A Microeconometric Dynamic Structural Model
of Copper Mining Decisions

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Abstract

This paper proposes and estimates a dynamic structural model of the operation of copper mines using a unique dataset with rich information at the mine level from 330 mines that account for more than 85% of the world production during 1992-2010. Descriptive analysis of the data reveals several aspects of this industry that have been often neglected by previous econometric models using data at a more aggregate level. First, there is a substantial number of mines that adjust their production at the extensive margin, i.e., temporary mine closings and re-openings that may last several years. Second, there is very large heterogeneity across mines in their unit costs. This heterogeneity is mainly explained by differences across mines in ore grades (i.e., the degree of concentration of copper in the rock) though differences in capacity and input prices have also relevant contributions. Third, at the mine level, ore grade is not constant over time and it evolves endogenously: it declines with the depletion of the mine reserves, and it increases as a result of (lumpy) investment in exploration. Fourth, there is high concentration of market shares in very few mines, and evidence of market power and strategic behavior. We propose and estimate a dynamic structural model that incorporates these features of the industry. Our estimates show that the proposed extensions of the standard model contribute to explain the observed departures from Hotelling's rule. We use the estimated model to study the dynamic response of prices and quantities permanent and temporary changes in demand and costs.

Keywords: Copper mining; nonrenewable resources; dynamic structural model; industry dynamics; Euler equations.

JEL classifications: Q31, L72, L13, C57, C61.

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1 Introduction

Mineral natural resources, such as copper, play a fundamental role in our economies. They are key inputs in important industries like construction, electric materials, electronics, ship building, or automobiles, among many others. This importance has contributed to develop large industries for the extraction and processing of these minerals. In 2008, the world consumption of copper was approximately 15 million tonnes, grossing 105 billion dollars in sales, and employing more than 360.000 people (source: US Geological survey). The evolution and the volatility of the price of these commodities, the concern for the socially optimal exploitation of non-renewable resources, or the implications of cartels, are some important topics that have received substantial attention of researches in Natural Resource economics at least since the 70s. More recently, the environmental regulation of these industries and the increasing concern on the over-exploitation of natural resources have generated a revival of the interest in research in these industries.

Hotelling model (Hotelling, 1931) has been the standard framework to study topics related to the dynamics of extraction of natural resources. In that model, a firm should decide the optimal production or extraction path of the resource to maximize the expected and discounted flow of profits subject to a known and finite stock of reserves of the non-renewable resource. The Euler equation of this model establishes that, under the optimal extraction path, the price-cost margin of the natural resource should increase over time at a rate equal to the interest rate. This prediction, described in the literature as Hotelling's rule, is often rejected in empirical applications (Farrow, 1985, Young, 1992). Different extensions of the basic model have been proposed to explain this puzzle. Pindyck (1978) included exploration decisions: a firm should decide every period not only the optimal extraction rate but also investment in exploration. In contrast to Hotelling's rule, Pindyck's model can predict that prices follow a U-shaped path. Gilbert (1979) and Pindyck (1980) introduce uncertainty in reserves and demand. Slade and Thille (1997) propose and estimate a model that integrates financial and output information and finds a depletion effect that is consistent with Hotelling model. Krautkraemer (1998) presents a comprehensive review of the literature, theoretical and empirical, on extensions of the Hotelling model.

Hotelling model and the different extensions are models for the optimal behavior production and investment decisions of a mine. The predictions that these models provide should be tested at the mine level because they involve mine specific state variables. An important limitation in the literature comes from the data that has been used to estimate these models. The type of data most commonly used in applications consists of aggregate data on output and reserves at the country or

firm level with very limited information at the mine level. These applications assume that the 'in situ' depletion effects at the mine level can be aggregated to obtain similar depletion effects using aggregate industry data. However, in general, the necessary conditions for this "representative mine" model to work are very restrictive and they do not hold. This is particularly the case in an industry, such as copper mining, characterized by huge heterogeneity across mines in key state variables such as reserves, ore grade, and unit costs. Using aggregate level data to test Hotelling rule can be misleading. Perhaps most importantly, the estimation of aggregate industry models can generate important biases in our estimates of short-run and long-run responses to demand and supply shocks, and in the evaluation of the effects of public policies and investment projects.

In this paper, we propose and estimate a dynamic structural model of the operation of copper mines using a unique dataset with rich information at the mine level from 330 mines that account for more than 85% of the world production during 1992-2010. Our descriptive analysis of the data reveals several aspects of this industry that have been often neglected in previous econometric models using data at a more aggregate level. First, there is a substantial number of small and medium size mines that adjust their production at the extensive margin, i.e., they go from zero production to positive production or vice versa. In most of the cases, these decisions are not permanent mine closings or new mines but re-openings and temporary closings that may last several years. Second, there is very large heterogeneity across mines in their unit costs. This heterogeneity is mainly explained by substantial differences across mines in ore grades (i.e., the degree of concentration of copper in the rock) though differences in capacity and input prices have also relevant contributions. Third, at the mine level, ore grade is not constant over time and it evolves endogenously. Ore grade declines with the depletion of the mine reserves, and it may increase as a result of (lumpy) investment in exploration. Fourth, there is high concentration of market shares in very few mines, and evidence of market power and strategic behavior.[1]

We present a dynamic structural model that incorporates these features of the industry and the operation of a mine. In the model, every period (year) a mine manager makes four dynamic decisions: the decision of being active or not; if active, how much output to produce; investments in capacity (equipment); and investments in explorations within the mine. Related to these decisions, there are also four state variables at the mine level that evolve endogenously and can have important impacts on the mine costs. The amount of reserves of a mine, that determines the expected

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[1]Some of the these features have been acknowledged in the theoretical literature as important factors to take into account in the valuation of investment projects in natural resources (Brennan and Schwartz, 1985). Nevertheless, they have not been fully incorporated in empirical structural models.

remaining life time and may also affect operating costs. A second state variable is the indicator that the firm was active at previous period. This variable determines whether the firm has to pay a (re-) start-up cost to operate. The ore grade of a mine is an important state variable as well because it determines the amount of copper per volume of extracted ore. This is the most important determinant of a mine average cost because it can generate large differences in output for given amounts of (other) inputs. The cross-sectional distribution of ore grades across mines has a range that goes from 0.1% to more than 10%. There is also substantial variation over time in ore grade within a mine. This variation is partly exogenous due to heterogeneity in ore grades in different sections of the mine that are unpredictable to managers and engineers. However, part of the variation is endogenous and depends on the depletion/production rate of the mine. Sections of the mine with high expected ore grades tend to be depleted sooner than areas with lower grades. As a result, the (marginal) ore grade of a mine declines with accumulated output. Finally, the capacity or capital equipment of a mine is an important state variable. Capacity is measured in terms of the maximum amount of copper that a mine can produce in a certain period (year), and it is determined by the mine extracting and processing equipment, such as hydraulic shovels, transportation equipment, crushing machines, leaching plants, mills, smelting equipment, etc.[2] The model includes also multiple exogenous state variables such as input prices, productivity shocks, and demand shifters. As a result, the model accounts for multiple sources of uncertainty (e.g., not only uncertainty on output price but also on the price of important inputs, such as energy, and ore grade) and multiple investment decisions for mine manager.

The set of structural parameters or primitives of the model includes the production function, demand equation, the functions that represent start-up costs and (capacity) investment costs, the endogenous transition rule of ore grade, and the stochastic processes of the exogenous state variables. The production function includes as inputs labor, capital, energy, ore grade and reserves. Our dataset has several features that are particularly important in the estimation of the production function: data on the amounts of output and inputs are in physical units; we have data on input prices at the mine level; data on output distinguishes two stages, output at the extraction stage (i.e., amount of extracted ore), and output at the final stage (i.e., amount of pure copper produced). We present estimates of a production function using alternative methods including dynamic panel data methods (Arellano and Bond, 1991, and Blundell and Bond, 1999), and control function methods (Olley and Pakes, 1994, Levinshon and Petrin, 2003). For the estimation of the transition rule of ore grade, we also present estimates based on dynamic panel data and control function methods.

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[2] Capacity is equivalent to capital equipment but it is measured in units of potential output.

The estimation of the structural parameters in the functions for start-up costs, investment costs, and fixed costs, is based on the mine's dynamic decision model. The large dimension of the state space, with twelve continuous state variables, makes computationally very demanding the estimation of the model using full solution methods (Rust, 1987) or even two-step / sequential methods that involve the computation of present values (Hotz and Miller, 1993, Aguirregabiria and Mira, 2002). Instead, we estimate the dynamic model using moment conditions that come from Euler equations for each of the decision variables. For the discrete choice variables (i.e., entry/exit and investment/no investment decisions), we derive Euler equations using the approach in Aguirregabiria and Magesan (2013 and 2014). The Euler equation for the continuous choice of output is also no standard because there is a strictly positive probability of corner solutions (i.e., zero production) in the future. For the Euler equation of the output decision, we use results from Pakes (1994). Based on all these Euler equations, we construct moment conditions and a GMM estimator in the spirit of Hansen and Singleton (1982). Our Euler equations also provide a computationally simple approach to estimate the option value of the possibility of closing a mine, temporarily or permanently, and of the effect of uncertainty on investment decisions in a setting with a rich specification of the sources of uncertainty and of heterogeneity across mines.

The *GMM-Euler equation* approach for the estimation of dynamic discrete choice models has several important advantages. First, the estimator does not require the researcher to compute or approximate present values, and this results into substantial savings in computation time and, most importantly, in eliminating the bias induced by the substantial approximation error of value functions when the state space is large. Second, since Euler equations do not incorporate present values and include only optimality conditions and state variables at a small number of time periods, the method can easily accommodate aggregate shocks and non-stationarities without having to specify and estimate the stochastic process of these aggregate processes.

In this model, the derivation of Euler equations has an interest that goes beyond the estimation of the model. Hotelling rule is the Euler equation for output in a simple dynamic model for the optimal depletion of a non-renewable natural resource where the firm is a price taker, it is always active, ore grade is constant over time, reserves and ore grade do not affect costs, and there are no investments in capacity or/and explorations. Our Euler equations relax all these assumptions. The comparison of our Euler equations with Hotelling rule provides a relatively simple way to study and to measure how each of the extension of the basic model contribute to the predictions of the model.

Our [preliminary] estimates show that the proposed extensions of the standard model contribute to explain the observed departures from Hotelling rule. We also use the estimated model to study the short-run and long-run dynamics of prices and output under different types of shocks in demand and supply. Our model and method can be used to evaluate the effects on investment, employment, output, or prices of alternative taxation, subsidy, or environmental policies in the mining industry. It can be also a useful tool for industry analysts interested in the valuation of mining corporations and their investment projects.

The rest of this preliminary and incomplete version of the paper is organized as follows. Section 2 provides a description of copper mining industry (history, extraction of processing techniques, geographic location mines, market structure) and of the relevant literature in economics. We describe our dataset and present descriptive statistics in section 3. We focus on describing the stylized facts that motivate the different extensions in our model. Section 4 presents our model and derives Euler equations for the different decision variables, both continuous and discrete. Section 5 describes the structural estimation and presents our preliminary estimation results.

2   The copper mining industry

2.1   A brief history of the copper mining industry [3]

The earliest usage of copper dates from prehistoric times when copper in native form was collected and beaten into primitive tools by stone age people in Cyprus (where its name originates), Northern Iran, and the Lake region in Michigan (Mikesell, 2013). The use of copper increased greatly since the invention of smelting around the year 5000BP, where copper ore was transformed into metal, and the development of bronze, an alloy of copper with tin. Since then until the development of iron metallurgy around 3000BP, copper and bronze were widely used in the manufacture of weapons, tools, pipes and roofing. In the next millennium, iron dominated the metal consumption and copper was displaced to secondary positions. However, a huge expansion in copper production took place with the discovery of brass, an alloy of copper and zinc, in Roman times reaching a peak of 16 thousand tonnes per year in the 150-year period straddling the birth of Christ (Radetzki, 2009). Romans also improved greatly the extraction techniques of copper. For instance, they implement the pumping drainage and widened the resource base from oxide to sulfide ores by implementing basic leaching techniques for the sulfide ores. After the fall of the Roman Empire, copper and all metals consumption declined and production was sustained by the use of copper in the manufacture

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[3]This section is mainly based on material from Radetzki (2009) and Mikesell (2013).

of bronze cannons for both land and naval use, and as Christianity spreads for roofing and bells in churches (Radetzki, 2009).

The industrial revolution in the half eighteenth century marked a new era in mining and usage for all metals. However, copper did not emerge until 100 years later with the growth of electricity. The subsequent increased demand for energy and telecommunications led to an impressive growth in the demand for copper, e.g., in 1866 a telegraph cable made of copper was laid across the Atlantic to connect North America and Europe; ten years later the first message was transmitted through a copper telephone wire by Alexander Graham Bell; in 1878 Thomas Alva Edison produced an incandescent lamp powered through a copper wire (Radetzki, 2009). In 1913, the International Electrotechnical Commission (IEC) established copper as the standard reference for electrical conductivity. From that time until now, the use of copper has spread to many industrial and service sectors, but still half of the total consumption of copper is related to electricity. Copper wires have been used to conduct electricity and telecommunications across long distances as well as inside houses and buildings, cars, aircrafts and many electric devices. Copper's corrosion resistance, heat conductivity and malleability has made it an excellent material for plumbing and heating applications such as car radiators and air conditioners, among others (Radetzki, 2009).

Figure 1: World Copper Industry 1900 - 2010
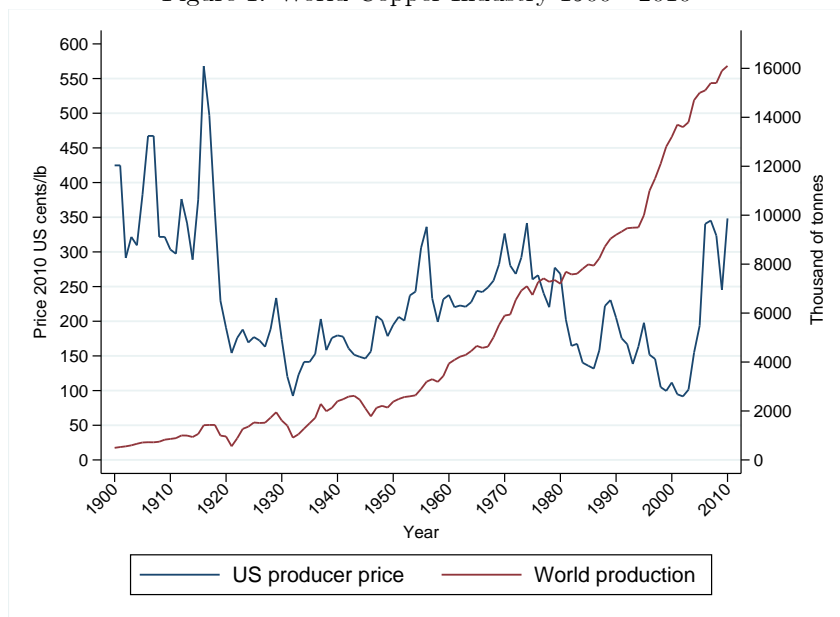
*Source: U.S. Geological Survey.

Deflator: U.S. Consumer Price Index (CPI). 2010 = 100

The evolution of the copper industry has also historically been closely related, from a macroeconomic point of view, to the economic activity in developed countries and the international political

scene. Figure 1 shows how the evolution of price and production has been affected by factors such as: world wars, political reasons (mainly in South America and Africa, which resulted in the nationalization of several U.S. copper operators in the 1960s and 1970s), the great depression, the Asian crisis and recently the subprime crisis.

Until the late 1970s, the United States dominated the global copper industry. In 1947 it accounted for 49% of the world copper consumption and 37% of the world copper mine production, whereas in 1970 it consumed 26% of world copper and produced 27%. Copper production controlled by American multinational companies outside the US declined because of successive strikes, the 1973 oil crisis, and the nationalization processes in Zambia, Zaire, Peru and Chile. Since 1978 the copper industry has been characterized by several changes in ownership and geographical location.[4] The London Metal Exchange (LME) price has been adopted as the international price reference by producers and the market structure has experienced a consolidation era, where a few large companies dominate this market.

2.2 Copper production technology

A copper mine is a production unit that vertically integrates the extraction and the processing (purification) of the mineral.[5] At the extraction stage, a copper mine is an excavation in earth for the extraction of copper ores, i.e., rocks that contain copper-bearing minerals. Copper mines can be underground or open-pit (at surface level), and this characteristic is pretty much invariant over time.[6] Most of the rock extracted from a copper mine is waste material. The ore grade of a mine is roughly the ratio between the pure copper produced and the amount of ores extracted. In our dataset, the average ore grade is 1.2% but, as we illustrate in section 3, there is large heterogeneity across mines, going from 0.1% to 11% ore grades.[7] Other important physical characteristic of a mine is the type of ore or minerals that copper is linked to: sulfide ores if copper is linked with sulfur, and oxide ores when copper is linked with either carbon or silicon, and oxygen. Although a mine may contain both types, the technological process typically depends on the main type of the ore. The type of ore is relevant because they have substantial differences in ore grades and volume

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[4]As deposits are depleted, mining shifts to countries with the next best deposits. In the absence of new discoveries and technological change, this tendency to exploit poorer quality ores tends to push productivity down and the prices of mineral commodities up over time.

[5]As explained below, mines differ on the level of vertical integration.

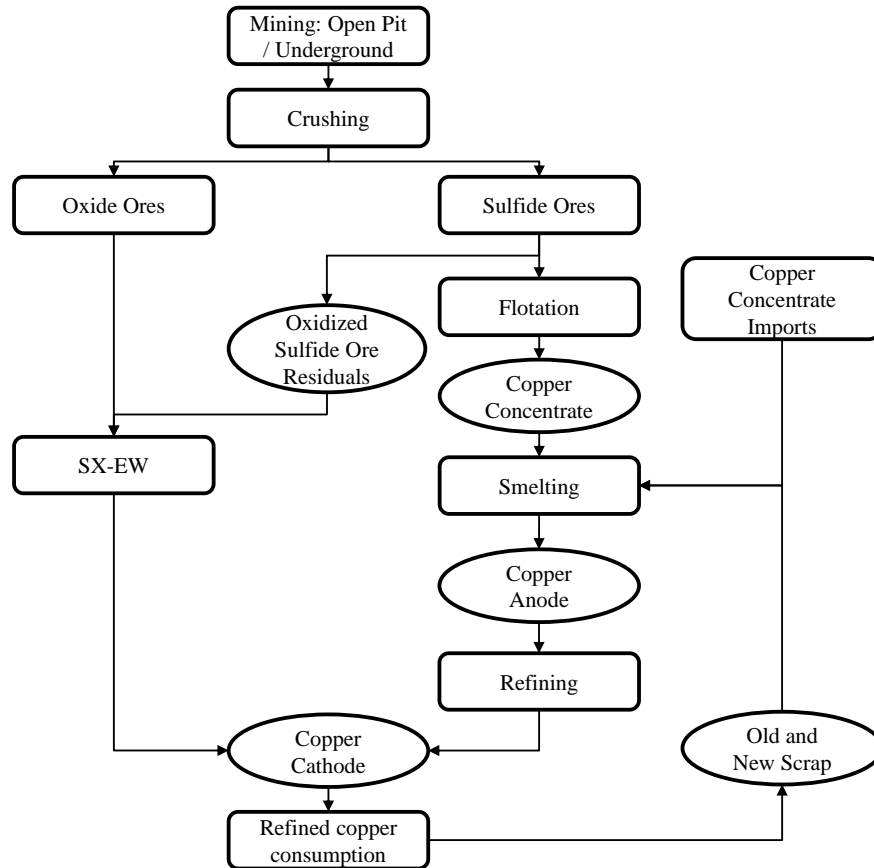[6]Some open-pit mines may eventually become underground, but this possible event occurs only once in the long lifetime of a copper mine.

[7]This final ore grade includes the recuperation rate that is the ratio between the ore grade at the end of production process and the ore grade after extraction and before the purification process. In our dataset the mean recuperation rate is 73.3%.

of the reserves and because the processing technology is very different. Oxide copper deposits have higher ore grade and their processing and purification implies a much lower cost than sulfide ores. Sulfide copper deposits, despite their lowest grade are also attractive for mining companies because their large volume, that allows exploiting economies of scale. Sulfide ores represent most of the world's copper production (80%).

Figure 2: Copper Production Technology

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          ┌─────────────────────┐
          │  Mining: Open Pit   │
          │    / Underground    │
          └─────────────────────┘
                    │
          ┌─────────────────────┐
          │      Crushing       │
          └─────────────────────┘
           │                  │
   ┌──────────────┐    ┌──────────────┐        ┌──────────────┐
   │  Oxide Ores  │    │ Sulfide Ores │        │    Copper    │
   └──────────────┘    └──────────────┘        │ Concentrate  │
                        │          │            │   Imports    │
                  ┌─────────┐  ┌──────────┐     └──────────────┘
                  │ Oxidized│  │ Flotation│
                  │ Sulfide │  └──────────┘
                  │   Ore   │        │
                  │Residuals│   ┌─────────┐
                  └─────────┘   │  Copper │
   ┌──────────────┐             │Concentrate│
   │    SX-EW     │             └─────────┘
   └──────────────┘        ┌──────────────┐
                           │   Smelting   │
                           └──────────────┘
                           ┌─────────┐
                           │  Copper │
                           │  Anode  │
                           └─────────┘
                           ┌──────────────┐
                           │   Refining   │
                           └──────────────┘
       ┌─────────┐                        ┌─────────┐
       │  Copper │                        │ Old and │
       │ Cathode │                        │New Scrap│
       └─────────┘                        └─────────┘
       ┌──────────────┐
       │Refined copper│
       │ consumption  │
       └──────────────┘
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The production process of copper can be described mainly in three stages: extraction, concentration, and a purification process. In the extraction process copper ore can be mined by either open pit or underground methods. Independently of the extraction method, copper ores and other elements are extracted from the mine through digging and blasting, then they are transported out of the mine and finally crushed and milled. The concentration and refining processes depend on whether the ore is sulfide or oxide. In the first case, sulfide ores are converted into copper concentrates with a purity varying from 20% to 50% by a froth flotation process. In the purification

stage, copper concentrates are melted removing unwanted elements such as iron and sulfur and obtaining a blister copper with a purity of 99.5%. Next, these blister copper are refined by electricity or fire eliminating impurities and obtaining a high-grade copper cathode with a purity of 99.9%. Typically, smelting and refining (or only refining) are carried out at smelter and refinery plants, different from the mine, either at the same country of the mine or in the final destination of the copper. High-grade copper is more easily extracted from oxide ores. In this case, refined copper is extracted in a two-stage hydrometallurgical process, so-called solvent extraction-electrowinning (SX-EW), where copper ores are first stacked and irrigated with acid solutions and subsequently cleaned by a solvent extraction process obtaining an organic solution. Next, in the refining process, copper with a grade of 99.9% is recovered from the organic solution by the application of electricity in a process called electrowinning. The final product for industrial consumption and sold in local or international markets is a copper cathode with a purity of 99.9%. As we describe in section 2.3, the SX-EW technology also allows to process residual ores (low ore grade) or waste dumps in mines from sulfide ores which have been oxidized by exposure to the air or bacterial leaching. Figure 2 describes the technological process of the copper production.

2.3   Technological change

As noted in section 2.1, the industrial revolution also had an impact on the technology of mining. There have been important breakthroughs in mining techniques that have allowed not only to reduce production costs but to increase the resource reserves, reducing the fear of exhaustion. Probably, the two most important breakthroughs took place in a very short time. First, by 1905 the mining engineer Daniel C. Jackling, first introduced the mass mining at the Bingham Canyon open-pit mine in Utah (Mikesell, 2013). Mass mining applied large scale machinery in the production process, e.g., the use of steam shovels, heavy blasting, ore crushers, trucks and rail made profitable the exploitation of low-grade sulfide ores through economies of scale. The second most important development was the flotation process, created in Britain and first introduced in copper in Butte, Montana in 1911 (Slade, 2013). This process, which is used to concentrate sulfide ores, improved significantly the recovery rates of metal and in turn lowered the processing costs. By 1935, recovery rates increased to more than 90% from the 75% average recovery rate observed in 1914 (Mcmahon, 1965).

Once open-pit mining, heavy blasting and flotation techniques were more practicable, the exploitation of low-grade sulfide deposits became economically profitable. By the beginning of the twentieth century most of the copper exploited came from selective mining where high grade veins

were extracted and mass mining was not possible because of high loss of metal. The average grade of copper ore decreased greatly as large scale mining was introduced, while at the beginning of the twentieth century the average grades were close to 4%, by 1920s they had fallen to less than 2%. Despite this decrease in ore grades, production costs also declined in this period. The costs in 1923 decline at least 20% compared with those in 1918. Moreover, between 1900 and 1950 world copper output was quintupled, raising from 490 Kt. in 1900 to 2490 Kt. in 1950, in response to the explosive demand and the new mining techniques that increased mining production (Radetzki, 2009).

A third important breakthrough was the improvement in leaching techniques for oxide ores by the introduction in 1968 of the SX-EW process for copper at the Bluebird mine in Arizona. This process, as described above, allows to extract high-grade copper by applying acid solutions to oxide ores. Before the SX-EW process were introduced oxide ores were treated by a combination of leaching and smelting processes. The SX-EW process presents a number of advantages compared with the more traditional pyrometallurgical process, e.g., it requires a lower capital investment and faster start-up times, allow to process lower grade ores and mining waste dumps (Radetzki, 2009). The application of this process has spread greatly in recent decades. Between 1980 and 1995, the U.S. production by this method increased from 6% to 27% (Tilton, 1999). The SX-EW has also spread at international level. In 1992, this process accounted for the 8% of the world production and by 2010 its participation increased to 20% (Cochilco, 2001 and 2013).

2.4   Geographical distribution of world production

As noted above, since the industrialization of mining until the late 1970s, the United States dominated the world industry. In the decade of 1920s, the U.S. copper industry reached its peak. By 1925, the United States produced 52% of the world's copper, while developing countries in Latin America, Africa and eastern Europe, produced 31%. This proportion was gradually reversed over time and by 1960 the U.S. world production rate had declined to 24% while that developing countries produced 40%. Africa accounted only about 7% by 1925, but by 1960 Africa, mainly by Zambia (14%), produced 56% (Mikesell, 2013). In 1982, the United States produced 16.23% while Chile, that between 1925 and early 1970s had accounted for 15% of the world production, produced 16.39% becoming the new world leader in the industry until today. The relative importance of the main producer countries for the period between 1985 and 2010 can be seen in table 1.

Copper deposits are distributed throughout the world in a series of extensive and narrow metallurgical regions. Most of copper deposits are concentrated in the so-called "Ring of Fire" around

the western coast of the Pacific Ocean in South and North America and in some copper belts located in eastern Europe and southern Asia. The geographical distribution of large and medium size copper deposits is shown in figure 3. As noted above, Chile is the major producer of copper and it accounts for 10 of the biggest 20 world copper mines, followed far behind by China, Peru, United States, and Indonesia with 2 world class mines each. The biggest 10 mines in the world for the period between 1992 and 2010 are shown in table 2.

Table 1: Producer Countries Market Shares (%) 1985 - 2010

Country[1]		1985	1990	1995	2000	2005	2010
1.	Chile	16	18	25	35	36	34
2.	China	3	3	4	4	5	8
3.	Peru	5	3	4	4	7	7
4.	USA	13	18	19	11	8	7
5.	Indonesia	1	2	5	8	7	5
6.	Australia	3	3	4	6	6	5
7.	Zambia	6	5	3	2	3	4
8.	Russia	0	0	5	4	4	4
9.	Canada	9	8	7	5	4	3
10.	Congo DR	6	4	0	0	0	3

Source: Codelco

Note (1): Ranking is based on output in 2010.
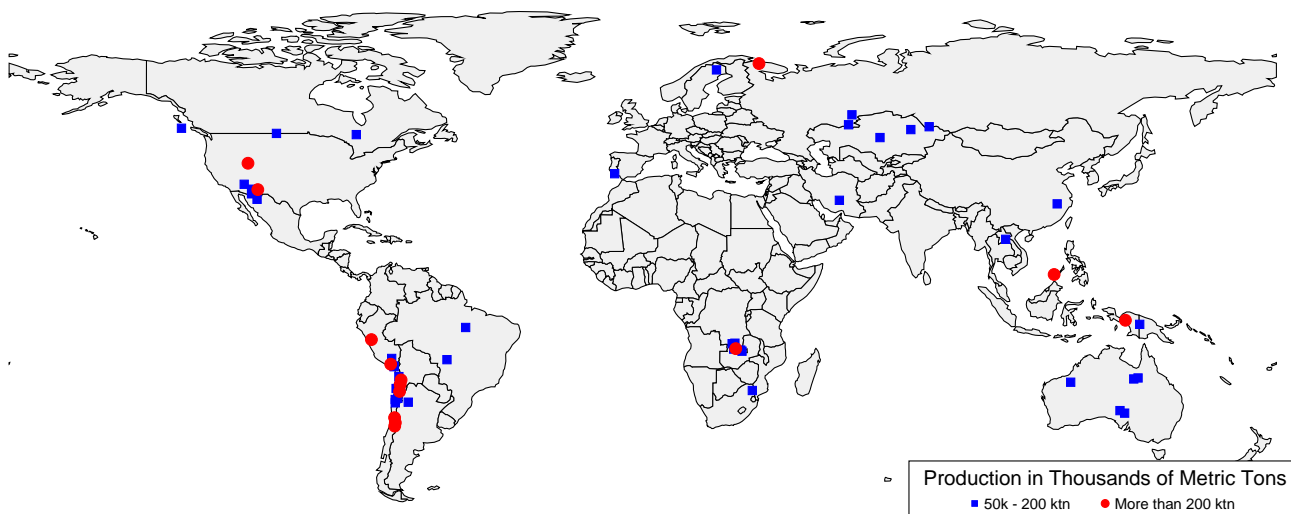
Figure 3: World Copper Mines 1992 - 2010

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Table 2: The Biggest 10 Mines in the World 1992 - 2010

	Mine name[1]	Country	Operator	Annual production (thousand Mt)
1.	**Escondida**	Chile	BHP Billiton	1443.5
2.	**Grasberg**	Indonesia	Freeport McMoran	834.1
3.	**Chuquicamata**	Chile	Codelco	674.1
4.	**Collahuasi**	Chile	Xstrata Plc	517.4
5.	**Morenci**	USA	Freeport McMoran	500.9
6.	**El Teniente**	Chile	Codelco	433.7
7.	**Norilsk**	Russia	Norilsk Group	392.7
8.	**Los Pelambres**	Chile	Antofagasta Plc	379.0
9.	**Antamina**	Peru	BHP Billiton	370.2
10.	**Batu Hijau**	Indonesia	Newmont Mining	313.8

Source: Codelco.

Note (1) Ranking is based on maximum annual production during 1992-2010.

2.5 The industry today

Prices. Copper is a commodity traded at spot prices which are determined in international auction markets such as the London Metal Exchange (LME) and the New York Commodity Exchange (Comex).[8] However, from the end of the Second World War until the late 1970s, the international copper market was spatially segregated in two main markets: The U.S. local market and a market for the rest of the world. In the US market the price was set by the largest domestic producers. In contrast, in the rest of the world, copper was sold at LME spot prices. This period, known as the "two-price system", officially ended in 1978, when the largest US producers announced that they would use the Exchange prices as reference to set their contracts.

Figure 4 depicts both LME and US producer copper prices (in constant 2010 US dollars) from 1950 to 2010. A glance at this figure shows that prices present a slightly declining trend. However, it is possible to identify at least three major booms in this period. Radetzki (2006) states that the post war booms of the early 1950s, early 1970s and 2004 onwards can be explained by demand shocks. Furthermore, he explains that the first boom was caused by inventory build up in response to the Korean War, the second boom in turn was triggered by the price increases instituted by the oil cartel, while the third boom has been a consequence of the explosive growth of China's and India's row materials demand. In an attempt to give a deeper understanding of the current boom, Radetzki (2008) state that increasing demand is not a full explanation for the high prices observed

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[8] A typical contract between producers and consumers specifies the frequency and point of deliveries. However, price is not specified in contracts, but is determined as the spot price in either COMEX or LME at the time of delivery.

in the last period. Hence, they postulate three possible explanations for the 2004 onwards boom: firstly, it now takes much longer time to build new capacity than in previous booms. Secondly, investors could have failed to predict the increasing demand, underestimating needed capacity. Finally, exploring costs may have increased, pushing up in turn prices to justify investment in new capacity. However, there is very little econometric evidence that measures the contribution of each of these factors.

Figure 4: Copper Price 1950 - 2009
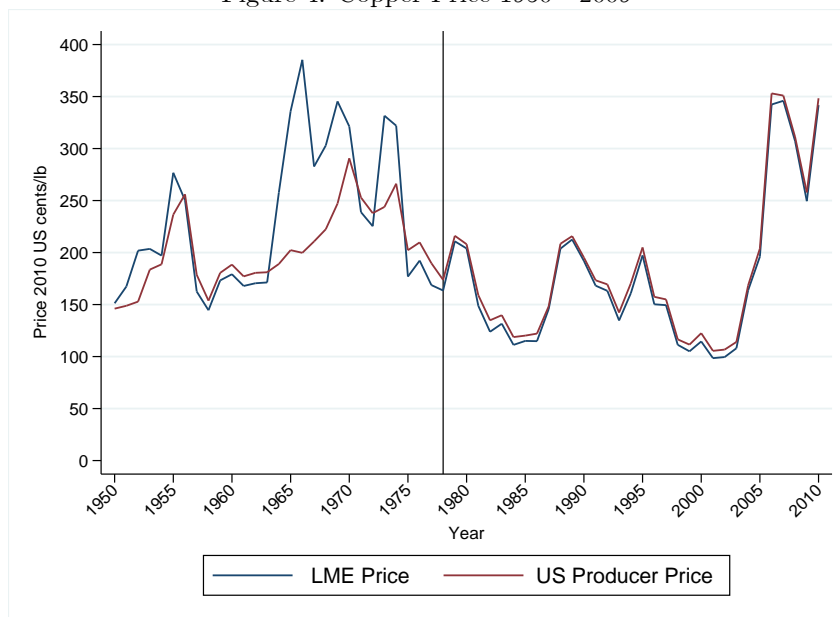
*Source: U.S. Geological Survey.

Deflator: U.S. Consumer Price Index (CPI). 2010 = 100

Consumption. Copper is the world's third most widely used metal, after iron and aluminum. Its unique chemical and physical properties (e.g., excellent heat and electricity conductivity, corrosion resistance, non-magnetic and antibacterial) make it a very valuable production input in industries such as electrical and telecommunications, transportation, industrial machinery and construction, among others. Fueled by the strong economic development in East Asia, and specially in China, the consumption of copper has grown rapidly. In 2008, world copper consumption was approximately 15 million tonnes, grossing roughly $105 billion in sales. Table 5 shows the consumption shares of the top ten consumer countries starting in 1980[9]. In this period China began an economic reform process, where the market rather the state has driven the Chinese economy, which has been very successful and it has led China to an important period of economic growth and industrial

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[9]Ranking list is elaborated in base of the top ten consumer countries in 2009.

development. This China's economic success has permitted it to overcome the United States' consumption since 2002. Moreover, in the period of 2005 to 2009 China has almost tripled the U.S. consumption, accounting roughly for 28% of world copper consumption.

Table 3: World Consumption Shares (%) of Refined Copper 1980 - 2009

Country[1]		1980-84	1985-89	1990-94	1995-99	2000-04	2005-09
1.	China	5.92	6.09	7.19	10.07	17.32	28.00
2.	USA	20.63	20.43	20.9	21.07	16.34	11.59
3.	Germany	-	3.85	9.11	8.16	7.20	7.40
4.	Japan	13.59	12.49	13.48	10.49	7.89	6.62
5.	South Korea	1.51	2.41	3.54	4.73	5.71	4.59
6.	Italy	3.85	3.99	4.41	4.24	4.31	3.88
7.	Russia	-	2.86	3.69	1.24	2.33	3.39
8.	Taiwan	1.07	1.99	3.94	4.49	4.02	3.36
9.	India	0.90	1.14	1.06	1.62	1.93	2.73
10.	France	4.51	3.98	4.33	4.16	3.53	2.42

Source: Codelco

Note (1): Ranking is based of consumption in 2009.

Supply. The supply of refined copper originates from two sources, primary production (mine production) and secondary production (copper produced from recycling old scrap). As figure 6 shows primary production has almost tripled whereas secondary production has increased much more modestly. Some tentative explanations for this fact can be found in the existing literature of mineral economics. An important factor to explain this poor growth of the secondary production is that the cost of recycling copper scrap has remained high, especially when copper scrap is old (Gamez, 2007). Other important factor is the effort of primary copper producers to reduce their production costs over this period that has contributed to a decline in the real price of copper since the early 1970s.

Figure 5: World Primary and Secondary Copper Production 1966 - 2009
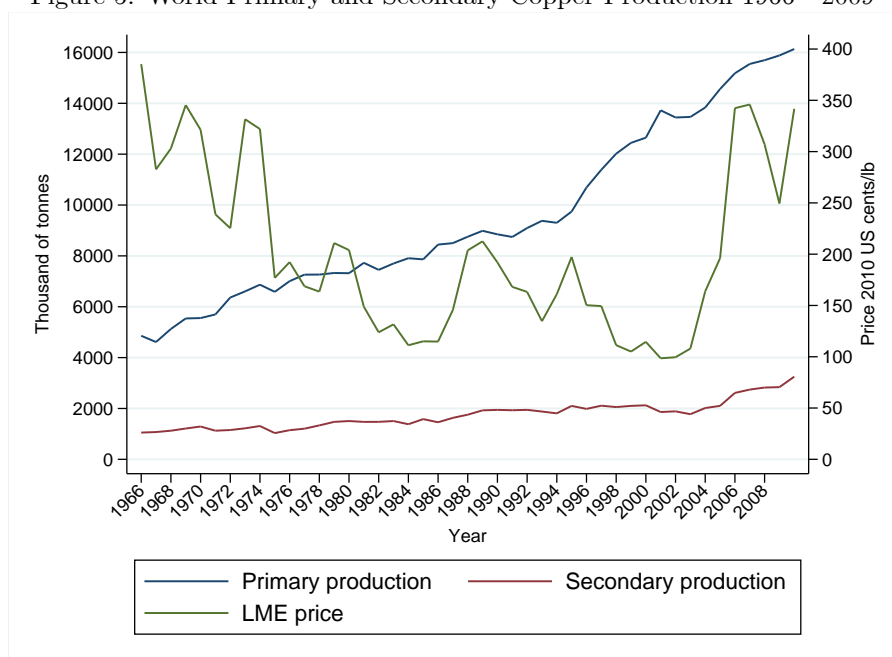
Copper costs have been extensively studied in the literature, e.g., Foley (1982), Davenport (2002), Crowson (2003, 2007), and Agostini (2006), as well as reports from companies and agencies. In mineral economics, costs are mainly classified in cash costs, operating costs and total costs. Cash costs (C1) represent all costs incurred at mine level, from mining through to recoverable copper delivered to market, less net by-product credits. Operating costs (C2) are the sum of cash costs (C1) and depreciation and amortization. Finally, total costs (C3) are operating costs (C2) plus corporate overheads, royalties, other indirect expenses and financial interest. Figure 6 shows world average copper costs and copper price in 2010 real terms from 1980 onwards. Both price and costs moved cyclically around a declining trend. However, since 2003 price has increased steadily while costs, with a certain lag, have increased since 2005. Part of the decrease in costs can be explained by management improvement (Perez, 2010), the introduction of SX-EW technology and geographical change in the production, from high-cost regions to low-cost regions (Crowson, 2003). The increase in costs in the last period can be explained by an increase in input prices and a decline in ore grades (Perez, 2010).

Figure 6: World Average Copper Costs 1980 - 2010
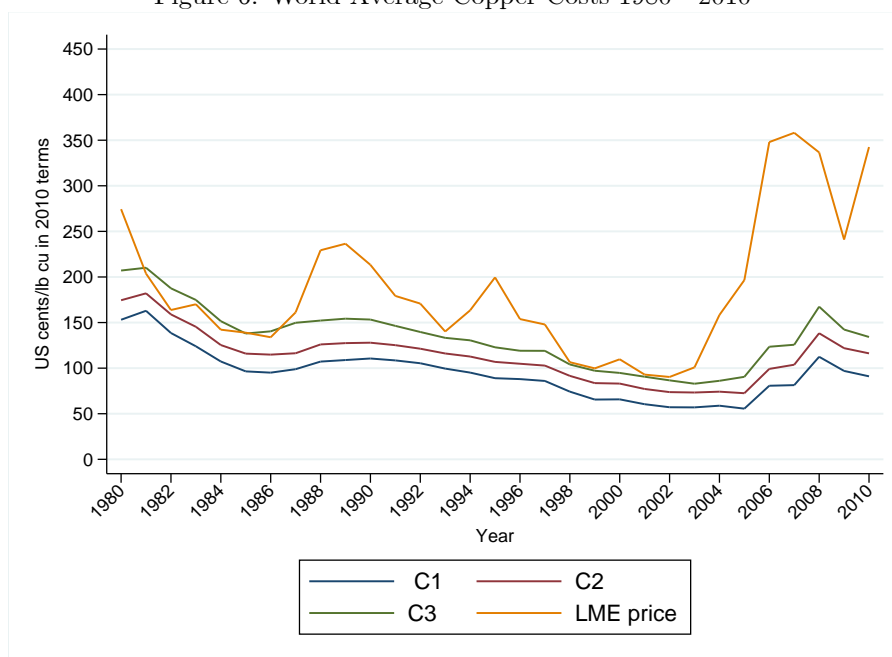
*Source: Brook Hunt.

Table 4 compares weighted average costs between top ten producer countries in the period from 1980 to 2010. Chile, Indonesia and Peru present the lowest costs for most of the period. Interestingly, USA has experienced the most dramatic decline in average costs. These three countries have become the most cost efficient places to produce copper.

Table 4: Weighted Average Cost (C1) by Country 1980-2010. In US dollars per pound (Deflated 2010)

Country[1]		1980-84	1985-89	1990-94	1995-99	2000-04	2005-10
1.	Indonesia	1.05	0.78	0.67	0.26	0.22	0.22
2.	Peru	1.21	1.16	1.05	0.74	0.54	0.40
3.	USA	1.72	1.15	1.03	0.86	0.74	0.71
4.	Chile	1.03	0.76	0.91	0.71	0.54	0.82
5.	China	-	-	-	0.72	0.79	1.02
6.	Russia	-	-	-	0.89	0.61	1.09
7.	Australia	1.59	1.04	1.00	0.89	0.64	1.26
8.	Poland	-	-	0.62	1.06	0.84	1.27
9.	Canada	1.35	1.10	1.23	0.96	0.77	1.34
10.	Zambia	1.52	0.82	0.89	1.16	1.02	1.54
	World Average	1.37	1.01	1.03	0.80	0.59	0.86

Source: Brook Hunt.

Note (1): Ranking is based on average costs in 2010.

Figure 7: Average Cost by component 1987 - 2010
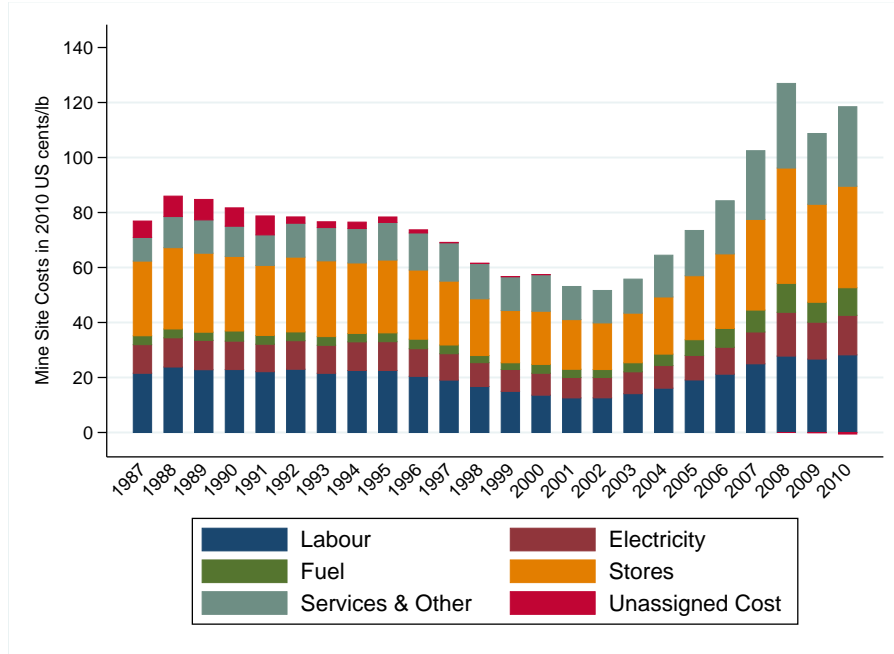
*Source: Brook Hunt.

Figure 7 presents the main cost components of copper production, in constant (2010) US dollars. The biggest contributors to production costs are the storage costs, which accounted for roughly 33%, on average, during this period. Labor costs are the second most important component in production costs. Labor costs increased, in real terms, from 0.21 $/lb to 0.28 $/lb between 1987 and 2010, but this represented a reduction from 28% to 24% of total production costs, as other costs, such as fuel and services, experienced larger increases. Electricity, that is intensively used at the SX-EW and refining stages, represents on average roughly 13% of the production costs of a pound of copper.

2.6   Related literature

This paper builds upon the natural resources literature. Natural resource industries have been little explored in the modern Industrial Organization literature. Most of the research had focused in commodity price fluctuations and the Hotelling's rule, and cartel behavior. New tools in empirical industrial organization and new and better data sets developed in recent years are leading to a revival of the interest on these old and somewhat forgotten models of natural resources. For instance, Slade (2013a) explores investment decisions under uncertainty in the U.S. copper industry. Lin (2013) estimates a dynamic game of investment in the offshore petroleum industry. Huang and Smith (2014) estimate a dynamic entry game of a fishery resource.

The dynamics of the extraction of natural resources has been analyzed by economists since Hotelling's seminal paper. The basic and well known Hotelling model considers the extraction path that maximizes the expected and discounted flow of profits of a firm given a known and finite stock of reserves of a nonrenewable resource. An important prediction of the model is that, under the optimal depletion of the natural resource, the price-cost margin should increase at a rate equal to the interest rate, i.e., Hotelling rule. Hotelling's paper also first introduced the concept of depletion effect which reflects the increasing cost associated with the scarcity of the resource.

Subsequent literature on natural resources has extended Hotelling model in different directions. Pindyck (1978) includes exploration in Hotelling model, and uses this model to derive the optimal production and exploration paths in the competitive and monopoly cases. He finds that the optimal path for price is U-shaped. Gilbert (1979) and Pindyck (1980) introduce uncertainty in reserves and demand. There has been also substantial amount of empirical work testing Hotelling rule. Most empirical studies have found evidence that contradicts Hotelling's rule. Farrow (1985) and Young (1992), using a sample of copper mines, reject the Hotelling rule. In contrast, Slade and Thille (1997) finds a negative and significant depletion effect and results more consistent with theory using a model of pricing of a natural-resource that integrates financial and output information. Krautkraemer (1998) and Slade and Thille (2009) provide comprehensive reviews of the theoretical and empirical literature, respectively, extending Hotelling model.

The copper industry has been largely examined by empirical researches since the well known study of competition by Herfindahl in 1956. In general, the literature on the copper industry can be divided into four groups according to their main interest. Most of these studies have used this industry to test more general theories on prices, uncertainty, tax effects and efficiency.

A first group include those studies in which the main purpose is to examine the behavior of prices and investment under uncertainty in the industry. A seminal paper in this branch is the work by Fisher et al (1972) who uses aggregate yearly data on prices, output and market characteristics for the period 1948-1956 and several countries to estimate the effect on the LME copper price of an exogenous increase in supply either from new local policies or new discovery. They found that these increases in supply will be mainly absorbed by offsetting reductions in the supply from other countries. Harchaoui and Lasserre (2001) study capacity decisions of Canadian copper mines during 1954-1980 using a dynamic investment model under uncertainty. They found that the model explains satisfactorily the investment behavior of mines. Slade (2001) estimates a real-option model to evaluate the managerial decision of whether to operate a mine or not also

using a sample of Canadian copper mines. More recently, Slade (2013a) investigates the relationship between uncertainty and investment using a extensive data series of investment decisions of U.S. copper mines. She uses a reduced form analysis to estimate the investment timing to go forward and the price thresholds that trigger this decision. Interestingly, she finds that with time-to-build, the effect of uncertainty on investment is positive. In a companion paper, Slade (2013b) studies the main determinants of entry decisions using a reduced form analysis. She extends the previous analysis adding concentration of the industry and resource depletion. Here, copper is considered as a common pool resource and depletion is measured as cumulative discoveries in all the industry rather than depletion at single mine level. Slade finds that technological change and concentration of the industry has a positive effect on entry decisions whereas resource depletion affect negatively the new entry of mines. Interestingly, in contrast to the companion paper, Slade finds a negative effect of uncertainty on entry decisions. The provided explanation is that an increase in uncertainty (with time-to-build) may encourage the implementation of investment projects that are at the planning stage, but it has also a negative effect in the long-run by moving resources towards industries with lower levels of uncertainty.

A second group of papers have studied the conditions for dynamic efficiency in mines output decisions in the spirit of the aforementioned Hotelling's model. Most of these studies use a structural model where the decision variable is the amount of output. Young (1992) examines Hotelling's model using a panel of small Canadian copper mines for the period 1954-1986. She estimated the optimal output path in a two stage procedure. In a first step, she estimates a translog cost function, and in a second step the estimated marginal cost is plugged into the Euler equation of the firm's intertemporal decision problem for output, and the moment conditions are tested in the spirit of the GMM approach in Hansen and Singleton (1982). The results showed that her data is no consistent with Hotelling model. Slade and Thille (1997) uses the same data as in Young (1992) to analyze the expected rate of return of a mine investment by combining Hotelling model with a CAPM portfolio choice model. Haudet (2007) explores copper price behavior and survey the factors that characterize the rate of return on holding an exhaustible natural resource stock and determine their implications in the context of the Hotelling's model.

A third group of papers study the effects of taxes and / or environmental policies (certifications) on the decisions of copper mines. Slade (1984) studies the effect of taxes on the decision of ore extraction and metal output. Foley (1982) evaluates the effects of potential state taxes on price and production in 47 U.S. copper mines using proprietary cost data for the period 1970-1978. Tole

and Koop (2013) studies the implications on costs and operation output decisions of the adoption of environmental ISO using a panel of 99 copper mines from different producing countries for the period 1992-2007. They find evidence that ISO adoption increases costs.

Our paper and empirical results emphasize the importance of the extensive margin, and the hetergogeneity and endogeneity of ore grade to explain the joint dynamics of supply and prices in the copper industry. Krautkraemer (1988, 1989) and Farrow and Krautkraemer (1989) present seminal theoretical models on these topics.

Finally, a reduced group of papers has studied the competition and strategic interactions in the copper industry. Agostini (2006) estimates a static demand and supply and a conjectural variation approach *a la* Porter (1983) to measure the nature or degree of competition in the U.S. copper industry before 1978. He finds evidence consistent with competitive behavior.

3 Data and descriptive evidence

3.1 Data

We have built a unique dataset of almost two decades for this industry. We have collected yearly data for 330 copper mines from 1992 to 2010 using different sources. The dataset contains detailed information at the mine-year level on extraction of ore and final production of copper and by-products (all in physical units),[10] reserves, ore and mill grades, recuperation rate, capacity, labor, energy and fuel consumption (in physical units), input prices, total production costs, indicators for whether the mine is temporarily or permanently inactive, and mine ownership.[11] Mine level data is compiled for active mines by Codelco. This data set represents, approximately, 85% of the industry output. Price at the LME is collected by USGS. Capacity and consumption data are from ICSG.

Table 5 presents the summary statistics for variables both at the mine level and market level. On average, there are 172 active mines per year, with a minimum of 144 in year 1993 and a maximum of 226 in 2010. We describe the evolution of the number of mines, entry, and exit in section 3.2 below. On average, an active mine produces 64 thousand tonnes of copper per year. The average copper concentration or grade is 1.21%. There is large heterogeneity across mines in production, capacity, reserves, ore grade, and costs. We describe this heterogeneity in more detail in section 3.2.

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[10]By-products include cobalt, gold, lead, molybdenum, nickel, silver, and zinc.

[11]We are especially grateful to Juan Cristobal Ciudad and Claudio Valencia of Codelco, Daniel Elstein of USGS, Carlos Risopatron and Joe Pickard of ICGS, and Victor Garay of Cochilco for providing the data for this analysis.

Table 5: Copper Mines Panel Data 1992-2010. Summary Statistics

Variable (measurement units)	Units[1]	Obs.	Mean	Std. Dev.	Min	Max
Mine-Year level data						
Number active mines	mines	19	172.8	26.4	144.0	226.0
Capacity	kt of cu	2672	86.36	146.89	0.25	1500.00
Copper Production	kt of cu	3284	64.51	127.53	0.00	1443.54
By-products production[2]	kt of equivalent cu	3284	23.75	61.42	0.00	809.10
Ore mined	mt of ore	3261	11.48	25.68	0.006	314.21
Reserves	mt of ore	2687	253.16	533.14	0.02	5730.15
Ore grade	%	2630	1.21	1.22	0.02	11.42
Mill grade	%	3279	1.23	1.30	0.00	12.45
Realized grade	%	3279	2.25	2.03	0.01	15.17
Number of workers	workers	3270	1584.49	3499.01	18.00	48750.00
Labor cost	US $ / t cu equiv.	3270	445.68	559.55	38.90	21277
Electricity consumption	Kwh/t treated ore	3011	78.70	299.26	1.13	7530.96
Electricity unit cost	US Cents/Kwh	3276	5.27	2.88	0.26	35.00
Fuel consumption	litres/t treated ore	3253	1.60	1.36	0.00	21.52
Fuel unit cost	US Cents/Litre	3260	44.48	26.16	0.70	156.00
Market-Year level data						
LME Price	US$/t	19	3375	2097	1559	7550
World consumption	mt	19	13.04	2.17	9.46	16.33
World production	mt	19	13.10	2.24	9.50	16.17
World capacity	mt	19	14.78	2.87	10.82	19.81
Total production in our sample	mt	19	11.14	2.34	7.28	13.95
Total capacity in our sample	mt	19	12.84	2.49	9.11	16.92

Source: Codelco

Note (1):t represents metric tonnes (1,000 Kg), kt thousand of metric tonnes, and mt million of metric tonnes.

Note (2): By-product production is transformed to copper equivalent production using 1992 copper price.

3.2 Descriptive Evidence

In this section, we use our dataset to present descriptive evidence on four features in the operation of copper mines that have been often neglected in previous econometric models: (a) the importance of production decisions at the extensive, i.e., active / inactive decision; (b) the very large heterogeneity across mines in unit costs, geological characteristics and the important role of ore grade in explaining this heterogeneity; (c) ore grade is not constant over time and it evolves endogenously; and (d) the high concentration of market shares in very few mines, and indirect evidence of market power and strategic behavior. The appendix contains a description of the variables in the dataset.

3.2.1 Active / inactive decision

Figure 8 presents the evolution of the number of active mines and the LME copper price during the period 1992-2010. The evolution of the number of active mines follows closely the evolution of copper price in the international market, though the series of price shows more volatility. The correlation between the two series is 0.89. However, market price and aggregate market conditions are not the only important factors affecting the evolution of the number of active mines. Mine idiosyncratic factors play an important role too. As shown in figure 9 and in table 6, this adjustment in the number of active mines is the result of very substantial amount of simultaneous entry (re-opening) and exit (temporary closing). decisions.

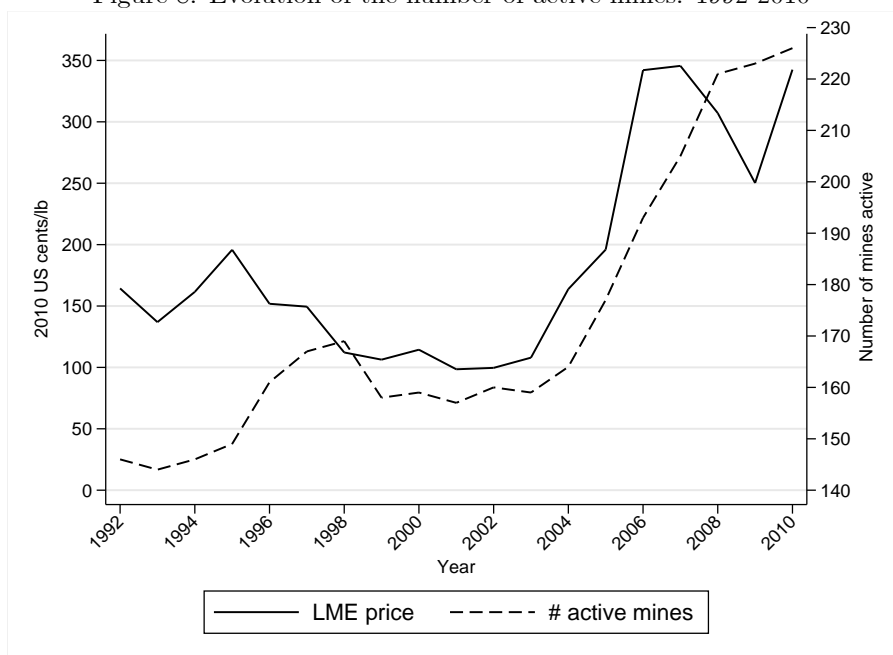Figure 8: Evolution of the number of active mines: 1992-2010

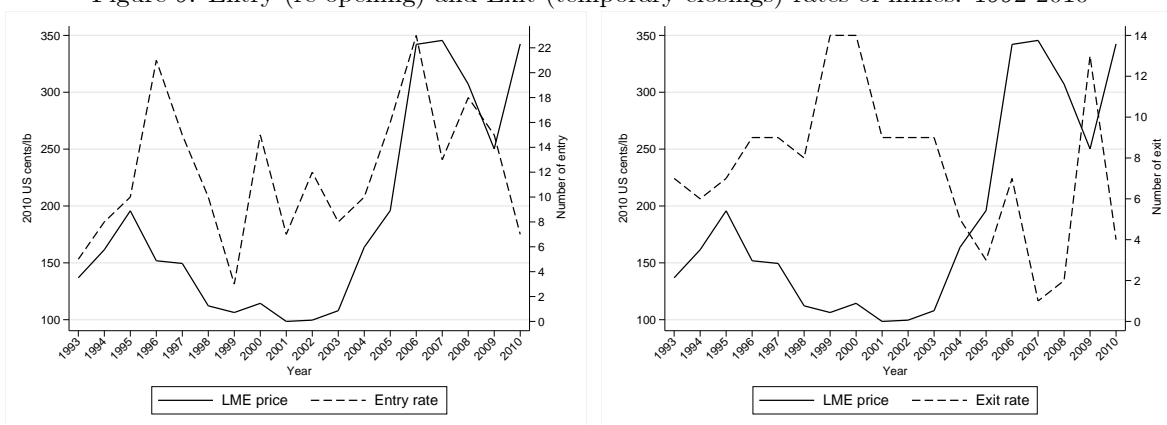Figure 9: Entry (re-opening) and Exit (temporary closings) rates of mines: 1992-2010

22

Table 6: Number of Mines, Entries, and Exits

Variable	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
# Active mines	146	144	146	149	161	167	169	158	159	157
Entries	-	5	8	10	21	15	10	3	15	7
Exits	-	7	6	7	9	9	8	14	14	9

Variable	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
# Active mines	160	159	164	177	193	205	221	223	226	-
Entries	12	8	10	16	23	13	18	15	7	
Exits	9	9	5	3	7	1	2	13	4	-

Source: Codelco

Note (1): Mt represents Metric tonnes = 1,000 Kg. Note (2). Observations with active mines.

Table 7 presents estimates of a probit model for the decision of exit (closing) that provide reduced form evidence on the effects of different market and mine characteristics on this decision. We present estimates both from standard (pooled) and fixed effect estimations, and report coefficients and marginal effects evaluated at sample means. The fixed effect Probit provides more sensible results: the signs of all the estimated effects are as expected, in particular, the effect of market price is positive and significant; and the marginal effects of the mine cost and ore grade variables become stronger. The smaller marginal effect of ore reserves in the FE Probit has also an economic interpretation: the mine fixed effect is capturing most of the "expected lifetime" effect (see the very substantial increase in the standard error), and the remaining effect captured by reserves is mainly through current costs. The estimates show that mine-specific state variables play a key role in the decision of staying active. The effect of ore grade is particularly important: doubling ore grade from the sample average 1.23% to 2.66% (percentile 85) implies an increase in the probability of staying active of almost 12 percentage points.

Figure 10 shows the estimated probability of an incumbent staying active varies with ore grade. Figure 11 presents this probability as a function of the mine average cost (C1). In this case, the higher the cost, the less likely an incumbent mine remain active.

Table 7: Reduced Form Probit for "Stay Active"[1]

Variable	Probit	Marginal effect	FE Probit	Marginal effect
ln(Price LME)[t]	-0.0681	-0.0201	0.520***	0.101***
	(0.0473)	(0.0139)	(0.140)	(0.0270)
ln(mine Avg. cost)[t-1]	-0.290***	-0.0857***	-1.940***	-0.377***
	(0.0442)	(0.0128)	(0.214)	(0.0401)
ln(Ore reserves)[t-1]	0.125***	0.0368***	0.235***	0.0456***
	(0.0108)	(0.00303)	(0.0697)	(0.0135)
ln(Ore grade)[t-1]	0.0242	0.00714	0.591	0.115
	(0.0276)	(0.00815)	(0.167)	(0.0322)
Number of obs.		3243		2233
Log-likelihood		-1697.4		-784.9

Note (1): Subsample of mines active at year t-1. Dependent variable: Dummy "Mine active at year t".

Note (2): * = significant at 10%; ** = significant at 5%; *** = significant at 1%;

Figure 10: Probability for Incumbent Staying Active by Ore Grade Level
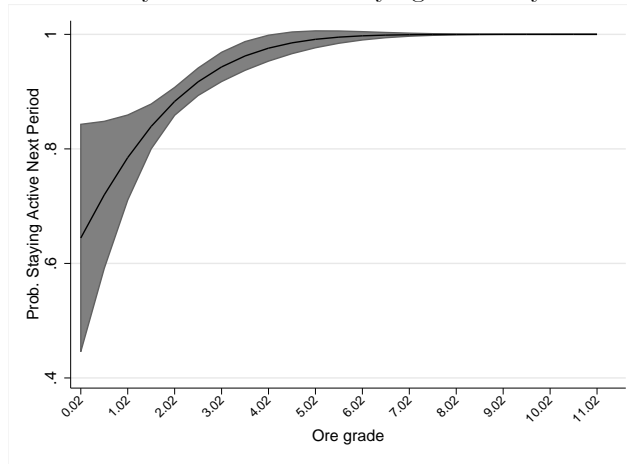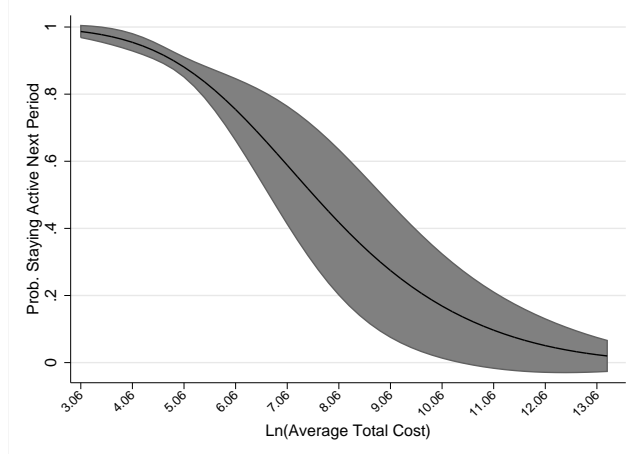

Figure 11: Probability for Incumbent Staying Active by Average Cost Level

3.2.2 Large heterogeneity across mines

There is very large heterogeneity across mines in geological characteristics, such as reserves, metal ores and ore grade, but also in capacity, production, and average costs. The degree of this heterogeneity is larger than what we typically find in manufacturing industries. Nature generates very different endowments of metals, ore grade and reserves across mines, and investment decisions tend to be complementary with these endowments such that they amplify differences across mines.

Table 8: Metal Share Mix Across Mines

Percentile[1]	Copper	Cobalt	Nickel	Lead	Zinc	Molybdenum	Gold	Silver
Pctile 1%	0.0038	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pctile 5%	0.0234	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pctile 10%	0.0465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pctile 25%	0.1612	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Pctile 50%	0.7600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0047	0.0104
Pctile 75%	0.9655	0.0000	0.0000	0.0117	0.4904	0.0000	0.0660	0.0417
Pctile 90%	1.0000	0.0000	0.0000	0.0959	0.7650	0.0017	0.2174	0.1253
Pctile 95%	1.0000	0.0530	0.0000	0.1430	0.8383	0.0259	0.4332	0.2525
Pctile 99%	1.0000	0.5148	0.7322	0.3804	0.9315	0.1548	0.6641	0.3768
Mean	0.6016	0.0134	0.0197	0.0290	0.2211	0.0045	0.0678	0.0429
Std. Dev.	0.3820	0.0719	0.1190	0.0855	0.3155	0.0211	0.1412	0.0855
Min	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Max	1.0000	0.6317	0.9079	0.8769	0.9676	0.1928	0.8007	0.6754
Obs	330	330	330	330	330	330	330	330

Mines With Positive Production of Metal								
	Copper	Cobalt	Nickel	Lead	Zinc	Molybdenum	Gold	Silver
# Mines	330	18	9	93	130	36	191	248
Main metal	205	4	9	3	93	0	12	4

Source: Codelco

Note (1): Cross-sectional distribution of mean values for each mine.

Copper mines frequently produce a variety set of metals as by-products of copper. The metal mix of by-products could include cobalt, nickel, lead, zinc, molybdenum, gold and silver depending on geological and geographical characteristics. Therefore, there is a high degree of heterogeneity in the mix of metal production across mines. In our sample, a mine produces typically two by-products and the maximum is four. However, the mine's by-product mix remains relatively constant over time. Ignoring the importance of by-products in the final output results in misleading estimates of both variable and marginal costs. Therefore, in order to take into account the role of by-products in mine's output, we transform the production of each metal in copper equivalent units. Table 8

presents the mean share distributions of production for each metal across mines, it also presents the number of mines producing each metal and the number of mines for which is their main metal. Most of the mines produce silver and gold as by-products and 284 mines produce at least one by-product. Surprisingly, for 125 mines a single metal different than copper was their main product and for 129 mines the 50% or more of their production is coming from aggregate metals other than copper.

Table 9: Heterogeneity Across Mines

Percentile[1]	Realized Grade (%)	Reserves (million t ore)	Production (thousand t)	Capacity (thousand t)	Avg. Total Cost ($/t cu)	Avg. Cost C1 ($/t cu)
Pctile 1%	0.14	0.11	0.01	0.05	908.80	450.94
Pctile 5%	0.26	0.60	0.04	0.42	1047.11	1021.50
Pctile 10%	0.39	1.11	0.16	0.95	1196.84	1287.78
Pctile 25%	0.75	3.66	1.09	2.79	1686.25	1632.94
Pctile 50%	1.71	13.48	5.10	8.58	2658.43	2067.43
Pctile 75%	3.42	120.61	22.72	38.00	8633.27	2638.71
Pctile 90%	5.16	541.99	96.50	151.37	33972.39	3867.84
Pctile 95%	6.28	978.48	156.23	203.79	80417.01	4968.07
Pctile 99%	8.99	2039.28	426.18	653.84	806386.06	7691.77

Source: Codelco

Note (1): Cross-sectional distribution of mean values for each mine.

We have measures of ore grade for each mine-year observation both for copper only (i.e., copper output per extracted ore volume) and for the copper equivalent output measure that takes into account by-products (i.e., copper equivalent output per extracted ore volume). Both measures show similar heterogeneity across firms. The differences in realized ore grade imply that two mines with exactly the same amount of inputs but different ore grades produce very different amount of output: a mine in percentile 75 would produce double than a median mine (i.e., 3.42/1.71), and five times the amount of output of a mine in percentile 25.

3.2.3   Endogenous ore grade

There is substantial time variation of realized ore grade within a mine. Figure 12 presents the empirical distribution for the change in realized ore grade (truncated at percentiles 2% and 98%). The median and the mode of this distribution is zero (almost 30% of the observations are zero), but there are substantial deviations from this median value. To interpret the magnitude of these changes, it is useful to take into account that the mean realized grade is 2.25%, and therefore

changes in realized grade with magnitude -0.25 and -0.02 represent, ceteris paribus, roughly 10% and 1% reductions in output and productivity. Since the 10th percentile is -0.25, we have that for one-tenth of the mine-year observations the decline in ore grade can generate reductions in productivity of more than 10%.

Figure 12: Empirical Distribution of Time Change in Ore Grade
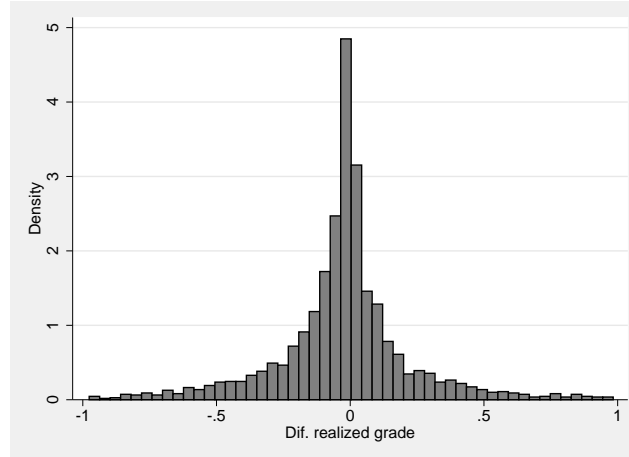
A question that we study in this paper is how much of these changes in grades are endogenous in the sense they depend on the depletion or production rate of a mine. More specifically, current production decreases the quality or grade of the mine and this, in turn, increases future production costs. On the other hand, investments in exploration can improve not only the amount of reserves but also the grade levels. Table 10 presents estimates of our dynamic model that support this hypothesis. We estimate the regression:

$$\ln\left(\text{Grade}_{it}\right) - \ln\left(\text{Grade}_{it-1}\right) = \beta_1\,\ln\left(1 + \text{Output}_{it-1}\right) + \beta_2\,\text{Discovery}_{it} + \alpha_i + \gamma_t + u_{it} \qquad (1)$$

where "Grade" is our realized measure of ore grade, "Output" is the mine production in copper equivalents units and "Discovery" is a binary indicator that is equal to 1 if the mine reserves increase by 20% or more, and it is zero otherwise. The estimates show a significant relationship between the change in the realized ore grade at period t and depletion (production) at $t-1$ after controlling for mine fixed effects and time effects. Doubling output is related to a reduction in almost 7% points in realized ore grade. As we show later, this implies a very relevant increase in the production cost of a mine. For the moment, using the "back of the envelope" calculation of the previous paragraph we have that increasing today's output by 100% implies a 7% reduction in the mine productivity next year. This is a non-negligible dynamic effect. We provide further details on the dynamics of realized ore grade in the results section.

Table 10: Estimation for Dynamics of Realized Ore Grade[1]

	OLS			
Variable	ln(grade) [t]	ln(grade) [t]	Dif. ln(grade) [t]	Dif. ln(grade) [t]
ln(grade) [t-1]	0.9812***	0.9812***	1.000	1.000
	(0.007)	(0.007)	(-)	(-)
ln(output)[t-1]	-0.0181***	-0.0181***	-0.0159***	-0.0159***
	(0.005)	(0.005)	(0.004)	(0.004)
Discovery[t]		0.0004		-0.0028
		(0.008)		(0.008)
Number of obs.	2918	2918	2918	2918
m1 p-value	0.9180	0.9177	0.8020	0.8006
m2 p-value	0.3802	0.3811	0.1173	0.1137

	Fixed Effect			
Variable	ln(grade) [t]	ln(grade) [t]	Dif. ln(grade) [t]	Dif. ln(grade) [t]
ln(grade) [t-1]	0.6937***	0.6936***	1.000	1.000
	(0.040)	(0.040)	(-)	(-)
ln(output)[t-1]	-0.0458***	-0.0458***	-0.0973***	-0.0972***
	(0.013)	(0.013)	(0.015)	(0.015)
Discovery[t]		0.0064		0.0054
		(0.009)		(0.010)
Number of obs.	2918	2918	2918	2918
m1 p-value	0.2747	0.2769	0.0289	0.0288
m2 p-value	0.3417	0.3543	0.0003	0.0003

	Blundell and Bond			
Variable	ln(grade) [t]	ln(grade) [t]	Dif. ln(grade) [t]	Dif. ln(grade) [t]
ln(grade) [t-1]	0.9091***	0.9062***	1.000	1.000
	(0.051)	(0.052)	(-)	(-)
ln(output)[t-1]	-0.0710**	-0.0708**	-0.0686***	-0.0684***
	(0.029)	(0.029)	(0.026)	(0.026)
Discovery[t]		0.0126		0.0008
		(0.016)		(0.011)
Number of obs.	2918	2918	2918	2918
m1 p-value	0.0001	0.0001	0.0001	0.0001
m2 p-value	0.9952	0.9884	0.9631	0.9666
Hansen p-value	0.1969	0.1955	0.1547	0.1503
RW[3]	0.0754	0.0733		

Note (1): Subsample of mines active at years t-1 and t.

Note (2): * = significant at 10%; ** = significant at 5%; *** = significant at 1%.

Note (3): RW is Wald test for random walk in the lagged dependent variable.

Note (4): Year dummies included in all models.

Figure 13 and table 11 present reduced form evidence on the evolution of realized grade over time. Time is number of years active (production > 0).Grades tend to decrease over active periods. This evidence is consistent with a depletion effect. Moreover, grade levels depreciate at different rates across mines according to size , which is mainly given by geological characteristics. For example, large mines present lower grades and lower grade depreciation rates whereas small and medium size mines have higher grades and higher grade depreciation rates. In general, there is no evidence to support that better mines (high initial grades) leave longer as this also depends on reserves and technology. This would indicate that small and medium size mines are exhausted faster which is consistent with the average years being active. However, older mines present a higher grade in the tail of the sample for all sizes.
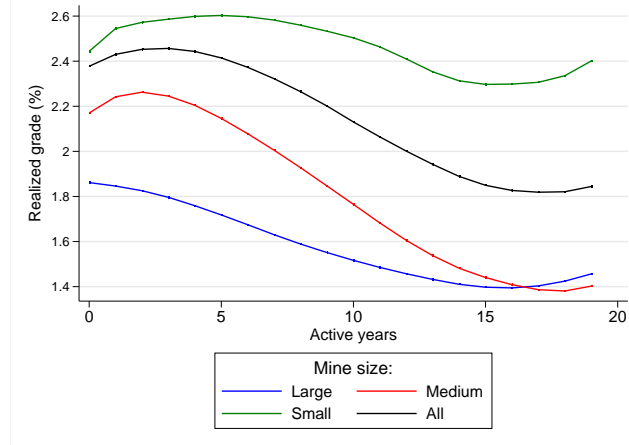
Figure 13: Evolution of ore grade by years of production


Table 11: Realized Ore Grade Descriptive Statistics

	Mine Size			All
	Large	Medium	Small	
# Mines	18	81	231	330
(%)	5.5	24.5	70	100
Realized grade (%)	1.6	1.89	2.52	2.25
Real. grade dep. rate (%)	-0.40	-1.42	-0.94	-1.29
Years active	14.83	12.91	8.53	9.95

Mean Realized Grade for all Mines				
	Years Active			
Age of the Mine	1-5	6-10	11-15	15-19
1-5	2.38	-	-	-
6-10	2.99	2.54	-	-
11-15	2.29	2.34	1.91	-
15-19	2.20	2.10	1.96	1.82

Figure 15 presents reduced form evidence on the relationship between the total average production costs, demand for inputs and ore grade (truncated at percentiles 2% and 98%). Mines with lower grades presents higher production costs, f.i. the lower the ore grade the more processing of ore is needed to produce the same amount of copper and therefore the higher is the cost. Moreover, the aging of mines or the depletion effect, as described above, implies an increase in demand for inputs such as electricity, fuel and other inputs, as shown in the right graph of figure 15 for the case of electricity consumption.

Figure 15: Production cost, electricity consumption and ore grade
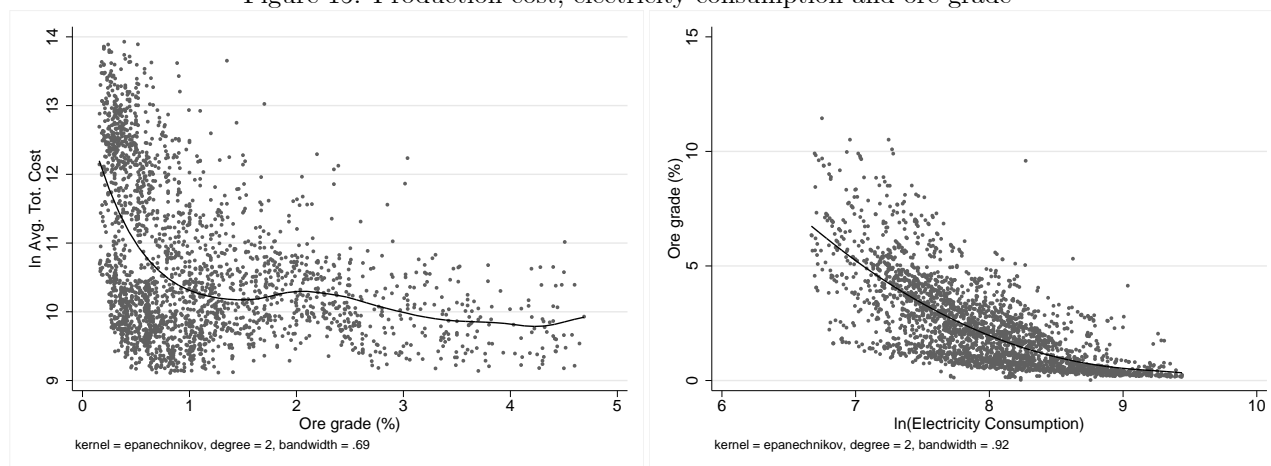
3.2.4 Concentration of market shares

The international copper market structure, as many other mineral industries, is characterized by a reduced number of mines that account for a very large proportion of world production. Table 12 presents the market shares of the leading copper mines in 1996. Escondida (BHP-Billiton) and the Chilean state-owned mine, Chuquicamata (Codelco), have dominated the market with the 16% of world copper production. Some changes in the industry have undergone in the last decade as new mines are discovered. For example, Collahuasi (Xstrata) and Los Pelambres (Antofagasta Minerals), two world-class mines located in Chile have been developed since then and were ranked fourth and seventh, respectively, in 2010. However, in general, market shares and concentration ratios have remained relatively stable over the sample period.

Table 12: Market Shares and Concentration Ratios: Year 1996

Rank in 1996	Mine (Country)	Annual production (thousand Mt)	Share %	Con. Ratio CR(n) %
1.	Escondida (Chile)	825	9.1	9.1
2.	Chuquicamata (Chile)	623	6.9	16.0
3.	Grasberg (Indonesia)	507	5.5	21.5
4.	Morenci (Arizona, USA)	462	5.1	26.6
5.	KGHM (Poland)	409	4.4	31.0
6.	El Teniente (Chile)	344	3.8	34.8
7.	ZCCM (Zambia)	314	3.4	38.2
8.	Bingham C. (Utah, USA)	290	3.2	41.4
9.	Ok Tedi (Papua)	179	2.0	43.4
10.	La Caridad (Mexico)	176	2.0	45.4

3.2.5 Lumpy investment in capacity

Table 13 present the empirical distribution of investment rate in capacity, $i_{it} \equiv (k_{it} - k_{it-1})/k_{it-1}$, for the subsample of observations where the firm is active at two consecutive years. Investment is very lumpy, with a high proportion of observations with zero investment, and large investment rates when positive.

Table 13: Empirical Distribution Investment Rate in Capacity

Statistic	1993	1997	2001	2005	2009
% Obs. Zero Investment	80.0%	41.6%	45.0%	60.3%	69.8%
Conditional on positive					
Pctile 25%	12.5%	6.2%	6.2%	4.0%	7.8%
Pctile 50%	20.0%	16.2%	20.0%	21.7%	37.5%
Pctile 75%	100.0%	31.6%	100.0%	80.0%	116.6%

Source: Codelco

4 A dynamic model of copper mining

4.1 Basic framework

A copper firm, indexed by f, consists of a set of mines $\mathcal{M}_f$, indexed by $i \in \mathcal{M}_f$ each one having its own specific characteristics. Time is discrete and indexed by t. A firm's profit at period t is the sum of profits from each of its mines, $\Pi_{ft} = \sum_{i \in \mathcal{M}_f} \pi_{it}$. For the moment, we assume that profits are separable across mines and we focus on the decision problem of a single mine. Every period (year) t, the managers of the mine make three decisions that have implications on current and future profits:

(1) whether to be active or not next period ($a_{it+1} \in \{0, 1\}$); (2) how much output to produce (q_{it}); and (3) investment in capital equipment (i_{it}). Let $\mathbf{d}_{it} \equiv (a_{it+1}, i_{it}, q_{it})$ be the vector with these decision variables, and let $\mathbf{y}_{it} \equiv (a_{it}, k_{it}, g_{it}, r_{it})$ be the vector of endogenous state variables, where k_{it} represents capital equipment, r_{it} represents ore reserves, and g_{it} is ore grade. Similarly, let $(\mathbf{z}_{it}, \boldsymbol{\varepsilon}_{it})$ be the vector with all the exogenous state variables in demand, productivity, and input prices, where $\mathbf{z}_{it}$ represents exogenous state variables that are observable to the researcher, and $\boldsymbol{\varepsilon}_{it}$ represents unobservables. We use $\mathbf{x}_{it} \equiv (\mathbf{y}_{it}, \mathbf{z}_{it})$ to represent the vector with all the observable state variables, and $\mathbf{s}_{it} \equiv (\mathbf{x}_{it}, \boldsymbol{\varepsilon}_{it})$ to represent all the state variables. Then, the dynamic decision problem of a mine manager can be represented using the Bellman equation:

$$
V_i(\mathbf{s}_{it}) = \max_{\mathbf{d}_{it}} \left\{ \pi_i(\mathbf{d}_{it}, \mathbf{s}_{it}) + \beta \int V_i(\mathbf{y}_{it+1}, \mathbf{z}_{it+1}, \boldsymbol{\varepsilon}_{it+1}) \ f_y(\mathbf{y}_{it+1}|\mathbf{d}_{it}, \mathbf{y}_{it}) \ f_z(\mathbf{z}_{it+1}|\mathbf{z}_{it}) \ f_\varepsilon(\boldsymbol{\varepsilon}_{it+1}|\boldsymbol{\varepsilon}_{it}) \right\}
$$
(2)

where $\beta \in (0, 1)$ is the discount factor, and f_y, f_z, and f_ε are the transition probability functions of the state variables.

The rest of this subsection describes in detail the economics of the primitive functions π_i, f_y, f_z, and f_ε. Section 4.2 presents the parametric specification of these functions.

(a) Profit function. The one-period profit function of a mine is:

$$
\pi_{it} = P_t \ q_{it} - VC_{it}(q_{it}) - FC_{it} - EC_{it} - XC_{it} - IC_{it}
$$
(3)

where P_t is the price of copper in the international market, VC_{it} is the variable cost function, FC_{it} represents fixed operating costs, EC_{it} is the cost of entry (re-opening), XC_{it} is the cost of closing the mine, and IC_{it} is the cost of investments in capital equipment.

(b) Markets, competition, and demand function. The market of copper is global and its price P_t is determined by aggregate world demand and supply. World inverse demand function is:

$$
P_t = p\left(Q_t, \mathbf{z}_t^{(d)}, \varepsilon_t^{(d)}\right)
$$
(4)

where Q_t is the aggregate industry output at period t, and $\left(\mathbf{z}_t^{(d)}, \varepsilon_t^{(d)}\right)$ are exogenous variables that enter in the demand function. $\mathbf{z}_t^{(d)}$ is a vector of exogenous demand shifters observable to the researcher, such as GDP growths of US, EU, and China, and the price of aluminium (i.e., the closest substitute of copper). $\varepsilon_t^{(d)}$ is a demand shock that is unobservable to the researcher. We assume that firms (mines) in this industry compete a la Nash-Cournot. When a mine decides its optimal amount of output at period t, q_{it}, it takes as given the output choices of the rest of the mines in the industry, Q_{-it}. Note that our game of Cournot competition is dynamic. As we describe below,

a mine output decision has effects of future profits. Mine managers are forward looking and take into account this dynamic effects. We assume that mines are price takers in input markets.

(c) Active / no acitve choice. Every year t, the managers of a mine decide whether to operate the mine next period or not. We represent this decision using the binary variable $a_{it+1} \in \{0, 1\}$, where $a_{it+1} = 1$ indicates that the mine decides to be active (operating) at period $t + 1$. Based on information from conversations with industry experts, we assume that there is time-to-build in the decision of opening or closing a mine: a decision taken at year t is not effective until year $t + 1$. Our conversations with industry experts also indicate that almost all the mine closings during our sample period were not permanent closings. This is reinforced by evidence in our data showing a substantial amount of reopenings during our sample period. Therefore, in our model, we consider that mine closings are reversible decisions. Though re-opening is reversible, it is costly. If the mine is not active at t, there is a fix cost EC_{it} of starting-up at $t + 1$. This cost may depend on state variables such as mine size as measured by reserves, and the price of fixed inputs. Closing a mine is costly too. If the mine is active at year t and the managers decide to stop operations at $t + 1$, there is a cost of closing the mine. Start-up and closing costs are paid at period t but the decision is not effective until $t + 1$. Start-up cost has the following form:

$$EC_{it} = c_i^{(e)}(\mathbf{x}_{it}) + \varepsilon_{it}^{(e)}, \tag{5}$$

and similarly, the closing cost is:

$$XC_{it} = c_i^{(x)}(\mathbf{x}_{it}) + \varepsilon_{it}^{(x)}, \tag{6}$$

where $c_i^{(e)}(.)$ and $c_i^{(x)}(.)$ are functions of the vector of observable state variables $\mathbf{x}_{it}$ that we specify below. $\varepsilon_{it}^{(e)}$ and $\varepsilon_{it}^{(x)}$ are state variables that are observable to mine managers but unobservable to the researcher.

(d) Production decision / Production function / Variable costs. An active mine at year t (i.e., $a_{it} = 1$) should decide how much copper to produce during the year, q_{it}. This is a dynamic decision because current production has two important implications on future profits. First, current production depletes reserves and then reduces the expected lifetime of the mine. Reducing reserves can also increase future production costs. As shown in table 10, a second dynamic effect is that the depletion of the mine has a negative impact on ore grade. We capture these effects through the specification of the variable cost function (or equivalently, the production function), and through the transition rule of ore grade. The production function of copper in a mine is Cobb-Douglas with

33

six different production inputs:

$$q_{it} = (\ell_{it})^{\alpha_\ell} \ (e_{it})^{\alpha_e} \ (f_{it})^{\alpha_f} \ (k_{it})^{\alpha_k} \ (r_{it})^{\alpha_r} \ (g_{it})^{\alpha_g} \ \exp\{\omega_{it}\} \tag{7}$$

where ℓ_{it} is labor, e_{it} and f_{it} represent electricity and fuel, respectively, k_{it} is capital equipment, r_{it} represents ore reserves, g_{it} is ore grade, ω_{it} is a productivity shock, and α's are parameters. Capital, reserves, ore grade, and productivity shock, $(k_{it}, r_{it}, g_{it}, \omega_{it})$, are predetermined variables for the output decision at year t.

The inclusion of ore reserves as an input in the production function deserves some discussion. Following the influential work by Herfindahl (1969), the literature on the estimation of production functions and cost functions in extractive industries has typically included ore reserves as input. The justification for this at the mine level is that higher grade ore is extracted first, so that costs fall as reserves dwindle due to lower quality ore being available. This is the so called *stock effect* on extraction costs. Since our data and specification of the production function includes ore grade as an input, Herfindahl's interpretation of the stock effect would suggest that conditional on grade reserves should not affect output. In other words, if reserves still have an effect on output after controlling for ore grade, the channel of this effect should be other than the depletion of ore grade. Our inclusion of reserves in the production function can be considered as a way of testing Herfindahl's hypothesis on the channel of the stock effect on extraction costs.

The mine chooses the amount of labor, electricity, and fuel at year t, $(\ell_{it}, e_{it}, f_{it})$, and this decision is equivalent to the choice of output q_{it}. The first order conditions of optimality for the three variable inputs are the following: for every variable input $v \in \{\ell, e, f\}$:

$$MR_{it} \ \frac{\partial q_{it}}{\partial v_{it}} - p_{it}^v + \beta \ \mathbb{E}_t \left[\frac{\partial V_{it+1}}{\partial \mathbf{y}_{it+1}} \frac{\partial \mathbf{y}_{it+1}}{\partial q_{it}} \right] \frac{\partial q_{it}}{\partial v_{it}} \ = \ 0 \tag{8}$$

where: $MR_{it} \equiv [p'_t(Q_t) q_{it} + P_t]$ is the marginal revenue of mine i; $\partial q_{it}/\partial v_{it}$ is the marginal productivity of variable input $v \in \{\ell, e, f\}$; p_{it}^v is the price of this input at the input markets where mine i operates; $\partial \mathbf{y}_{it+1}/\partial q_{it}$ represents the dynamic effects of current output on next period reserves and ore grade, that we describe below; and $\partial V_{it+1}/\partial \mathbf{y}_{it+1}$ is the marginal value of reserves and ore grade. Though the choice of these variable inputs have dynamic implications, the dynamic effect operates only through current output. Therefore, it is clear that these dynamic marginal conditions of optimality imply the standard static condition that the ratio between marginal productivity of inputs should equal the ratio between input prices:

$$\frac{\partial q_{it}/\partial \ell_{it}}{\partial q_{it}/\partial e_{it}} = \frac{p_{it}^\ell}{p_{it}^e} \quad \text{and} \quad \frac{\partial q_{it}/\partial \ell_{it}}{\partial q_{it}/\partial f_{it}} = \frac{p_{it}^\ell}{p_{it}^f} \tag{9}$$

Using these conditions, the production function, and the definition of variable cost as $VC_{it} \equiv p_{it}^{\ell}$ $\ell_{it} + p_{it}^{e} e_{it} + p_{it}^{f} f_{it}$, we can derive the variable cost function:

$$VC_{it} = \alpha_v \left[\frac{(p_{it}^{\ell}/\alpha_\ell)^{\alpha_\ell} \ (p_{it}^{e}/\alpha_e)^{\alpha_e} \ \left(p_{it}^{f}/\alpha_f\right)^{\alpha_f}}{(k_{it})^{\alpha_k} \ (r_{it})^{\alpha_r} \ (g_{it})^{\alpha_g} \ \exp\{\omega_{it}\}} \ q_{it} \right]^{(1/\alpha_v)} \tag{10}$$

where α_v is the sum of the coefficients of all the variable inputs, i.e., $\alpha_\ell + \alpha_e + \alpha_f$. The marginal cost is equal to $MC_{it} = (1/\alpha_v) \ (VC_{it}/q_{it})$.

(e) Transition rule for reserves and ore grade. The endogenous evolution of ore reserves is described by the equation:

$$r_{i,t+1} = r_{it} - \left(\frac{q_{it}}{g_{it}} \right) + z_{i,t+1}^{(r)} \tag{11}$$

q_{it}/g_{it} represents the amount of ore that was extracted from the mine at period t to produce q_{it} units of copper. $z_{i,t+1}^{(r)}$ is an stochastic shock that represents updates in ore reserves due to new discoveries or reviews in the estimated value of reserves. For the moment, we assume that $z_{i,t+1}^{(r)}$ follows an exogenous stochastic process. More specifically, $z_{i,t+1}^{(r)}$ follows a first order Markov process, i.e., new discoveries have positive serial correlation. We assume that $z_{i,t+1}^{(r)}$ is unknown to the mine managers when they make their decisions at year t, but they know z_{it}^{r}, and this is a state variable of the model. Note that $z_{it}^{(r)}$ is observable to the researcher, i.e., for every mine i and year t, we can construct $z_{it}^{(r)} = r_{it} - r_{it-1} + q_{it-1}/g_{it-1}$. For periods where a mine is not active, we have that $z_{it}^{(r)} = 0$.

The transition for ore grade captures the depletion effect on the quality of the mine. Following Caldentey et al. (2006) and consistent with mining practices, we assume that a mine is divided into a collection of blocks each one having particular geological characteristics, i.e., its own ore grades and extraction costs. These blocks represent minimal extraction units so that the miner's production decisions are made at block level. Usually, blocks with the highest ore grade are extracted first which determine the block extraction path of the mine. As deposits are depleted, mining shifts to blocks with the next best quality. However, given the physical characteristics of the mine, factors other than ore grade, such as the depth level of the block, hardness of the rocks, and distance to the processing plant, play also a role in the optimal path of block extraction. We have tried different specifications for the transition rule of ore grade. The following equation describes our favored specification:

$$\ln g_{i,t+1} = \ln g_{it} - \delta_q^{(g)} \ \ln(1 + q_{it}) + \delta_z^{(g)} z_{it}^{(r)} + \varepsilon_{it+1}^{(g)} \tag{12}$$

The model imposes the restriction that all the effects, exogenous or endogenous, on ore grade are permanent. We have tried specifications where the parameter of g_{it} is smaller than 1, and the

estimate of this parameter, though precise, is not significantly different to 1. The parameter $\delta_q^{(g)}$ (depletion elasticity of ore grade) is positive and $-\delta_q^{(g)} \ln q_{it}$ captures the depletion effect on ore grade. The term $\delta_z^{(g)} z_{it}^{(r)}$ takes into account that new discoveries may imply changes in ore grade.

(f) Fixed cost and investment cost. The operation of a copper mine is very intensive in specialized and expensive capital equipment, i.e., extraction machinery, transportation equipment, and processing/refining equipment. These inputs are typically fixed within a year, but they imply costs of amortization, leasing, and maintenance. These costs depend on the size of the mine as measured by reserves and capital.

$$FC_{it} = p_{it}^k \; k_{it} + \theta_1^{(fc)} \; k_{it} + \theta_2^{(fc)} \; r_{it} + \theta_3^{(fc)} \; (k_{it})^2 + \theta_4^{(fc)} \; (r_{it})^2, \tag{13}$$

where p_{it}^k price of capital, as measured by interest costs, and $\{\theta_j^{(fc)}\}$ are parameters that capture the relationship between mine size and fixed costs. Parameters $\theta_3^{(fc)}$ and $\theta_4^{(fc)}$ can be either positive or negative, depending on whether there economies or diseconomies of scale associated to the size of the mine (in contrast to the more standard economies of scale associated to the level of output, that are captured by the fixed cost itself).

The specification of the investment cost function tries to capture the lumpy behavior of this decision as shown in table 11 above. It has fixed (lump-sum), linear, and quadratic components, that are asymmetric between positive and negative investments.

$$
\begin{aligned}
IC_{it} \;\; = \;\; & 1\{i_{it+1} < 0\} \; \left[\theta_0^{(ic-)} + \varepsilon_{it}^{(ic-)} + p_{it}^k \; i_{it+1} + \theta_1^{(ic-)} i_{it+1} + \theta_2^{(ic-)}(i_{it+1})^2 \right] \\
+ \;\; & 1\{i_{it+1} > 0\} \; \left[\theta_0^{(ic+)} + \varepsilon_{it}^{(fc+)} + p_{it}^k \; i_{it+1} + \theta_1^{(ic+)} i_{it+1} + \theta_2^{(ic+)}(i_{it+1})^2 \right]
\end{aligned}
\tag{14}
$$

The transition rule for the capacity stock is $k_{t+1} = k_t + i_t$.

4.2   Euler equations

In this section, we derive the dynamic conditions of optimality that we use for the estimation of model parameters and for testing some specification assumptions. These optimality conditions, that we generally describe as Euler equations, involve decisions and state variables, at a small number of consecutive years. We derive four different types of Euler equations: (a) for output when output is positive; (b) for investment when investment is positive; (c) for the binary choice of being active or not; and (d) for the binary decision of investment in capacity. Euler equation (a) is standard and it can be derived by combining marginal conditions of optimality at two consecutive periods with the application of the Envelope Theorem in the Bellman equation. The other three

Euler equations are not standard. The Euler equation for investment is not standard because, for the mine manager, it is not a probability 1 event that after an interior solution (i.e., non-zero investment) at period t there will be an interior solution next period $t + 1$. For the derivation of standard Euler equations, interior solutions should occur every period with probability one. To deal with these non-standard case, we derive Euler equations following Pakes (1994). Conditions (c) and (d) are even less standard because they involve discrete choices and, in principle, these choices do not involve marginal conditions of optimality. Following Aguirregabiria and Magesan (2013), we show that our dynamic decision model has a representation where discrete choices for output and investment are described in terms of Conditional Choice Probabilities (CCPs). Then, we show that a mine optimal decision rule for these discrete decisions implies marginal conditions of optimality. Finally, we show that we can combine these marginal conditions at two consecutive periods two derive Euler equations.

For notational simplicity, in this section we omit the mine subindex i.

4.2.1 Euler equation for output

Consider a mine that is active at two consecutive years, t and $t + 1$. Note that the time-to-build assumption, on opening and closing decisions, implies that when the managers of the mine make output decision at year t they know that the mine will be active at year $t + 1$ with probability one. The managers know that the marginal condition of optimality with respect to output will hold at period $t + 1$ with probability one. Under this condition, we can derive a standard Euler equation for output. We show in Appendix B that the Euler equation for output is:

$$\frac{\partial \pi_t}{\partial q_t} + \beta \, \mathbb{E}_t \left(-\frac{\partial \pi_{t+1}}{\partial q_{t+1}} - \frac{\partial \pi_{t+1}}{\partial r_{t+1}} \frac{1}{g_{t+1}} - \frac{\partial \pi_{t+1}}{\partial g_{t+1}} \, \delta_q^{(g)} \right) = 0 \tag{15}$$

Or taking into account the form of the profit function π_t:

$$MR_t - MC_t = \beta \, \mathbb{E}_t \left(MR_{t+1} - MC_{t+1} + \frac{\alpha_r}{\alpha_\ell + \alpha_m} \frac{VC_{t+1}}{r_{t+1}} \frac{1}{g_{t+1}} + \frac{\alpha_g}{\alpha_\ell + \alpha_m} \, \delta_q^{(g)} \frac{VC_{t+1}}{g_{t+1}} \right) \tag{16}$$

where $\dfrac{\alpha_r}{\alpha_\ell + \alpha_m} \dfrac{VC_{t+1}}{r_{t+1}}$ and $\dfrac{\alpha_g}{\alpha_\ell + \alpha_m} \dfrac{VC_{t+1}}{g_{t+1}}$ represent the increase in variable cost from a unit in reserves and ore grade, respectively. This Euler equation already contains several extensions with respect to Hotelling's rule. In a simple dynamic decision model for the exploitation of a nonrenewable resource, where firms do not have market power and the marginal production cost does not depend on reserves and ore grade, the Euler equation of the model becomes $P_t - MC_t = \beta$ $\mathbb{E}_t \left(P_{t+1} - MC_{t+1} \right)$, that often is represented as Hotelling's rule as:

$$\frac{\mathbb{E}_t \left(P_{t+1} - MC_{t+1} \right)}{P_t - MC_t} - 1 = \frac{1 - \beta}{\beta} \tag{17}$$

This equation implies that, on average, the price-cost margin increases over time at an annual rate equal to $(1 - \beta)/\beta$, e.g., for $\beta = 0.95$, this rate is equal to 5.2%. This prediction is typically rejected for most non-renewable resources, and for copper in particular. The Euler equation in (16) introduces two extensions that modify this prediction. First, a unit increase in output at period t implies an increase in the marginal cost at $t + 1$ equal to $(\alpha_r/(\alpha_\ell + \alpha_m))\ VC_{t+1}/(r_{t+1}\ g_{t+1}) + (\alpha_g/(\alpha_\ell + \alpha_m))\ \delta_q^{(g)}\ VC_{t+1}/g_{t+1}$. Depletion increases future marginal cost. This effect may offset, partly or even completely, the standard depletion effect on price-cost margin in Hotelling model. To see this, we can write the Euler

$$\frac{\mathbb{E}_t\left(P_{t+1} - MC_{t+1}\right)}{P_t - MC_t} - 1 = \frac{1 - \beta}{\beta} - \frac{\mathbb{E}_t\left(\frac{\alpha_r}{\alpha_\ell + \alpha_m} \frac{VC_{t+1}}{r_{t+1}} \frac{1}{g_{t+1}} + \frac{\alpha_g}{\alpha_\ell + \alpha_m} \delta_q^{(g)} \frac{VC_{t+1}}{g_{t+1}}\right)}{P_t - MC_t} \tag{18}$$

The second term in the equation is negative and it can be larger, in absolute, than $(1 - \beta)/\beta$. In section 5, we present our estimates of production function parameters and show that, while the parameter α_r is very small and, in some cases, not significantly different to zero, α_g is relatively large, i.e., point estimates between 0.59 and 0.77, depending on the estimation method.

4.2.2 Euler equation for investment in capacity

Let $i^*(\mathbf{s}_t)$ be the optimal decision rule for capacity investment in the dynamic decision model defined by Bellman equation (2). Suppose that at period t the optimal decision is to make a non-zero investment such that $i^*(\mathbf{s}_t) \neq 0$. Suppose that we modify marginally this optimal decision rule at periods $\{t, t + 1, ..., t + \tau_t\}$, where $\tau_t \in \{1, 2, ...\}$ is the number of periods until the next interior solution (with the optimal decision rule), i.e., τ_t is such that $i^*(\mathbf{s}_{t+\tau_t}) \neq 0$ and $i^*(\mathbf{s}_{t+j}) = 0$ for $j < \tau_t$. Let $i(\mathbf{s}, t + j, \kappa)$ be the the perturbed decision rule, that is defined as:

$$i(\mathbf{s}, t + j, \kappa) = \begin{cases} i^*(\mathbf{s}) - \kappa & \text{for} \quad j = 0 \\ 0 & \text{for} \quad 0 < j < \tau_t \text{ if } \tau_t > 1 \\ i^*(\mathbf{s}) + \kappa & \text{for} \quad j = \tau_t \\ i^*(\mathbf{s}) & \text{for} \quad j > \tau_t \end{cases} \tag{19}$$

In words, the modified rule reduces investment in κ units at period t and increases investment also in κ units at the next period with positive investment, $t + \tau_t$. For the rest of the periods, $i(\mathbf{s}, t + j, \kappa) = i^*(\mathbf{s})$. The capital stock is κ units smaller between $t + 1$ and $t + \tau_t$, and then it returns to its optimal path after $t + \tau_t$.

Suppose that we solve the decision rule $i(\mathbf{s}, t + j, \kappa)$ in the expected and discounted intertemporal profit of the mine at period t. This intertemporal profit function is continuously differentiable in κ, and we show in Appendix B that, by construction, the value of κ that maximizes this function

is $\kappa = 0$. [12] The first order condition of optimality with respect to κ, evaluated at $\kappa = 0$, provides the Euler equation:

$$\frac{\partial \pi_t}{\partial i_t} + \mathbb{E}_t \left(\beta^{\tau_t} \left[-\frac{\partial \pi_{t+\tau_t}}{\partial i_{t+\tau_t}} + \frac{\partial \pi_{t+\tau_t}}{\partial k_{t+\tau_t}} \right] \right) = 0 \tag{20}$$

Or taking into account the form of the profit function π_t:

$$-\frac{\partial IC_t}{\partial i_t} + \mathbb{E}_t \left(\beta^{\tau_t} \left[-\frac{\partial IC_{t+\tau_t}}{\partial i_{t+\tau_t}} - \alpha_k \frac{VC_{t+\tau_t}}{k_{t+\tau_t}} \right] \right) \tag{21}$$

where $\partial IC_t / \partial i_t$ is the marginal investment cost, and $\alpha_k VC_t / k_t$ is the marginal effect of capacity on the variable cost.

4.2.3 Euler equation for discrete choice active/non active

Let $\pi^*(a_{t+1}, \mathbf{x}_t) + \varepsilon_t(a_{t+1})$ be the one-period profit function such that: (a) it is conditional to the hypothetical choice of a_{t+1} for the active/no active decision; and (b) we have already solved in this function the optimal decisions for output and investment. By definition, we have that $\varepsilon_t(0) = -a_t$ $\varepsilon_t^{(x)}$ and $\varepsilon_t(1) = -(1 - a_t)\,\varepsilon_t^{(e)}$, and:

$$\pi^*(a_{t+1}, \mathbf{x}_t) \;=\; \pi\left(a_{t+1}, q^*[\mathbf{x}_t], i[\mathbf{x}_t], \mathbf{x}_t\right) = \begin{cases} \Pi^*(\mathbf{x}_t) - a_t\, c_i^{(x)}(\mathbf{x}_t) & \text{if} \quad a_{t+1} = 0 \\[2mm] \Pi^*(\mathbf{x}_t) - (1 - a_t)\, c_i^{(e)}(\mathbf{x}_t) & \text{if} \quad a_{t+1} = 1 \end{cases} \tag{22}$$

where $\Pi^*(\mathbf{x}_t) \equiv VP^*(\mathbf{x}_t) - FC(\mathbf{x}_t) - IC^*(\mathbf{x}_t)$ is the part of the profit function that does not depend on a_{t+1}. We can use the profit function to define a dynamic binary choice model that represents the part of our model related to the mine decision to be active or not. The Bellman equation of this problem is:

$$V\left(\mathbf{x}_t, \varepsilon_t\right) = \max_{a_{t+1} \in \{0,1\}} \left\{ \pi^*(a_{t+1}, \mathbf{x}_t) + \varepsilon_t(a_{t+1}) + \beta \int V(\mathbf{x}_{t+1}, \varepsilon_{t+1})\, f_x^*(\mathbf{x}_{t+1} | a_{t+1}, \mathbf{x}_t)\, f_\varepsilon(\varepsilon_{t+1} | \varepsilon_t) \right\} \tag{23}$$

Let $a_{t+1} = \alpha^*(\mathbf{x}_t, \varepsilon_t)$ be the optimal decision rule of this DP problem. Under the assumption that $\varepsilon_t = \{\varepsilon_t(0), \varepsilon_t(1)\}$ is i.i.d. over time, this optimal decision rule has a threshold structure, i.e., there is a real-valued function $\mu^*(\mathbf{x}_t)$ such that:

$$a_{t+1} = \alpha^*(\mathbf{x}_t, \varepsilon_t) = 1\left\{\varepsilon_t(0) - \varepsilon_t(1) \leq \mu^*(\mathbf{x}_t)\right\} \tag{24}$$

Therefore, to characterize this optimal decision rule, we can concentrate in the class of decision rules with the structure $\alpha(\mathbf{x}_t, \varepsilon_t) = 1\left\{\varepsilon_t(0) - \varepsilon_t(1) \leq \mu(\mathbf{x}_t)\right\}$, for arbitrary $\mu(\mathbf{x}_t)$. Given the CDF

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[12]Note that by the Envelope Theorem we can ignore how the change in the capital stock between $t + 1$ and $t + \tau_t$ affects intertemporal profits through the change in output choice, i.e., this marginal effect is zero.

of $\varepsilon_t(0) - \varepsilon_t(1)$, i.e., $F(.)$, and an arbitrary real-valued function $\mu(\mathbf{x}_t)$, we can uniquely define a Conditional Choice Probability (CCP) function:

$$P(\mathbf{x}_t) \equiv F(\mu(\mathbf{x}_t)) \tag{25}$$

This CCP function represents the probability of being active at period $t + 1$ given the observable state $\mathbf{x}_t$ and given the decision rule μ.

It is clear that there is a one-to-one relationship between the three representations of a decision rule: (1) the representation in action space, $\alpha(\mathbf{x}_t, \varepsilon_t)$; (2) the threshold function $\mu(\mathbf{x}_t)$; and (3) the CCP function $P(\mathbf{x}_t)$. Following Aguirregabiria and Magesan (2013), we consider a representation of the model in terms of the CCP function. This representation has the following (integrated) Bellman equation:

$$V^P(\mathbf{x}_t) = \max_{P(\mathbf{x}_t) \in [0,1]} \left\{ \Pi^P(P(\mathbf{x}_t), \mathbf{x}_t) + \beta \int V^P(\mathbf{x}_{t+1}) \, f_x^P(\mathbf{x}_{t+1} | P(\mathbf{x}_t), \mathbf{x}_t) \, d\mathbf{x}_{t+1} \right\} \tag{26}$$

with:

$$\Pi^P(P(\mathbf{x}_t), \mathbf{x}_t) \equiv (1 - P(\mathbf{x}_t)) \left[\pi^*(0, \mathbf{x}_t) + e(0, \mathbf{x}_t, P) \right] + P(\mathbf{x}_t) \left[\pi^*(1, \mathbf{x}_t) + e(1, \mathbf{x}_t, P) \right], \tag{27}$$

where $e(a, \mathbf{x}_t, P)$ is the expected value of $\varepsilon_t(a)$ conditional on alternative a being chosen under decision rule $P(\mathbf{x}_t)$; and

$$f_x^P(\mathbf{x}_{t+1} | P(\mathbf{x}_t), \mathbf{x}_t) \equiv (1 - P(\mathbf{x}_t)) \, f_x^P(\mathbf{x}_{t+1} | 0, \mathbf{x}_t) + P(\mathbf{x}_t) \, f_x^P(\mathbf{x}_{t+1} | 1, \mathbf{x}_t). \tag{28}$$

Aguirregabiria and Magesan (2013, Proposition 2(i)) show that the optimal CCP function $P^*(\mathbf{x}_t)$ that solves Bellman equation (26) is the CCP function that corresponds to the optimal decision rule in our original problem in equation (23), i.e., $a_{t+1} = \alpha^*(\mathbf{x}_t, \varepsilon_t) = 1\left\{ \varepsilon_t(0) - \varepsilon_t(1) \leq F^{-1}[P(\mathbf{x}_t)] \right\}$.

Using this representation property of our dynamic binary choice model, we can derive the following Euler equation that involves CCPs at periods t and $t + 1$. We provide the details of this derivation in Appendix B.

$$\left[\frac{\pi^*(a_{t+1} = 1, \mathbf{x}_t) - \pi^*(a_{t+1} = 0, \mathbf{x}_t)}{\sigma_\varepsilon} - \ln\left(\frac{P(\mathbf{x}_t)}{1 - P(\mathbf{x}_t)} \right) \right] +$$

$$\beta \, \mathbb{E}_t \left[\frac{\pi^*(a_{t+2} = 1, a_{t+1} = 1, \mathbf{x}_{t+1}) - \pi^*(a_{t+2} = 1, a_{t+1} = 0, \mathbf{x}_{t+1})}{\sigma_\varepsilon} - \ln\left(\frac{P(a_{t+1} = 1, \mathbf{x}_{t+1})}{P(a_{t+1} = 0, \mathbf{x}_{t+1})} \right) \right] = 0 \tag{29}$$

Or taking into account the form of the profit function π_t:

$$\left[\frac{a_t\ c^{(x)}(\mathbf{x}_t) + (1-a_t)c^{(e)}(\mathbf{x}_t)}{\sigma_\varepsilon} - \ln\left(\frac{P(\mathbf{x}_t)}{1-P(\mathbf{x}_t)}\right)\right] \ +$$

$$\beta\ \mathbb{E}_t\left[\frac{\Pi^*(\mathbf{x}_{t+1}) + c^{(e)}(\mathbf{x}_{t+1})}{\sigma_\varepsilon} - \ln\left(\frac{P(a_{t+1}=1, \mathbf{x}_{t+1})}{P(a_{t+1}=0, \mathbf{x}_{t+1})}\right)\right] \ =\ 0$$

5   Estimation results

This section presents estimates of our structural model. We estimate the parameters in two steps. In a first step, we estimate the parameters of the transition function of the realized grade and the production function. In a second step, we estimate our Euler equations using as inputs the estimates from the first step.

5.1   Transition rule of ore grade

Equation (12) above presents our specification of the transition rule of ore grade. If the mine is inactive ($q_{it} = 0$), the evolution of the logarithm of ore grade is governed by a random walk $\ln(g_{i,t+1}) - \ln(g_{it}) = \delta_z^{(g)} z_{it}^{(r)} + \varepsilon_{it+1}^{(g)}$, where $\delta_z^{(g)} z_{it}^{(r)} + \varepsilon_{it+1}^{(g)}$ represent new discoveries and natural events affecting the mine. The term $-\delta_q^{(g)} \ln(1 + q_{it})$ captures the depletion effect on ore grade, where $\delta_q^{(g)} > 0$ is the depletion elasticity. We have presented estimates of this equation in Table 10 above to emphasize the importance of dynamics and depletion in ore grade. Here we comment these estimates in more detail.

The main econometric concern in the estimation of equation (12) comes from the potential correlation between output q_{it} and the unobservable shock $\varepsilon_{it+1}^{(g)}$. For instance, suppose that new discoveries have positive serial correlation (i.e., $cov(\varepsilon_{it}^{(g)}, \varepsilon_{it+1}^{(g)}) > 0$) and that new discoveries at period t have a positive impact on output (as we would expect from our dynamic decision model). Under these conditions, OLS estimation of equation (12) provides an under-estimation of the depletion effect, i.e., the OLS estimate of $\delta_q^{(g)}$ is upward biased.[13] Our OLS estimates of the depletion elasticity, in the top panel of table 10, are between -0.016 and -0.018. However, we find strong positive serial correlation in the OLS residuals, what indicates that these estimates are inconsistent. To control for potential changes in grade due to new discoveries in reserves, we include a discovery dummy, $z_{it}^{(r)}$, for changes in the level of reserves of at least 20%. However, in none of our

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[13]Another potential source of bias in our estimates is because of measurement error. Since our measure of output contains a conversion of by-products to a copper equivalents units of output, any error in this measurement, which is uncorrelated with the fixed characteristic of the mine, would imply that the depletion effect is further underestimated.

specifications new discoveries seems to play a role for changes in realized grades.

We deal with this endogeneity problem using standard methods in the econometrics of dynamic panel data models. We assume that the error has the following variance-components structure: $\varepsilon_{it+1}^{(g)} = \alpha_i^{(g)} + \gamma_{t+1}^{(g)} + u_{i,t+1}^{(g)}$, where the term $\alpha_i^{(g)}$ denotes time-invariant differences in realized grades across mines such as geological characteristics. $\gamma_{t+1}^{(g)}$ is an aggregate shock affecting all mines and $u_{i,t+1}^{(g)}$ is a mine idiosyncratic shock that is assumed not serially correlated over time. Under the assumption of no serial correlation in the transitory shock, we have that output at period t is not correlated with $u_{i,t+1}^{(g)}$. Since we have a relatively large number of mines in our sample, we can control for the aggregate shocks $\gamma_{t+1}^{(g)}$ using time dummies. In principle, if our sample included also a large number of years for each mine, we could also control for the individual effects $\alpha_i^{(g)}$ by using mine dummies. This is the approach in the Fixed Effects estimation presented in the second panel in Table 10. The fixed-effect estimates of the depletion elasticity are between -0.046 and -0.097 that are substantially larger than the OLS estimates. As we expected, controlling for persistent unobservables in innovation $\varepsilon_{it+1}^{(g)}$ contributes to reduce the OLS downward bias in depletion elasticity. However, as it is well known in the dynamic panel data literature, this fixed effects estimator can be seriously biased when the number of time periods in the sample is smaller than $T = 20$ or $T = 30$. The most common approach to deal with this problem is the GMM estimators proposed by Arellano and Bond (1991) and Blundell and Bond (1998). Since the parameter for the lagged ore grade is very close to one and the Arellano-Bond estimator suffers of a weak instruments problem in that situation, here we use the System GMM estimator proposed by blundell and Bond. Our GMM estimates of the depletion elasticity, in the bottom panel in Table 10, are between -0.068 (s.e. $= 0.026$) and -0.071 (s.e. $= 0.029$). These estimates are very robust to imposing or not the restriction that lagged ore grade has a unit coefficient. As expected, the magnitude of the System GMM estimates of the effect of output in the evolution of realized grade is higher than those in the OLS estimates. The Arellano-Bond test of autocorrelation in the residuals cannot reject zero second-order autocorrelation in first differences. This evidence support the key assumption for identification of no serial correlation in the error term. The Hansen test for over-identifying restrictions does not reject the validity of the instruments. The estimated coefficient for lagged ore grade is close to 1 and, using a Wald test, we cannot reject the null hypothesis that it is equal to 1.

The main finding in our estimates of the transition function of the realized grade is that current output has a substantial negative effect on future ore grades. This dynamic depletion effect is

not negligible. Our favorite specification states that increasing current output by 100% leads to a depreciation of 7% in the mine realized grades next period. Note that this is a long-run effect.

5.2 Production function

We estimate a Cobb-Douglas production function in terms of physical units for output, capital, labor, reserves, ore grade, and intermediate inputs electricity and fuel. The log-linearized production function is:

$$\ln q_{it} = \alpha_k \ \ln k_{it} + \alpha_\ell \ \ln \ell_{it} + \alpha_e \ \ln e_{it} + \alpha_f \ \ln f_{it} + \alpha_g \ \ln g_{it} + \alpha_r \ \ln r_{it} + \omega_{it} + e_{it} \qquad (30)$$

where the input variables are capital, k, labor, ℓ, electricity, e, fuel, f, ore grade, g, and reserves, r. ω_{it} is a productivity shock and e_{it} represents a measurement error in output or any shock that is unknown to the mine when it decides the quantity of inputs to use. The estimation of the parameters in this function should deal with the well-known endogeneity problem due to the simultaneous determination of inputs and output. Here we present estimates from two different methods that deal with this problem: dynamic panel data methods proposed by Arellano and Bond (1991) and Blundell and Bond (1998); and control function approach methods proposed Olley and Pakes (1996) and Levinsohn and Petrin (2003).

Table 14 presents our estimations of production function parameters. We report estimates from six different specifications and methods. All the specifications include time dummies. Column (1) reports fixed effect estimates (OLS with mine dummies) based on the assumption that the productivity shock follows a variance-components structure $\omega_{it} = \eta_i + \gamma_t + \omega_{it}^*$, where η_i is a time-invariant mine specific effect such as some geological characteristics, and ω_{it}^* is not serially correlated and it is realized after the miner decides the amount of inputs to use at period t. Of course, the conditions for consistency of the fixed effects estimator are very strong.

Column (2) provides estimates using Arellano-Bond GMM method, based on the same covariance-structure for productivity, $\omega_{it} = \eta_i + \gamma_t + \omega_{it}^*$, and the assumption that ω_{it}^* is not serially correlated, but allowing for correlation between inputs and the productivity shock ω_{it}^*. In the equation in first differences at period t, $\Delta \ln q_{it} = \Delta \ln \mathbf{x}_{it} \ \boldsymbol{\alpha} + \Delta \omega_{it}^*$, inputs and output variables dated at $t-2$ and before are valid instruments for endogenous inputs: i.e., $\mathbb{E}(\ln \mathbf{x}_{it-2} \ \Delta \omega_{it}^*) = 0$ and $\mathbb{E}(\ln q_{it-2} \ \Delta \omega_{it}^*) = 0$. The assumption of no serial correlation in ω_{it}^* is key for the consistency of this estimator, but this assumption is testable: i.e., it implies no serial correlation of second order in the residuals in first differences, $\mathbb{E}(\Delta \omega_{it}^* \ \Delta \omega_{it-2}^*) = 0$.

Column (3) presents also estimates using Arellano-Bond GMM estimator but of a model where

the productivity shock ω_{it}^* follows an AR(1) process, $\omega_{it}^* = \rho\,\omega_{it-1}^* + \xi_{it}$, where ξ_{it} is not serially correlated. In this model, we can apply a quasi-time-difference transformation, $(1 - \rho\mathbb{L})$, that accounts for the AR(1) process in ω_{it}^*, and then a standard first difference transformation that eliminates the time-invariant individual effects. This transformation provides the equation, $\Delta \ln q_{it} = \rho$ $\Delta \ln q_{it-1} + \Delta \ln \mathbf{x}_{it}\,\boldsymbol{\alpha} + \Delta \ln \mathbf{x}_{it-1}\,(-\rho\boldsymbol{\alpha}) + \Delta\xi_{it}$, where inputs and output variables dated at $t-2$ and before are valid instruments: i.e., $\mathbb{E}(\ln \mathbf{x}_{it-2}\,\Delta\xi_{it}) = 0$ and $\mathbb{E}(\ln q_{it-2}\,\Delta\xi_{it}) = 0$.

Column (4) reports estimates using Blundell-Bond System GMM. As we have mentioned above for the estimation of the transition rule of ore grade, in the presence of persistent explanatory variables, the First-Difference GMM may suffer a weak instruments problem that implies substantial variance and finite sample bias of the estimator. In this case, Blundell-Bond System GMM is preferred to Arellano-Bond method. This system GMM is based on two sets of moment conditions: the Arellano-Bond moment conditions, i.e., input variables in levels at $t-2$ and before are valid instruments in the equation in first differences at period t; and the Blundell-Bond moment conditions, i.e., input variables in first-differences at $t-1$ and before are valid instruments in the equation in levels at period t, $\mathbb{E}(\Delta \ln \mathbf{x}_{it-1}\,[\eta_i + \omega_{it}^*]) = 0$ and $\mathbb{E}(\Delta \ln q_{it-1}\,\omega_{it}^*) = 0$. As in the Arellano-Bond estimator, the assumption of no serial correlation in ω_{it}^* is fundamental for the validity of these moment conditions and the consistency of the estimator. Column (5) provides estimates using Blundell-Bond System GMM for the model where ω_{it}^* follows an AR(1) process.

Column (6) presents estimates using the control function approach of Olley and Pakes (1996) based in the extension proposed by Levinsohn and Petrin (2003). We use materials rather than investment as proxy for the productivity shock given the high degree of lumpiness in our investment measure. We also allow for adjustment costs in labor and introduce formally lagged labor as an state variable in the control function. A mine demand for materials, m_{it}, is given by $m_{it} = m_t(k_{it}, \ell_{it-1}, g_{it}, r_{it}, \omega_{it})$. Since this demand is strictly monotonic in the productivity shock, ω_{it}, there is an inverse function $\omega_{it} = m_t^{-1}(m_{it}, k_{it}, \ell_{it-1}, g_{it}, r_{it})$ and we can control for the unobserved productivity in the estimation of the production function by including a nonparametric function (i.e., high order polynomial) of the observables $(m_{it}, k_{it}, \ell_{it-1}, g_{it}, r_{it})$, such that, $\ln q_{it} = \alpha_\ell \ln \ell_{it} + \alpha_e$ $\ln e_{it} + \alpha_f \ln f_{it} + \phi_t(m_{it}, k_{it}, \ell_{it-1}, g_{it}, r_{it}) + e_{it}$. In the first step of this method, parameters α_ℓ, α_e, and α_f, and the parameters in the polynomials ϕ_t are estimated by least squares. We use a third order polynomial function to approximate our control function. The parameters α_k, α_g, and α_r are estimated in a second step by exploiting the assumption that the productivity shock evolves following a first-order Markov process. For instance, if we assume that ω_{it} follows an AR(1)

process,[14] $\omega_{it} = \rho\,\omega_{it-1} + \xi_{it}$, then the model implies the equation $\phi_{it} = \rho\,\phi_{it-1} + \alpha_k\,\ln k_{it} + (-\rho\alpha_k)$ $\ln k_{it-1} + \alpha_g\,\ln g_{it} + (-\rho\alpha_g)\,\ln g_{it-1} + \alpha_r\,\ln r_{it} + (-\rho\alpha_r)\,\ln r_{it-1} + \xi_{it}$. All the regressors in this equation are pre-determined before period t and therefore not correlated with ξ_{it}. Given that ϕ_{it} has been estimated in the first step, this equation can be estimated by nonlinear least squares to obtain consistent estimates of α_k, α_g, α_r, and ρ.[15]

Table 14: Production Function Estimates

	(1) FE no AR(1)	(2) FD-GMM no AR(1)	(3) FD-GMM AR(1)	(4) SYS-GMM no AR(1)	(5) SYS-GMM AR(1)	(6) LP
Capital	0.2417***	0.3728***	0.2431***	0.1265***	0.2206***	0.3328***
	(0.054)	(0.064)	(0.065)	(0.035)	(0.060)	(0.020)
Labor	0.1418***	0.1841*	0.0860	0.0319	0.0936*	0.0355
	(0.038)	(0.096)	(0.096)	(0.065)	(0.051)	(0.046)
Electricity	0.2229***	0.1335*	0.1835**	0.3217***	0.2316***	0.2063***
	(0.078)	(0.069)	(0.079)	(0.078)	(0.084)	(0.047)
Fuel	0.4001***	0.4755***	0.4547***	0.3582***	0.4372***	0.1609***
	(0.048)	(0.055)	(0.073)	(0.055)	(0.067)	(0.022)
Grade	0.7432***	0.6120***	0.7688***	0.6657***	0.6999***	0.5860***
	(0.069)	(0.142)	(0.122)	(0.055)	(0.068)	(0.032)
Reserves	0.0011	-0.0621**	-0.0211	0.0693***	0.0159	0.0062
	(0.014)	(0.030)	(0.026)	(0.017)	(0.015)	(0.012)
Output(t-1) (ρ)	-	-	0.5467***	-	0.5660***	-
	-	-	(0.074)	-	(0.060)	-
Obs.	2150	1906	1684	2150	1906	1719
m1-pvalue	0.0000	0.0296	0.0000	0.0459	0.0000	
m2-pvalue	0.0000	0.0112	0.4438	0.0264	0.4514	
Hansen -pvalue		0.9105	0.7569	1.0000	1.0000	
RTS	1.0076	1.1037	0.9464	0.9076	0.9989	0.7355
Null CRS	0.8449	0.2076	0.5706	0.0320	0.9825	

In Table 14, several important empirical results are robust across the different specifications and estimation methods. First, the production technology is very intensive in energy, both electricity and fuel. The sum of the parameters for energy and fuel, $\alpha_e + \alpha_f$, is always between 0.61 and 0.68

[14]Note that this assumption is different to the specification of the productivity shock in dynamic panel models. In the Olley-Pakes model, the whole productivity shock, ω_{it}, follows a Markov process. In dynamic panel data models, we have that $\omega_{it} = \eta_i + \omega_{it}^*$, and ω_{it}^* follows an AR(1) process.

[15]We experiment with many alternative specifications, however, results does not vary too much. We also allow for selection bias in the LP method but results are very similar.

and represents approximately two thirds of the returns to scale of all the inputs. The technology is also relatively intense in capital, with a capital coefficient between $0,13$ and 0.37. In contrast, the technology presents a low coefficient for labor, between 0.03 and 0.18. Second, the coefficient of ore grade is large an very significant, between 0.61 and 0.77. Ceteris paribus, we would expect a grade elasticity equal to one, i.e., keeping all the inputs constant, an increase in the ore grade should imply a proportional increase in output. The estimated elasticity, though high, is significantly lower than one. This could be explained by heterogeneity across mines. Mines with different ore grades may be also different in the type of mineral, hardness of the rock, depth of the mineral, or distance to the processing plant. Third, the estimated coefficient for reserves is always very small and not economically significant. This result supports Herfindahl's interpretation of the effect of reserves on output as coming from the depreciation of ore grade. Once we control for ore grade, the amount of reserves does not any significant effect on output. Fourth, tests of serial correlation in the residuals in columns (2) and (4) reject the null hypothesis of no serial correlation of second order, and therefore reject the hypothesis that the shock ω_{it}^* is not serially correlated. The same test for the models in columns (3) and (5) (with an AR(1) process for ω_{it}^*) cannot reject the null hypothesis that the shock ξ_{it} in the AR(1) process is not serially correlated. Therefore, these tests clearly favor the specification with an AR(1) process for ω_{it}^*.

Based on the specification tests and on the economic interpretation of the results, our preferred specification and estimates are the ones in column (5). These estimates imply an industry very intensive in energy ($\alpha_e + \alpha_f = 0.67$) and capital ($\alpha_k = 0.23$) but not in labor ($\alpha_\ell = 0.09$), with constant returns to scale (i.e., $\mu = 0.99$, the hypothesis of CRS cannot be rejected), a sizeable effect of ore grade ($\alpha_g = 0.70$), and very persistent idiosyncratic productivity shocks.

Table 15: Distribution of Estimated Marginal and Variable Costs[1]

Pctile.	Marginal Cost	Ex. Marginal Cost	Variable Cost	AVC
Pctile 1%	99.09	17.59	937.58	75.55
Pctile 5%	156.17	40.90	1,741.60	119.07
Pctile 10%	197.11	57.25	3,076.57	150.29
Pctile 25%	306.30	89.88	7,209.35	233.54
Pctile 50%	541.56	154.21	20,506.95	412.90
Pctile 75%	1,008.46	285.06	80,177.51	768.89
Pctile 90%	1,932.51	561.43	181,830.00	1,473.42
Pctile 95%	2,632.80	832.48	335,716.40	2,007.34
Pctile 99%	4,901.29	1,619.28	1,026,301.00	3,736.93
Mean	878.70	262.37	83,047.93	669.95
Std. Dev.	1104.82	354.02	189,853.30	842.36
Min	26.47	7.81	140.53	20.18
Max	15298.46	5,591.10	2,657,656.00	11,664.12
Obs	2102	2102	2102	2102

Note (1): values in US$ per ton.

5.3 Marginal costs and variable costs

Given the estimated parameters of the production function, and the information on variable input prices, we calculate variable costs and marginal costs using the formula in equation (10). Table 15 presents the empirical distributions of variable cost, average variable cost, marginal cost, and the exogenous part (or predetermined part) of the marginal cost ($ExMC$) defined as the marginal cost of producing the first unit of output (i.e., the first ton of copper). We can interpret this exogenous marginal cost as the intercept of the marginal cost curve with the vertical axis at $q = 1$.

There is very substantial heterogeneity across mines in all measures of variable cost. For the exogenous marginal cost, the interquartile difference is 217% (i.e., $(285.06 - 89.88)/89.88$), and the difference between the 90th and 10th percentiles is 880% (i.e., $(561.43 - 57.25)/57.25$). Interestingly, the degree of heterogeneity in marginal cost is similar to the heterogeneity in its exogenous component. This clearly contradicts the hypothesis of static perfect competition.

Table 16: Variance Decomposition of Marginal Cost

Variance - Covariance Matrix[1],[2]

(all variables in logarithms)

	Wage	Elect. p.	Fuel p.	Ore grade	Reserves	Capital	Product.
Wage	1.0526						
Electricity price	0.0469	0.2289					
Fuel price	0.0658	0.1069	0.3983				
Ore grade	0.0215	0.0127	0.0538	0.9265			
Reserves	0.1497	0.0015	-0.1834	-1.4527	5.5826		
Capital	0.3470	0.0139	-0.0864	-0.7434	2.9898	2.7069	
Productivity	0.0598	-0.0254	-0.0037	-0.0276	0.0162	-0.1177	0.1377

Variance Decomposition: Exogenous Part of Marginal Cost[1],[2],[3]

x	Variance	Covariance	Weight (%)	Obs.
Marginal cost (Ex)	0.8384	0.8384	100.0	2102
Wage	0.0159	-0.0025	-0.30	2102
Electricity price	0.0211	0.0469	5.6	2102
Fuel price	0.1310	0.1452	17.3	2102
Ore grade	0.7806	0.4879	58.2	2102
Reserves	0.0024	-0.0051	-0.6	2102
Capital	0.2266	0.0033	0.4	2102
Productivity	0.2368	0.1627	19.4	2102

Note (1): All variables in logs.

Note (2): Weight is computed as $cov(\beta_x \ln[x], \ln[ExMC])/Var(\ln[ExMC])$.

Note (3): The number of observations is restricted to observed productivity from the estimation
of the production function.

Table 16 provides a decomposition of the variance of the logarithm of the exogenous marginal cost into the contribution of its different components. The top-panel reports the variance-covariance matrix of the seven components of the (exogenous) marginal cost. The bottom panel presents the variance decomposition. By definition, the logarithm of the exogenous marginal cost (i.e., $\ln[ExMC]$) is equal to $\beta_\ell \ln(p_{it}^\ell) + \beta_e \ln(p_{it}^e) + \beta_f \ln(p_{it}^f) + \beta_g \ln(g_{it}) + \beta_r \ln(r_{it}) + \beta_k \ln(k_{it}) + \beta_\omega \omega_{it}$, where $\beta_\ell = \alpha_\ell/(\alpha_\ell + \alpha_e + \alpha_f)$, $\beta_g = -\alpha_g/(\alpha_\ell + \alpha_e + \alpha_f)$, and so on. For each of its additive components, we calculate its covariance with $\ln[ExMC]$. This decomposition provides a measure of the contribution of each component to the heterogeneity in marginal costs. There are several interesting results. First, ore grade is, by far, the factor with the most important contribution to the heterogeneity in marginal costs across mines. If we eliminate that source of heterogeneity, keeping the rest of the elements constant, the variance of marginal costs would decline by 58%. Second, total factor productivity with 19% and fuel prices with 17% are the other most important sources of heterogeneous marginal costs. These three variables together account for 95% of the

variance. Electricity prices have also a non-negligible contribution of 5.6%. Third, the contribution of capital to the dispersion of marginal costs is basically null. This is despite, our estimation of the production function, implies that capital has an important contribution to output and marginal cost, and also despite the variance of capital across mines is quite important (see top panel in this table). The explanation for this result comes from the correlation between capital and ore grade. Capital and ore grade have a strong negative correlation (i.e., correlation coefficient -0.47). Mines with poorer ore grades require typically more equipment both in extraction and in the processing stages. The contribution of capital to the variance of marginal costs is mostly offset by the fact that larger mines in terms of capital are typically associated with lower ore grades. Fourth, interestingly, the contribution of wages is zero.

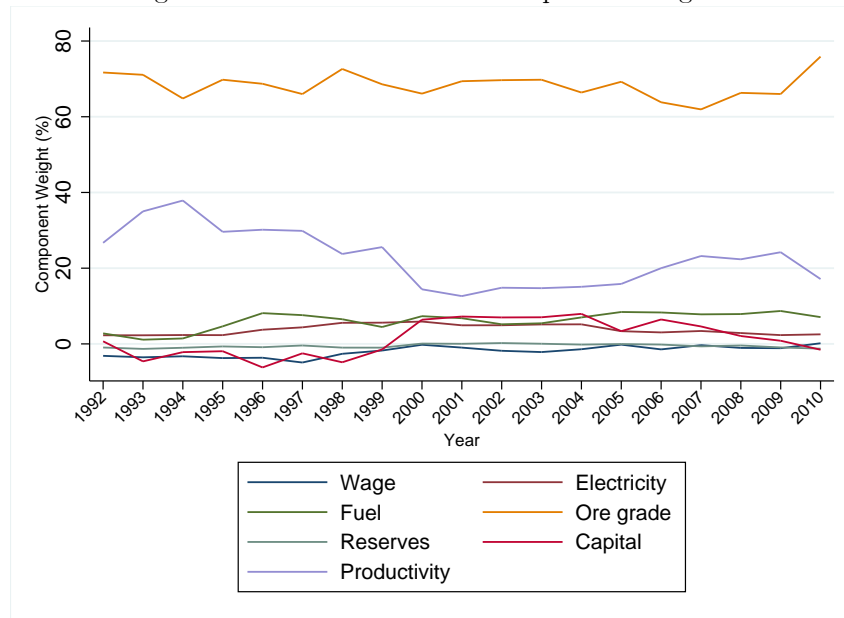Figure 16: Evolution of ExMC components weights

Figure 16 presents the evolution over time of the contribution of each component in the variance of the exogenous marginal costs. Weights remain relatively stable over the period. However, the contribution of the variance of productivity is decreasing over time. This decreasing effect of productivity could be capturing the extensive and intensive changes effects during the boom, as described below.

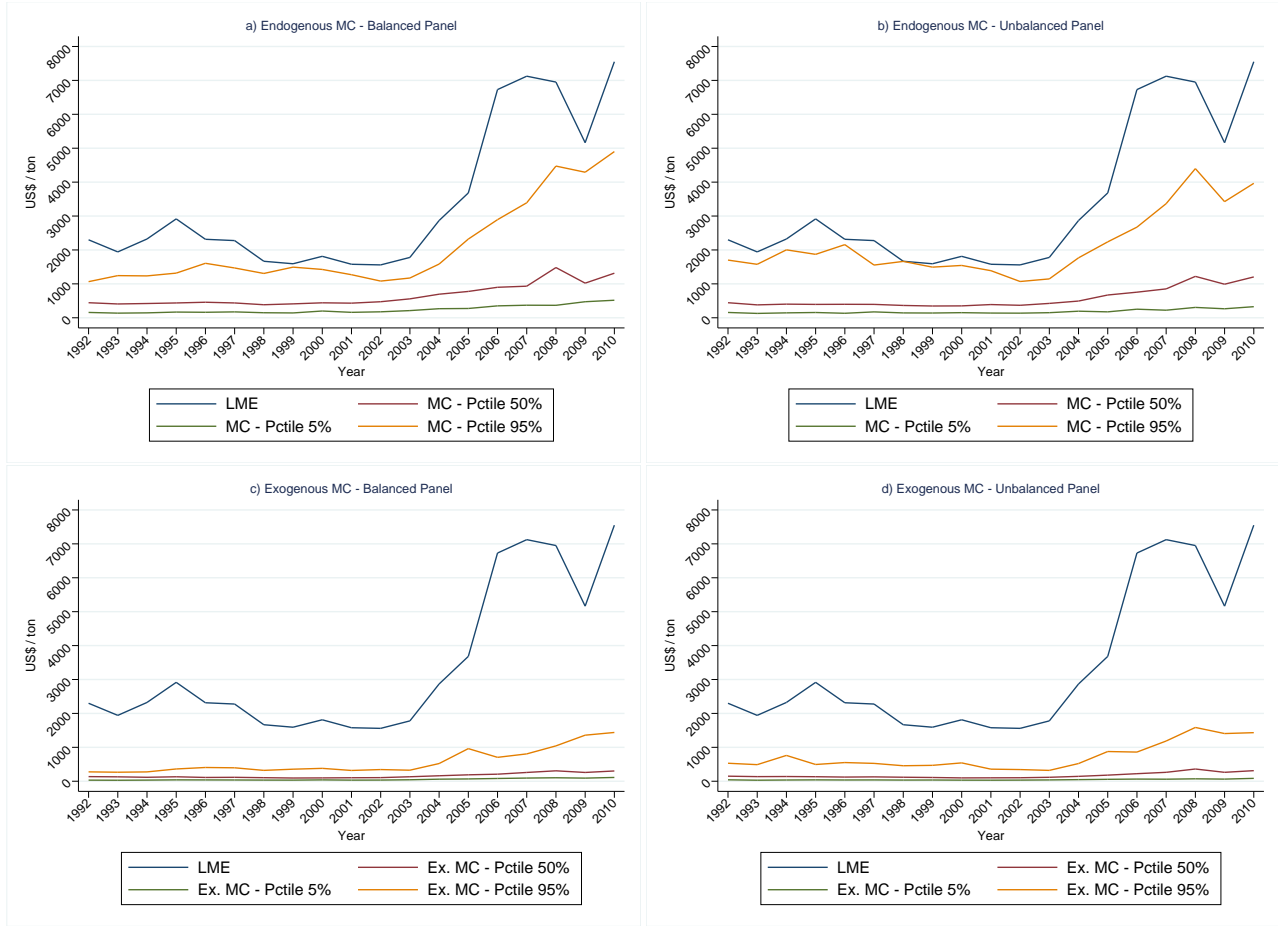Figure 17: Evolution of Est. Marginal Cost and Exogenous Marginal Cost

Figure 17 provides evidence on the relationship between the evolution of the LME copper price and the evolution of the 5th, 50th and 95th percentiles of the estimated marginal costs. We present figures both for the exogenous and for the total marginal cost, and for a balanced panel of 43 mines and an unbalanced panel of 212 mines. These figures provide an interesting description of the relationship between copper prices, marginal costs, and demand. First, panel (c) presents the evolution of $ExMC$ for the balanced panel of mines. This figure represents changes in marginal costs that are not associated to changes in the composition of active firms and are not related to the amount of output produce, i.e., to demand. We can see that there is a relatively modest increase in the marginal cost at the 95th percentile between 2003 and 2010. This modest increase can account only for a small portion of the observed increased in copper price during this period. Panel (d) presents the evolution of $ExMC$ for the unbalanced panel. The evidence provided by this figure is similar as the one from panel (c): even if we take into account changes in the composition of mines, and more specifically the entry of less efficient mines due to increasing prices, the increase in the exogenous marginal cost accounts at most for one-fourth of the increase in price. Second, the

50

comparison of panels (c) and (d) provides evidence that positive demand shocks promote entry of less efficient mines. For instance, the exogenous marginal costs of the mines at the 95th percentile in the unbalanced panel is a 16% higher than the exogenous marginal costs of those mines in the balanced panel. Third, panels (a) and (b) present the evolution of total (endogenous) marginal cost. Interestingly, the $95th$ percentile follows very closely the evolution of copper price, though the 5th and 50th percentiles are still quite flat. This picture seems consistent with the story that most of the price increase comes from the combination of a positive demand shock, but also with the fact that mines with relatively higher marginal costs have increased their production share during this period. Fourth, these figures show that some mines in this industry enjoy large markups in terms of marginal costs. Price is mainly determined by the marginal cost of less efficient mines, and given the high heterogeneity in marginal costs, most efficient mines have large markups.

5.4   Euler equation for output

[TO BE INCLUDED]

5.5   Euler equation for investment (marginal choice)

[TO BE INCLUDED]

5.6   Euler equation for investment (discrete choice)

[TO BE INCLUDED]

5.7   Euler equation for active/no active choice

[TO BE INCLUDED]

6   Conclusion

A   Appendix A. Description of variables in the dataset

Industry Level:

LME price: Copper LME price in US$ per tonne.

Consumption: World total consumption of primary copper in thousands of tonnes.

Capacity: World annual production capability for copper units, whether contained in concentrate, anode, blister, or refined copper in thousands of metric tonnes.

Production: World total mine copper produced by mines in thousands of tonnes.

# of Mines: Number of active mines per year.

Mine Capacity: Capacity reflects a plant's annual production capability for copper units, whether contained in concentrate, anode, blister, or refined copper in thousands of metric tonnes. Capacity is usually determined by a combination of engineering factors, such as gross tonnage of milling capacity and feed grades that determine long-term sustainable production rates. Mine capacity is not generally adjusted to reflect short-term variations in ore grade but would reflect long-term trends in ore grade. Electrowinning capacity at both the mine and refinery level is usually determined by tankhouse parameters. (ICGS)

Mine Level:

Mine Production:

Total: Total payable copper produced in thousands of tonnes.

Concentrates: Thousands of tonnes of payable copper produced by concentrates. Production data in concentrates are presented in terms of the amount of metal contained in the concentrate.

Sx-Ew: Thousands of tonnes of payable copper produced by electro winning.

Production costs:

Concentrates: Total production costs incurred in concentrate production in US$ dollars per tonne.

Sx-Ew: Total production costs incurred in SxEw production (cathode costs) in US$ dollars per tonne.

C1 cash: C1 cash cost represents the cash cost incurred at each processing stage, from mining through to recoverable metal delivered to market (total production costs) less net by-product credits (if any) US$ dollars per tonne.

Labor: Total labor cost in thousands of US$ in concentrates process.

Services: Total third party services paid per year in thousands of US$.

Energy: Total energy costs in thousands of US$.

Interest: Total interests paid in thousands of US$.

# of Workers: Number of workers per year.

Geological data:

Reserves: Ore reserves in millions of tonnes. It accounts for part of the mineral resource for which appropriate assessments have been carried out to demonstrate at a given date that extraction could be reasonably justified in terms of mining, economic, legal and environmental factors.

Grade: percentage of copper content in the ore body.

B   Appendix B. Euler equations

B.1   Euler equation for output

The first order condition of optimality with respect to output is:

$$\frac{\partial \pi_t}{\partial q_t} + \beta \ \mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial r_{t+1}} \frac{-1}{g} + \frac{\partial V_{t+1}}{\partial g_{t+1}} \left(-\delta_q^{(g)} \right) \right) = 0$$

where $\mathbb{E}_t(.)$ represents the expectation over all the exogenous innovations at period $t+1$ and conditional on the information at year t. Differentiating the Bellman equation with respect to the endogenous state variables r_t and g_t, we have that:

$$\frac{\partial V_t}{\partial r_t} = \frac{\partial \pi_t}{\partial r_t} + \beta \ \mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial r_{t+1}} \right)$$

$$\frac{\partial V_t}{\partial g_t} = \frac{\partial \pi_t}{\partial g_t} + \beta \ \mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial g_{t+1}} \right)$$

Combining the two equations,

$$\left[\frac{\partial V_t}{\partial r_t} - \frac{\partial \pi_t}{\partial r_t} \right] \left(\frac{-1}{g} \right) + \left[\frac{\partial V_t}{\partial g_t} - \frac{\partial \pi_t}{\partial g_t} \right] \left(-\delta_q^{(g)} \right) = \beta \ \mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial r_{t+1}} \frac{-1}{g} + \frac{\partial V_{t+1}}{\partial g_{t+1}} \left(-\delta_q^{(g)} \right) \right) = -\frac{\partial \pi_t}{\partial q_t}$$

Solving for the marginal values:

$$\left(\frac{\partial V_t}{\partial r_t} \frac{-1}{g} + \frac{\partial V_t}{\partial g_t} \left(-\delta_q^{(g)} \right) \right) = -\frac{\partial \pi_t}{\partial q_t} + \frac{\partial \pi_t}{\partial r_t} \left(\frac{-1}{g} \right) + \frac{\partial \pi_t}{\partial g_t} \left(-\delta_q^{(g)} \right)$$

Combining this expression and the first order condition of optimality above, we get the Euler equation:

$$\frac{\partial \pi_t}{\partial q_t} + \beta \ \mathbb{E}_t \left(-\frac{\partial \pi_{t+1}}{\partial q_{t+1}} - \frac{\partial \pi_{t+1}}{\partial r_{t+1}} \frac{1}{g} - \frac{\partial \pi_{t+1}}{\partial g_{t+1}} \delta_q^{(g)} \right) = 0$$

B.2   Euler equation for investment in capacity

[TO BE INCLUDED]

B.3 Euler equation for discrete choice

[TO BE INCLUDED]

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