets, the next step is to calculate the infered fuzzy subset, given a certain implicand fuzzy subset. Knowing the rule: if 'input is big' then 'output is medium' the question arises what will be the output when the 'input is very big'? Here the following compositional rule of inference is used: given a fuzzy implication S: if A then B, the fuzzy subset B', infered from a given fuzzy input set A' (A and A' fuzzy subsets of X, B and B' of Y), has a membership function defined by [5]

$$\mu_{B'}(y) = \max_x \min [\mu_{A'}(x); \mu_S(y, x)] \quad (3)$$

*The extension to the case of an implication of the form: if A then (if B then C), is straightforward: $\min [\mu_A(x);$ $\min [\mu_B(y); \mu_C(z)]] = \min [\mu_A(x); \mu_B(y); \mu_C(z)]$.

The input to the system in this control application was considered to be precise, not fuzzy. There is no fuzzy input, hence there is no need to apply the compositional rule of inference. Using the intuitive meaning of a fuzzy implication: if A then B, the implied output can never obtain a higher degree of truth than that of the implying input. That would be contrary to the nature of an implication. Hence one obtains the fuzzy output B up to the degree of membership of the measured value x_0 in the fuzzy input A. This gives the fuzzy output set

$$\mu_{B'}(y) = \min [\mu_A(x_0); \mu_B(y)] = \mu_S(y, x_0)$$

An alternative way to obtain the same result is to interpret this input x_0 as a 'fuzzy' input set A' with all membership values $\mu_{A'}(x)$ equal to zero, except the value at the measured point $\mu_{A'}(x_0)$ which is equal to one. Equation (3)—the compositional rule of inference—reduces then to

$$\mu_{B'}(y) = \mu_S(y, x_0)$$

The representation of a fuzzy system is used as an algorithm for a fuzzy controller: a decision has to be made as to which particular action should be taken and fed into the process. The decision procedure applied here is to take that value y_0 at which the final membership function is a maximum, that is y_0 at which

$$\mu_{B'}(y_0) = \max_y \mu_{B'}(y) = \max_y \max_x \min [\mu_{A'}(x); \mu_S(y, x)] \tag{4}$$