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Yang-Baxter deformations of the $AdS_5 \times T^{1,1}$ superstring and their backgrounds

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Abstract: We consider three-parameter Yang-Baxter deformations of the $AdS_5 \times T^{1,1}$ superstring for abelian r-matrices which are solutions of the classical Yang-Baxter equation. We find the NSNS fields of two new backgrounds which are dual to the dipole deformed Klebanov-Witten gauge theory and to the nonrelativistic Klebanov-Witten gauge theory with Schrödinger symmetry.

Keywords: Gauge-gravity correspondence, Integrable Field Theories

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Contents

1 Introduction

The AdS/CFT correspondence conjectures that certain gauge theories have a dual description in terms of string theories. The first case of the AdS/CFT correspondence states that $\mathcal{N}=4$ supersymmetric Yang-Mills theory on a four-dimensional flat spacetime is dual to type IIB superstring theory propagating in $AdS_5 \times S^5$ [1]. One of the most important features of the AdS/CFT correspondence is its integrability which in the string theory side is associated to the existence of a Lax connection ensuring the existence of an infinite number of conserved charges. In the case of $AdS_5 \times S^5$ superstring, the theory is described by a σ-model on the supercoset $\frac{\mathrm{PSU}(2,2|4)}{\mathrm{SO}(1,4)\times\mathrm{SO}(5)}$ [2] and the $\mathbb{Z}_4$-grading of the $\mathfrak{psu}(2,2|4)$ superalgebra is an essential ingredient to get a Lax connection [3]. The same happens for the $AdS_4 \times \mathbb{CP}^3$ superstrings [4], partially described by the supercoset $\frac{\mathrm{UOSp}(2,2|6)}{\mathrm{SO}(1,3)\times\mathrm{U}(3)}$ [5, 6], which also has $\mathbb{Z}_4$-grading and is integrable [5].

Another way to get integrable theories is to start with an integrable model and then deformed it in such a way that integrability is preserved. This is accomplished by introducing r-matrices that satisfy the Yang-Baxter equation [7]. When applied to the $AdS_5 \times S^5$ case [8, 9] the superstring will propagate on what is called a η-deformed background which is not a solution of the standard type IIB supergravity equations [10, 11], leading to the proposal of generalized supergravities [12, 13].

Deformations based on r-matrices that satisfy the classical Yang-Baxter equation (CYBE) can also be considered [14]. When applied to superstrings in $AdS_5 \times S^5$ [15–19] they generate type IIB supergravity backgrounds like the Lunin-Maldacena-Frolov [20, 21], Hashimoto-Itzhaki-Maldacena-Russo [22, 23][1] and Schrödinger spacetimes [25–28] which

[1]This background was also obtained as the $\eta \to 0$ limit of the η-deformed $AdS_5 \times S^5$ background [10] after a rescaling [11, 24].

were previously obtained by TsT transformations [29].[2] In terms of TsT transformations, these backgrounds can be obtained by considering two-tori with directions either along the brane or transverse to it, or with a direction along the brane and the other transverse to it [35]. In the first case, the two-torus along the brane will be generated by momenta operators, which introduce noncommutativity in the dual field theory,[3] while the two-torus in the transverse space to the brane will be along the U(1) directions generated by the Cartan generators of the isometry group. These type of U(1) $\times$ U(1)-deformations are called β-deformations. Now, if the two-torus has directions one along the brane and the other transverse to it, taking a momentum and a Cartan generator, in the field theory side it leads to dipole field theories [40, 41]. Another possibility is to take a null direction along the brane and a U(1) direction transverse to it. This generates gravity duals of nonrelativistic field theories which have Schrödinger symmetry [26, 42–46]. The TsT procedure in this latter case is actually a TsssT transformation called Melvin twist [28, 47]. In general, having more than one U(1) direction in the transverse space allow us to construct the transverse two-torus in several ways so we get different deformations or a combinations of them. These Yang-Baxter deformations can also be applied to $AdS_4 \times \mathbb{CP}^3$ superstrings [48, 49] giving rise to gravity duals for the noncommutative, dipole and β-deformed ABJM theory [35]. It was also found an Yang-Baxter deformation that generates Schrödinger spacetimes which correspond to a family of gravity duals of nonrelativistic ABJM theory [49].

In this paper we will consider the duality between type IIB superstring theory in $AdS_5 \times T^{1,1}$ and $\mathcal{N} = 1$ SU$(N) \times$ SU(N) Yang-Mills theory in four dimensions, also known as Klebanov-Witten theory [50]. The internal $T^{1,1}$ manifold has SU(2) $\times$ SU(2) $\times$ U$(1)_R$ symmetry, instead of the SU$(4)_R$ symmetry of the $AdS_5 \times S^5$ case, leading to a less supersymmetric dual field theory. It has been argued that $AdS_5 \times T^{1,1}$ is non-integrable since some wrapping string configurations present chaotic behavior [51, 52].[4] Even so, an integrable Yang-Baxter deformation of $AdS_5 \times T^{1,1}$ was found [54] which agrees with the gravity dual of the β-deformed gauge theory obtained by TsT transformations [55]. In this paper we will discuss two other types of Yang-Baxter deformations generated by commuting r-matrices. One gives rise to a background dual to a three-parameter dipole deformed Klebanov-Witten gauge theory and the other is dual to a nonrelativistic Klebanov-Witten gauge theory on a Schrödinger spacetime.

This paper is organized as follows. In section 2 we build the coset for $AdS_5 \times T^{1,1}$ paying attention to the relevant subalgebras that will be used. In section 3 we discuss the new backgrounds obtained by deforming $AdS_5 \times T^{1,1}$. Finally, in section 4, we discuss our results and present future perspectives.

[2] It should be remarked that these deformations are generated by abelian r-matrices which, from the TsT side, involve commuting isometries. More general r-matrices, however, are associated to non-abelian T-dualities [30–34].

[3] It is argued in [36–39] that noncommutativity, in general, can be introduced by considering conformal twists, or in terms of Yang-Baxter deformations by taking generators of the conformal algebra.

[4] Notice, however, that a coset construction for $T^{1,1}$ manifolds based on affine Gaudin models was found to be integrable [53].

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2 Coset construction of $AdS_5 \times T^{1,1}$

The Klebanov-Witten gauge theory is obtained by putting N $D3$-branes on the singularity of $M_{1,4} \times Y_6$, where $M_{1,4}$ is the four-dimensional Minkowski space and Y_6 a Ricci flat Calabi-Yau cone $C(X_5)$ with base X_5 [50]. Near the horizon the geometry becomes $AdS_5 \times X_5$, where X_5 is a compact Sasaki-Einstein manifold, i.e., an odd-dimensional Riemannian manifold such that its cone $C(X_5)$ is a Calabi-Yau flat manifold [56]. Taking X_5 as $T^{1,1}$,[5] only 1/4 of the supersymmetries are preserved so that we have $\mathcal{N} = 1$ supersymmetry in four dimensions. The superpotential has a $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry, with U(1) being part of the R-symmetry that gives the $\mathcal{N} = 1$ supersymmetry, and $\mathrm{SU}(2) \times \mathrm{SU}(2)$ being a flavor symmetry which is not included in the $\mathcal{N} = 1$ superconformal group in four dimensions $\mathrm{PSU}(2,2|1)$ [58–60]. Thus, the full isometry group is $\mathrm{PSU}(2,2|1) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$. The bosonic part of the superalgebra $\mathfrak{g} = \mathfrak{psu}(2,2|1)$ on which we construct the σ-model is $\mathfrak{su}(2,4) \otimes \mathfrak{u}(1)$. The generators of $\mathfrak{psu}(2,2|1)$ can be written as supermatrices which are formed by blocks that correspond to bosonic (diagonal) and fermionic (anti-diagonal) generators,

$$M_{(4|1)\times(4|1)} = \left(\begin{array}{c|c} \mathfrak{so}(2,4) & \overline{Q} \\ \hline Q & \mathfrak{u}(1)_R \end{array}\right). \tag{2.1}$$

The isometry group of $AdS_5 \times T^{1,1}$ is given by the coset

$$AdS_5 \times T^{1,1} \equiv \frac{\mathrm{SO}(2,4)}{\mathrm{SO}(1,4)} \times \frac{\mathrm{SU}(2) \times \mathrm{SU}(2)}{\mathrm{U}(1)}, \tag{2.2}$$

which is not the bosonic part of any supercoset [61, 62]. Besides that, the coset for $T^{1,1}$ does not lead to the standard Sasaki-Einstein metric for $T^{1,1}$. This happens because neither the bosonic subalgebra $\mathfrak{su}(2) \otimes \mathfrak{u}(1)$ nor the isometry group (2.2) captures the full isometries of the theory. All this can be overcome by extending the coset (2.2) to [54]

$$AdS_5 \times T^{1,1} \equiv \frac{\mathrm{SO}(2,4)}{\mathrm{SO}(1,4)} \times \frac{\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R}{\mathrm{U}(1) \times \mathrm{U}(1)}, \tag{2.3}$$

where the $\mathrm{U}(1)_R$ now appears as part of the global symmetries and a second U(1) was added in order to preserve the number of parameters that describe the space. Thus, in terms of this extended $\mathbb{Z}_2$-graded algebra, the symmetric coset for $AdS_5 \times T^{1,1}$ is taken as

$$\mathfrak{so}(2,4)\oplus\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1) = \overbrace{(\mathfrak{so}(1,4)\oplus\mathfrak{u}(1)\oplus\mathfrak{u}(1))}^{\mathfrak{g}^{(0)}} \oplus \overbrace{\left(\frac{\mathfrak{so}(2,4)\oplus\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)_R}{\mathfrak{so}(1,4)\oplus\mathfrak{u}(1)\oplus\mathfrak{u}(1)}\right)}^{\mathfrak{g}^{(2)}=\mathfrak{g}/\mathfrak{g}^{(0)}}. \tag{2.4}$$

The supermatrix has the block structure

$$M_{(8|1)\times(8|1)} = \left(\begin{array}{c:c|c} \mathfrak{so}(2,4) & 0 & \overline{Q} \\ \hdashline 0 & \mathfrak{su}(2)\oplus\mathfrak{su}(2) & 0 \\ \hline Q & 0 & \mathfrak{u}(1)_R \end{array}\right), \tag{2.5}$$

[5] This space belongs to a general class of Einstein spaces called $T^{p,q}$ [57] described by the coset $\mathrm{SU}(2) \times \mathrm{SU}(2)/\mathrm{U}(1)$, where the U(1) is generated by $p\sigma_3^L + q\sigma_3^R$, where σ_i^L and σ_j^R are the generators of the left and right SU(2)'s, respectively.

where the dashed lines split the algebras corresponding to the subspaces AdS_5 and $T^{1,1}$, while the solid lines split the $M_{8\times 8}$ and $M_{1\times 1}$ bosonic blocks.

The basis of $\mathfrak{so}(2,4)\oplus\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$ that we will consider is composed of $\mathfrak{so}(2,4)$ generators denoted by Γ_μ, Γ_5, $\mathbf{M}_{\mu\nu}$ and $\mathbf{M}_{\mu 5}$, $\mu=0,1,2,3$, which, when written as supermatrices become

$$\Gamma_\mu=\left(\begin{array}{c:cc|c}\gamma_\mu & & & \\ \hdashline & 0 & & \\ & & 0 & \\ \hline & & & 0\end{array}\right),\quad \Gamma_5=\left(\begin{array}{c:cc|c}\gamma_5 & & & \\ \hdashline & 0 & & \\ & & 0 & \\ \hline & & & 0\end{array}\right),\quad \mathbf{M}_{\mu\nu}=\left(\begin{array}{c:cc|c}m_{\mu\nu} & & & \\ \hdashline & 0 & & \\ & & 0 & \\ \hline & & & 0\end{array}\right),\quad \mathbf{M}_{\mu 5}=\left(\begin{array}{c:cc|c}m_{\mu 5} & & & \\ \hdashline & 0 & & \\ & & 0 & \\ \hline & & & 0\end{array}\right), \tag{2.6}$$

and $\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$ generators denoted by $\mathbf{X}_a, \mathbf{Y}_a$, $a=1,2,3$ and $\mathbf{M}$, with supermatrices

$$\mathbf{X}_a=-\frac{i}{2}\left(\begin{array}{c:cc|c}0 & & & \\ \hdashline & \sigma_a & & \\ & & 0 & \\ \hline & & & 0\end{array}\right),\quad \mathbf{Y}_a=-\frac{i}{2}\left(\begin{array}{c:cc|c}0 & & & \\ \hdashline & 0 & & \\ & & \sigma_a & \\ \hline & & & 0\end{array}\right),\quad \mathbf{M}=-\frac{i}{2}\left(\begin{array}{c:cc|c}0 & & & \\ \hdashline & 0 & & \\ & & 0 & \\ \hline & & & 1\end{array}\right), \tag{2.7}$$

where γ_μ, γ_5, $m_{\mu\nu}$ and $m_{\mu 5}$ are the fifteen 4×4 matrices for the generators of isometries of AdS_5 (detailed in appendix A) and σ_a are the conventional 2×2 Pauli matrices of $\mathfrak{su}(2)$. The commutation rules and supertraces are then

$$[\mathbf{M}_{ij},\mathbf{M}_{k\ell}]=\eta_{i\ell}\mathbf{M}_{jk}+\eta_{jk}\mathbf{M}_{i\ell}-\eta_{ik}\mathbf{M}_{j\ell}-\eta_{j\ell}\mathbf{M}_{ik}, \tag{2.8}$$

where $i,j,k,\ell=0,1,\dots,5$, and

$$\begin{aligned} [\mathbf{X}_a,\mathbf{X}_b] &= \epsilon_{ab}{}^{c}\mathbf{X}_c, & [\mathbf{Y}_a,\mathbf{Y}_b] &= \epsilon_{ab}{}^{c}\mathbf{Y}_c,\\ \mathrm{Str}\,(\mathbf{X}_a\mathbf{X}_b) &= -\frac{1}{2}\delta_{ab}, & \mathrm{Str}\,(\mathbf{Y}_a\mathbf{Y}_b) &= -\frac{1}{2}\delta_{ab},\\ \mathrm{Str}\,(\mathbf{M}\mathbf{M}) &= \frac{1}{4}, \end{aligned} \tag{2.9}$$

with $\eta_{ij}=\mathrm{diag}\,(-,+,+,+,-,+,+,+,+,+)$.

The algebra for the global symmetry of the AdS_5 space is

$$\mathfrak{so}(2,4)=\mathfrak{so}(1,4)\oplus\frac{\mathfrak{so}(2,4)}{\mathfrak{so}(1,4)}, \tag{2.10}$$

with basis

$$\frac{\mathfrak{so}(2,4)}{\mathfrak{so}(1,4)}=\mathrm{span}\,\{\mathbf{K}_m\}\,,\qquad m=0,1,2,3,4, \tag{2.11}$$

where

$$\mathbf{K}_0=\frac{1}{2}\Gamma_0,\quad \mathbf{K}_1=\frac{1}{2}\Gamma_1,\quad \mathbf{K}_2=\frac{1}{2}\Gamma_2,\quad \mathbf{K}_3=\frac{1}{2}\Gamma_3,\quad \mathbf{K}_4=\frac{1}{2}\Gamma_5, \tag{2.12}$$

and

$$\mathrm{Str}\,(\mathbf{K}_m\mathbf{K}_n)=\eta_{mn},\qquad m,n=0,1,2,3,4. \tag{2.13}$$

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The $\mathfrak{so}(1,4)$ generators are $\{\mathbf{M}_{01}, \mathbf{M}_{02}, \mathbf{M}_{03}, \mathbf{M}_{12}, \mathbf{M}_{13}, \mathbf{M}_{23}, \mathbf{M}_{05}, \mathbf{M}_{15}, \mathbf{M}_{25}, \mathbf{M}_{35}\}$, and an appropriate coset representative for AdS_5 is

$$g_{AdS_5} = \exp\left(x^0\mathbf{p}_0 + x^1\mathbf{p}_1 + x^2\mathbf{p}_2 + x^3\mathbf{p}_3\right)\exp\left(\log z\mathbf{D}\right), \tag{2.14}$$

where

$$\mathbf{D} = \frac{1}{2}\Gamma_5, \qquad \mathbf{p}_\mu = \frac{1}{2}\Gamma_\mu + \mathbf{M}_{\mu 5}, \qquad \mu = 0,1,2,3. \tag{2.15}$$

The $T^{1,1}$ space can be written as the coset in

$$\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1) = \mathfrak{u}(1)\oplus\mathfrak{u}(1)\oplus\frac{\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)}{\mathfrak{u}(1)\oplus\mathfrak{u}(1)}. \tag{2.16}$$

with basis

$$\frac{\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)}{\mathfrak{u}(1)\oplus\mathfrak{u}(1)} = \text{span}\left\{\mathbf{K}_m\right\}, \qquad m = 5,\ldots,9, \tag{2.17}$$

where

$$\begin{aligned} &\mathbf{K}_5 = \sqrt{\frac{2}{3}}\mathbf{X}_1, \qquad \mathbf{K}_6 = \sqrt{\frac{2}{3}}\mathbf{X}_2, \qquad \mathbf{K}_7 = \sqrt{\frac{2}{3}}\mathbf{Y}_1, \\ &\mathbf{K}_8 = \sqrt{\frac{2}{3}}\mathbf{Y}_2, \qquad \mathbf{K}_9 = \frac{2}{3}\mathbf{H}, \end{aligned} \tag{2.18}$$

with

$$\mathbf{H} = \mathbf{X}_3 - \mathbf{Y}_3 + \mathbf{M}. \tag{2.19}$$

We also have

$$\text{Str}\left(\mathbf{K}_m\mathbf{K}_n\right) = -\frac{1}{3}\delta_{mn}, \qquad m,n = 5,\ldots,9. \tag{2.20}$$

The generators of $\mathfrak{u}(1)\oplus\mathfrak{u}(1)$ are $\{\mathbf{T}_1, \mathbf{T}_2\}$ with

$$\mathbf{T}_1 = \mathbf{X}_3 + \mathbf{Y}_3, \qquad \mathbf{T}_2 = \mathbf{X}_3 - \mathbf{Y}_3 + 4\mathbf{M}, \tag{2.21}$$

where T_1 generates the original U(1) in (2.2). An appropriate coset representative is then

$$g_{T^{1,1}} = \exp\left(\phi_1\mathbf{X}_1 + \phi_2\mathbf{X}_2 + 2\phi_3\mathbf{M}\right)\exp\left(\theta_1\mathbf{X}_2 + (\theta_2+\pi)\mathbf{Y}_2\right). \tag{2.22}$$

The coset representative that will allow use for $AdS_4\times T^{1,1}$ is then

$$g = g_{AdS_5}\times g_{T^{1,1}}. \tag{2.23}$$

The projector P_2 on $\mathfrak{g}^{(2)}$ can be defined as

$$P_2\left(X\right) = \sum_{m=0}^{4}\frac{\text{Str}\left(\mathbf{K}_m X\right)}{\text{Str}\left(\mathbf{K}_m\mathbf{K}_m\right)}\mathbf{K}_m - \frac{1}{3}\sum_{m=5}^{9}\frac{\text{Str}\left(\mathbf{K}_m X\right)}{\text{Str}\left(\mathbf{K}_m\mathbf{K}_m\right)}\mathbf{K}_m. \tag{2.24}$$

Applied to $A = g^{-1}dg$, the Maurer-Cartan one-form, we get

$$P_2\left(A\right) = E^m\mathbf{K}_m, \qquad m = 0,1,\ldots,9, \tag{2.25}$$

with

$$E^0 = \frac{dx^0}{z}, \qquad E^1 = \frac{dx^1}{z}, \qquad E^2 = \frac{dx^2}{z}, \qquad E^3 = \frac{dx^3}{z}, \qquad E^4 = \frac{dz}{z},$$
$$E^5 = \frac{1}{\sqrt{6}}\sin\theta_1 d\phi_1, \quad E^6 = -\frac{1}{\sqrt{6}}d\theta_1, \quad E^7 = -\frac{1}{\sqrt{6}}\sin\theta_2 d\phi_2, \quad E^8 = -\frac{1}{\sqrt{6}}d\theta_2,$$
$$E^9 = -\frac{1}{3}\left(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\phi_3\right). \tag{2.26}$$

Then, we can compute the $AdS_5 \times T^{1,1}$ metric from

$$\mathrm{Str}\left(A\, P_2\left(A\right)\right) = E^m \mathrm{Str}\left(A\mathbf{K}_m\right), \quad m = 0, \ldots, 9, \tag{2.27}$$

to get

$$ds^2 = ds^2_{AdS_5} + ds^2_{T^{1,1}}, \tag{2.28}$$

where

$$ds^2_{AdS_5} = \frac{1}{z^2}\left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2\right), \tag{2.29}$$

and

$$\begin{aligned} ds^2_{T^{1,1}} = &\frac{1}{6}\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \frac{1}{6}\left(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right) \\ &+ \frac{1}{9}\left(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\phi_3\right)^2, \end{aligned} \tag{2.30}$$

where (θ_1, ϕ_1) and (θ_2, ϕ_2) parametrize the two spheres of $T^{1,1}$ and $0 \leq \phi_3 \leq 2\pi$.

The metric (2.30) was first obtained in [63] and describes the basis of a six-dimensional cone. It can be understood as the intersection of a cone and a sphere in $\mathbb{C}^4$ such that its topology is $S^2 \times S^3$, and that the metric is a U(1) bundle over $S^2 \times S^2$. Besides that, $\mathrm{SO}(4) \cong \mathrm{SU}(2) \times \mathrm{SU}(2)$ acts transitively on $S^2 \times S^3$ and U(1) leaves each point of it fixed so that $T^{1,1}$ is described by the coset $(\mathrm{SU}(2) \times \mathrm{SU}(2))/\mathrm{U}(1)$.

3 Yang-Baxter deformed backgrounds

In this section we present some r-matrices satisfying the CYBE and build the corresponding deformed background identifying its gravity dual. As mentioned before, the background can be deformed partially by choosing generators on each subspace. The bosonic Yang-Baxter deformed action is [8]

$$S = -\frac{1}{2}\int d^2\sigma \left(\gamma^{\alpha\beta} - \varepsilon^{\alpha\beta}\right)\mathrm{Str}\left(A_\alpha P_2\left(J_\beta\right)\right), \tag{3.1}$$

where $A = g^{-1}dg \in \mathfrak{g}$, $\gamma^{\alpha\beta}$ is the worldsheet metric and $\varepsilon^{\alpha\beta}$ is the Levi-Civita symbol. P_2 was defined in (2.24) and the deformed current one-form is

$$J = \frac{1}{1 - 2\eta R_g \circ P_2}A, \tag{3.2}$$

where η is the deformation parameter. The dressed R operator R_g is defined as

$$R_g\left(M\right) = \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g\left(M\right) = g^{-1}R(gMg^{-1})g. \tag{3.3}$$

Moreover, we can compute $P_2(J)$ in (3.1) by defining the action of P_2 as

$$P_2(A) = E^m \mathbf{K}_m, \qquad P_2(J) = j^m \mathbf{K}_m. \tag{3.4}$$

The coefficients j^m can be calculated from

$$j^m = E^n C_n{}^m, \tag{3.5}$$

where the matrix components $C_m{}^n$ are those of

$$\mathbf{C} = (\mathbf{I} - 2\eta\mathbf{\Lambda})^{-1}. \tag{3.6}$$

The matrix $\mathbf{\Lambda}$ has components defined as

$$P_2(R_g(\mathbf{K}_m)) = \Lambda_m{}^n \mathbf{K}_n. \tag{3.7}$$

Then, from (3.1), we can read off the metric and the B-field as [49]

$$ds^2 = \mathrm{Str}(A\, P_2(J)) = j^m \mathrm{Str}(A\mathbf{K}_m) = E^m C_m{}^n \mathrm{Str}(A\mathbf{K}_n), \tag{3.8}$$
$$B = \mathrm{Str}(A \wedge P_2(J)) = -j^m \wedge \mathrm{Str}(A\mathbf{K}_m) = E^m C_m{}^n \wedge \mathrm{Str}(A\mathbf{K}_n). \tag{3.9}$$

The three-parameter β-deformed of $T^{1,1}$ was obtained in [54] by a Yang-Baxter deformation and in [55] by a TsT transformation in perfect agreement. In this case the r-matrix was

$$r = \mu_1 \mathbf{X}_3 \wedge \mathbf{M} + \mu_2 \mathbf{M} \wedge \mathbf{Y}_3 + \mu_3 \mathbf{X}_3 \wedge \mathbf{Y}_3. \tag{3.10}$$

In the following subsections we will introduce two more r-matrices and the corresponding deformations they produce.

3.1 Dipole deformed Klebanov-Witten theory

Let us first consider an Abelian r-matrix like

$$r = \mathbf{p}_2 \wedge (\mu_1 \mathbf{X}_3 + \mu_2 \mathbf{Y}_3 + \mu_3 \mathbf{M}), \tag{3.11}$$

where $\mathbf{X}_3$, $\mathbf{Y}_3$ and $\mathbf{M}$ are the Cartan generators of $\mathfrak{su}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$ and μ_i, $i = 1, 2, 3$, are the deformation parameters.[6] In this case (3.11) combines generators of both subspaces, which will lead to a deformation of the entire $AdS_5 \times T^{1,1}$ background. The nonzero components of $\Lambda_m{}^n$ in (3.7) are

$$\begin{aligned}
\Lambda_3{}^5 &= -\Lambda_5{}^3 = -\frac{1}{\sqrt{6}} \frac{\mu_1 \sin\theta_1}{z}, \\
\Lambda_3{}^7 &= -\Lambda_7{}^3 = \frac{1}{\sqrt{6}} \frac{\mu_2 \sin\theta_2}{z}, \\
\Lambda_3{}^9 &= -\Lambda_9{}^3 = \frac{1}{6} \frac{(\mu_3 - 2\mu_1 \cos\theta_1 - 2\mu_2 \cos\theta_2)}{z},
\end{aligned} \tag{3.12}$$

[6] The deformation parameter η can always be absorbed in the r-matrix such that it is present in the μ_i's.

while the nonzero elements of ${C_m}^n$, from (3.6), are

$$
\begin{aligned}
&{C_0}^0 = {C_1}^1 = {C_2}^2 = {C_4}^4 = {C_6}^6 = {C_8}^8 = 1, \\
&{C_3}^3 = \mathcal{M}, \\
&{C_5}^3 = -{C_3}^5 = \mathcal{M} f_1, \\
&{C_7}^3 = -{C_3}^7 = -\mathcal{M} f_2, \\
&{C_9}^3 = -{C_3}^9 = \mathcal{M} f_3, \\
&{C_5}^5 = \mathcal{M}\left(1 + f_2^2 + f_3^2\right), \\
&{C_7}^5 = {C_5}^7 = \mathcal{M} f_1 f_2, \\
&{C_9}^5 = {C_5}^9 = -\mathcal{M} f_1 f_3, \\
&{C_7}^7 = \mathcal{M}\left(1 + f_1^2 + f_3^2\right), \\
&{C_9}^7 = {C_7}^9 = \mathcal{M} f_2 f_3, \\
&{C_9}^9 = \mathcal{M}\left(1 + f_1^2 + f_2^2\right),
\end{aligned}
\tag{3.13}
$$

where

$$
\mathcal{M}^{-1} = 1 + f_1^2 + f_2^2 + f_3^2, \tag{3.14}
$$

with

$$
\begin{aligned}
f_1 &= \sqrt{\frac{2}{3}} \frac{\mu_1 \sin\theta_1}{z}, \quad f_2 = \sqrt{\frac{2}{3}} \frac{\mu_2 \sin\theta_2}{z}, \\
f_3 &= \frac{\mu_3 - 2\mu_1 \cos\theta_1 - 2\mu_2 \cos\theta_2}{3z}.
\end{aligned}
\tag{3.15}
$$

The deformed metric can be obtained from (3.8)

$$
\begin{aligned}
ds^2 = &\frac{1}{z^2}\left(-dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{M} dx_3^2 + dz^2\right) \\
&+ \frac{1}{6}\left(d\theta_1^2 + \mathcal{M}\left(1 + f_2^2 + f_3^2\right)\sin^2\theta_1 d\phi_1^2\right) + \frac{1}{6}\left(d\theta_2^2 + \mathcal{M}\left(1 + f_1^2 + f_3^2\right)\sin^2\theta_2 d\phi_2^2\right) \\
&+ \frac{\mathcal{M}}{9}\left(1 + f_1^2 + f_2^2\right)\left(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\phi_3\right)^2 \\
&+ \frac{\sqrt{6}\mathcal{M}}{9} f_3 \left(f_1 \sin\theta_1 d\phi_1 + f_2 \sin\theta_2 d\phi_2\right)\left(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\phi_3\right) \\
&- \frac{\mathcal{M}}{3} f_1 f_2 \sin\theta_1 \sin\theta_2 d\phi_1 d\phi_2,
\end{aligned}
\tag{3.16}
$$

and the B-field from (3.9),

$$
\begin{aligned}
B = &-\frac{\mathcal{M}}{3z}\left(2 f_3 \cos\theta_1 - \sqrt{6} f_1 \sin\theta_1\right) dx^3 \wedge d\phi_1 \\
&- \frac{\mathcal{M}}{3z}\left(2 f_3 \cos\theta_2 - \sqrt{6} f_2 \sin\theta_2\right) dx^3 \wedge d\phi_2 \\
&- \frac{2\mathcal{M}}{3z} f_3 dx^3 \wedge d\phi_3.
\end{aligned}
\tag{3.17}
$$

It is worth mentioning that the choice of generators in (3.11) is dictated by the place where we want put the two-tori from the TsT perspective. In the present case we have one coordinate in AdS_5 and a combination of the U(1)'s in $T^{1,1}$. The resulting metric (3.16) has deformations along the x^3-direction in AdS_5 and along the angles ϕ_1, ϕ_2 and ϕ_3 in $T^{1,1}$.

3.2 Nonrelativistic Klebanov-Witten theory

In order to construct this deformation we must write the AdS_5 space in light-cone coordinates. Thus, the coset representative is now

$$g_{AdS_5} = \exp\left(x_-\mathbf{p}_- + x_+\mathbf{p}_+ + x^1\mathbf{p}_1 + x^2\mathbf{p}_2\right)\exp\left(\log z\mathbf{D}\right), \tag{3.18}$$

with

$$\mathbf{p}_\pm = \frac{1}{\sqrt{2}}\left(\mathbf{p}_0 \pm \mathbf{p}_3\right), \qquad x_\pm = \frac{1}{\sqrt{2}}\left(x^0 \pm x^3\right), \tag{3.19}$$

while for the $T^{1,1}$ we keep the same form as in (2.22). The AdS_5 metric is then

$$ds^2 = \frac{1}{z^2}\left(-2dx_+dx_- + dx_1^2 + dx_2^2 + dz^2\right), \tag{3.20}$$

while the $T^{1,1}$ metric is given by (2.30).

Let us now consider the r-matrix (3.11) with $\mathbf{p}_2$ replaced by $\mathbf{p}_-$,[7]

$$r = \mathbf{p}_- \wedge \left(\mu_1\mathbf{X}_3 + \mu_2\mathbf{Y}_3 + \mu_3\mathbf{M}\right), \tag{3.21}$$

where $\mathbf{X}_3$, $\mathbf{Y}_3$ and $\mathbf{M}$ are Cartan generators of the algebra. Taking the same steps as in the previous case we find that the nonzero components of ${\Lambda_m}^n$ are

$$\begin{aligned}
{\Lambda_0}^5 &= {\Lambda_5}^0 = \frac{1}{2\sqrt{3}}\frac{\mu_1\sin\theta_1}{z}, \\
{\Lambda_0}^7 &= {\Lambda_7}^0 = -\frac{1}{2\sqrt{3}}\frac{\mu_2\sin\theta_2}{z}, \\
{\Lambda_0}^9 &= {\Lambda_9}^0 = \frac{1}{6\sqrt{2}}\frac{\mu_3 - 2\mu_1\cos\theta_1 - 2\mu_2\cos\theta_2}{z}, \\
{\Lambda_3}^5 &= -{\Lambda_5}^3 = \frac{1}{2\sqrt{3}}\frac{\mu_1\sin\theta_1}{z}, \\
{\Lambda_3}^7 &= -{\Lambda_7}^3 = -\frac{1}{2\sqrt{3}}\frac{\mu_2\sin\theta_2}{z}, \\
{\Lambda_3}^9 &= -{\Lambda_9}^3 = \frac{1}{6\sqrt{2}}\frac{\mu_3 - 2\mu_1\cos\theta_1 - 2\mu_2\cos\theta_2}{z},
\end{aligned} \tag{3.22}$$

[7]In this case we identify $x_- \sim x_- + 2\pi r^-$, such that $p^- = i\partial_{x_-}$ can be interpreted as the number operator $p^- = N/r^-$. Moreover, if we consider x_+ to be the time then p^+ is the energy [64].

while the nonzero elements of $C_m{}^n$ are now

$$
\begin{aligned}
&C_1{}^1 = C_2{}^2 = C_4{}^4 = C_5{}^5 = C_6{}^6 = C_7{}^7 = C_8{}^8 = C_9{}^9 = 1, \\
&C_0{}^0 = 1 + f_1^2 + f_2^2 + f_3^2, \\
&C_3{}^0 = -C_0{}^3 = -\left(f_1^2 + f_2^2 + f_3^2\right), \\
&C_5{}^0 = C_0{}^5 = f_1, \\
&C_7{}^0 = C_0{}^7 = -f_2, \\
&C_9{}^0 = C_0{}^9 = f_3, \\
&C_3{}^3 = 1 - \left(f_1^2 + f_2^2 + f_3^2\right), \\
&C_5{}^3 = -C_3{}^5 = -f_1, \\
&C_7{}^3 = -C_3{}^7 = f_2, \\
&C_9{}^3 = -C_3{}^9 = -f_3,
\end{aligned}
\tag{3.23}
$$

where

$$
\begin{aligned}
f_1 &= \frac{1}{\sqrt{3}} \frac{\mu_1 \sin\theta_1}{z}, \qquad f_2 = \frac{1}{\sqrt{3}} \frac{\mu_2 \sin\theta_2}{z}, \\
f_3 &= \frac{1}{3\sqrt{2}} \frac{\mu_3 - 2\mu_1 \cos\theta_1 - 2\mu_2 \cos\theta_2}{z}.
\end{aligned}
\tag{3.24}
$$

The deformed metric is then

$$
ds^2 = \frac{1}{z^2}\left(-2dx_+ dx_- + dx_1^2 + dx_2^2 + dz^2\right) - 2\mathcal{M}\frac{dx_+^2}{z^2} + ds^2_{T^{1,1}}, \tag{3.25}
$$

where now

$$
\mathcal{M} = f_1^2 + f_2^2 + f_3^2, \tag{3.26}
$$

while the deformed B-field is

$$
\begin{aligned}
B = &-\frac{2}{3z}\left(\sqrt{2} f_3 \cos\theta_1 + \sqrt{3} f_1 \sin\theta_1\right) dx_+ \wedge d\phi_1 \\
&-\frac{2}{3z}\left(\sqrt{2} f_3 \cos\theta_2 + \sqrt{3} f_2 \sin\theta_2\right) dx_+ \wedge d\phi_2 \\
&-\frac{2\sqrt{2}}{3z} f_3 dx_+ \wedge d\phi_3.
\end{aligned}
\tag{3.27}
$$

The first two terms in (3.25) is the metric of a Schrödinger spacetime.[8] The choice of generators in (3.21) is very similar to the one in (3.11). Now, however, the two-tori defined by the TsT transformation takes the x_- coordinate and a combination of the internal U(1)'s in $T^{1,1}$ and does not introduce any noncommutativity in the dual field theory. The metric (3.25) coincide with the $Sch_5 \times T^{1,1}$ obtained in [47] for $\mu_1 = n_1/2$, $\mu_2 = n_2/2$ and $\mu_3 = -n_3$, where n_i $(i = 1, 2, 3)$ are the deformation parameters.

[8]The Schrödinger symmetry is the maximal symmetry group of the free Schrödinger equation. It is the nonrelativistic version of the conformal algebra [42, 43]. This symmetry is realized geometrically as Schrödinger spacetimes.

The Schrödinger spacetime in (3.25) has dynamical exponent two [44, 45].[9] Schrödinger backgrounds with dynamical exponent z are argued to be integrable for $z = 1, 2, 3$, and non-integrable for $z = 4, 5, 6$ [65]. It has been argued that there are several nonrelativistic gravity duals with Schrödinger symmetry [64]. The number of $Sch_5 \times T^{1,1}$ spaces is equal to the degeneracy of a scalar harmonic function $\Phi_{(0)}^{(\ell_1,\ell_2,r)}$ on $T^{1,1}$, with quantum numbers (ℓ_1, ℓ_2, r), for which the Laplace-Beltrami equation is $-\nabla^2\Phi_{(0)}^{(\ell_1,\ell_2,r)} = \lambda_{(\ell_1,\ell_2,r)}\Phi_{(0)}^{(\ell_1,\ell_2,r)}$ with $\lambda_{(\ell_1,\ell_2,r)} = 6\left(\ell_1\left(\ell_1+1\right) + \ell_2\left(\ell_2+1\right) - r^2/8\right)$.[10] Since ℓ_1, ℓ_2 label standard spherical harmonics on the S^2's of $T^{1,1}$ the multiplicities of $\Phi_{(0)}^{(\ell_1,\ell_2,r)}$ are $(2|\ell_1|+1)$ and $(2|\ell_2|+1)$ [64]. Our background has[11] $\Phi = 2\mathcal{M}z^2$ and $-\nabla^2\Phi_{(0)}^{(\ell_1,\ell_2,r)} = 12\Phi_{(0)}^{(\ell_1,\ell_2,r)}$, so that (ℓ_1, ℓ_2, r) takes two values, $(1, 0, 0)$ and $(0, 1, 0)$. Then the total degeneracy of $\Phi_{(0)}$ is six so that we have a family of six $Sch_5 \times T^{1,1}$ spacetimes [64]. This kind of spacetimes was recently studied in [67, 68].

4 Conclusions

In this paper we have derived the metric and the B-field for the gravity duals of the dipole-deformed and the nonrelativistic Klebanov-Witten theory as Yang-Baxter deformations. We made use of an extended coset description of $AdS_5 \times T^{1,1}$ which simplified the computation of the undeformed background and its deformation. We considered two abelian r-matrices with three-parameter satisfying the classical Yang-Baxter equation. The first r-matrix was composed by a momentum generator in AdS and a combination of the three U(1)'s generators of the internal space which lead to the gravity dual of the dipole-deformed Klebanov-Witten theory which should be obtained by TsT transformation of the $AdS_5 \times T^{1,1}$ background. In second case we have also a momentum operator in AdS and a combination of the three U(1) generators in $T^{1,1}$. It produced the $Sch_5 \times T^{1,1}$ background which, having Schrödinger symmetry, corresponds to the nonrelativistic Klebanov-Witten theory [45].

The next step is to compute the RR fields of the deformed backgrounds. To get them we have to consider the fermionic sector as in [19]. The fact that we have not included the fermionic sector of the supercoset does not mean that we are unable to check the supergravity equations for the new backgrounds. Since the r-matrices that we used in the bosonic background are abelian they satisfy trivially the unimodularity condition, which is a sufficient for the background to satisfy the supergravity equations [12, 13, 33].

Another interesting case which deserves further study is the dual of the dipole deformation of $\mathcal{N} = 1$ SU$(N) \times$ SU(N) Yang-Mills theory as well as its nonrelativistic limits.

[9] The dynamical z factor is the exponent in the power of the radial direction in the $z^{-2z}dx_+^2$ term. To have Schrödinger symmetry we must have $z = 2$. The relativistic symmetry corresponds to $z = 1$.

[10] The harmonic function is denoted in general as $\Phi_{(q)}^{(\ell_1,\ell_2,r)}$, where ℓ_1, ℓ_2 are labels for the SU(2)'s, and r and q are U(1) charges [58, 66].

[11] In [47], the harmonic function Φ is defined as the non-negative length square of the Killing vector $\mathcal{K}$ on $T^{1,1}$, $\Phi = \|\mathcal{K}\|^2 = g_{ij}\mathcal{K}^i\mathcal{K}^j$ with $i, j = 1, 2, 3$, where $\mathcal{K} = (\mu_1\partial_{\phi_1}, \mu_2\partial_{\phi_2}, \mu_3\partial_{\phi_3})$.

A A basis for $\mathfrak{so}(2,4)$ algebra

Let us choose the following representation for γ_μ

$$\gamma_0 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix},$$
$$\gamma_2 = \begin{pmatrix} 0 & i\sigma_1 \\ -i\sigma_1 & 0 \end{pmatrix}, \qquad \gamma_3 = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}, \tag{A.1}$$

and

$$\gamma_5 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}. \tag{A.2}$$

We can also define

$$m_{\mu\nu} = \frac{1}{4}\left[\gamma_\mu, \gamma_\nu\right], \quad m_{\mu 5} = \frac{1}{4}\left[\gamma_\mu, \gamma_5\right], \tag{A.3}$$

and

$$\begin{aligned} p_\mu &= \frac{1}{2}\gamma_\mu - m_{\mu 5}, \\ k_\nu &= \frac{1}{2}\gamma_\mu + m_{\mu 5}, \\ D &= \frac{1}{2}\gamma_5. \end{aligned} \tag{A.4}$$

The conformal algebra $\mathrm{SO}(2,4)$ is then

$$\begin{aligned} [m_{\mu\nu}, m_{\rho\sigma}] &= \eta_{\mu\sigma} m_{\nu\rho} + \eta_{\nu\rho} m_{\mu\sigma} - \eta_{\mu\rho} m_{\nu\sigma} - \eta_{\nu\sigma} m_{\mu\rho}, \\ [m_{\mu\nu}, D] &= 0, \\ [D, p_\mu] &= p_\mu, \\ [D, k_\mu] &= -k_\mu, \\ [k_\mu, p_\nu] &= 2\eta_{\mu\nu} D + 2m_{\mu\nu}, \\ [m_{\mu\nu}, p_\rho] &= -\eta_{\mu\rho} P_\nu + \eta_{\nu\rho} P_\mu, \\ [m_{\mu\nu}, k_\rho] &= -\eta_{\mu\rho} k_\nu + \eta_{\nu\rho} k_\mu. \end{aligned} \tag{A.5}$$

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